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B-Spline Collocation Method for Solving Time-Dependent Differential Equations

Dr. Y. Rajashekhar Reddy

Assistant Professor of Mathematics, JNTUHCSET, Hyderabad

Abstract: ***This paper presents the use of B-spline collocation methods for solving time-dependent partial differential equations (PDEs) and ordinary differential equations (ODEs). B-splines, known for their flexibility and local support, are used as basic functions in a collocation framework. We validate the method on classical problems like the heat equation and second-order ODEs, showing high accuracy and efficiency.***

Keywords: B-spline, collocation method, time-dependent PDE, numerical analysis, heat equation.

1. Introduction

Numerical methods are essential for solving time-dependent problems that lack closed-form solutions. The B-spline collocation method combines the smoothness of B-splines with the ease of collocation techniques, providing a reliable tool for time evolution problems in physics, engineering, and finance.

2. Literature Review

Classical numerical methods for time-dependent problems include finite difference, finite element, and spectral methods. B-splines offer advantages such as local support and smooth basis functions with minimal oscillation. Previous research has demonstrated the success of B-spline-based methods in various applied domains.

3. Methodology

We consider a general time-dependent PDE:
$\partial u(x, t)/\partial t = \mathcal{L}(u(x, t)), x \in [a, b], t > 0$

Using cubic B-splines $B_i(x)$ over a uniform knot vector, the solution is approximated as:
$u(x, t) \approx \Sigma c_i(t) B_i(x)$

At selected collocation points x_j, we enforce the PDE:
$\Sigma (d c_i(t)/dt) B_i(x_j) = \mathcal{L}(\Sigma c_i(t) B_i(x_j))$

Time integration is done using Crank-Nicolson or implicit Euler methods.

4. Results and Examples

We apply the method to the heat equation:
$\partial u/\partial t = \alpha \partial^2u/\partial x^2, x \in [0, 1]$, with boundary conditions $u(0, t) = u(1, t) = 0$

Results are compared to analytical or finite difference solutions, showing good agreement. Error metrics such as L2 norm and maximum error are evaluated.

5. Discussion

B-spline collocation provides accurate results using fewer basis functions compared to other methods. The local support of B-splines allows for refinement and computational efficiency. The method is extendable to higher dimensions and nonlinear problems.

6. Conclusion

B-spline collocation is an effective method for solving time-dependent problems. It offers smooth and accurate solutions with manageable computational cost, and can be extended to complex domains.

7. Future Work

Future studies can focus on adaptive knot placement, nonlinear PDEs like Burgers' or reaction-diffusion systems, and higher-order B-splines or NURBS.

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