International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

On Topological Polynomials of Silicate Network

N. K. Raut

Ex.Head, Department of Physics, Sunderrao Solanke Mahavidyalaya Majalgaon Dist: Beed, (M.S.) India
Email: *rautnk87[at]gmail.com*

Abstract: ***The first and second Zagreb polynomials are defined as [1]:M1(G,x) $= \sum_{uv\in E(G)} x^{d_u+d_v}$ and M2(G,x) $= \sum_{uv\in E(G)} x^{d_u d_v}$, where d_u is degree of a vertex u. In this paper, Zagreb polynomials, fifth M-Zagreb polynomials, hyper fifth M-Zagreb polynomials, first, second K-Banhatti polynomials and general Zagreb polynomials of silicate network are studied.***

Keywords: Degree, degree-sum, fifth M-Zagreb polynomial, hyper fifth M-Zagreb polynomial, K-Banhatti polynomial, Zagreb polynomial

1. Introduction

Let G = (V,E) be a graph with vertex set V(G) and edge set E(G). The degree of a vertex u∈V(G) is denoted by d_u and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv [2].A topological index is a numerical parameter mathematically derived from the graph structure. Many topological polynomials appear in the molecular topology [3-4]. Hosoya, Schultz, Modified Schultz polynomials and their indices of PAHs were studied in [5]. The Zagreb polynomial was studied for different molecular graphs in [6-12].M-Zagreb polynomials and corresponding degree-based topological indices have been considered in many papers such as [13-16]. Augmented Revan index and its polynomial of certain families of benzenoid systems, hyper-Revan indices and their polynomials of silicate networks and arithmetic–geometric reverse indices of certain networks were studied by V. R. Kulli [17-19].Degree-based topological indices and topological polynomials of silicate network have been computed in many papers [20-24].

The degree-based Zagreb polynomials are defined as [25-26]

$$M_1(G,x) = \sum_{uv\in E(G)} x^{d_u+d_v}. \quad (1)$$
$$M_2(G,x) = \sum_{uv\in E(G)} x^{d_u\times d_v}. \quad (2)$$
$$M_3(G,x) = \sum_{uv\in E(G)} x^{|d_u-d_v|}. \quad (3)$$
$$M_4(G,x) = \sum_{uv\in E(G)} x^{d_u(d_u+d_v)}. \quad (4)$$
$$M_5(G,x) = \sum_{uv\in E(G)} x^{d_v(d_u+d_v)}. \quad (5)$$

In addition, two polynomials related to first Zagreb index are defined as

$$M_1^*(G,x) = \sum_{v\in V(G)} d_v x^{d_v} \text{ and } M_0(G,x) = \sum_{v\in V(G)} x^{d_v} \quad (6)$$

The fifth M-Zagreb polynomials are defined as [27-29]

$$M_1G_5(G,x) = \sum_{uv\in E(G)} x^{S_u+S_v}. \quad (7)$$
$$M_2G_5(G,x) = \sum_{uv\in E(G)} x^{S_u\times S_v}. \quad (8)$$

The fifth hyper M-Zagreb polynomials are

$$HM_1G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u+S_v)^2}. \quad (9)$$
$$HM_2G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u\times S_v)^2}. \quad (10)$$

And general fifth M-Zagreb polynomials are defined as

$$M_1^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u+S_v)^{\alpha}}. \quad (11)$$
$$M_2^{\alpha}G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u\times S_v)^{\alpha}}, \quad (12)$$

where α is a real number and $S_u = \sum_{v\in N_u} d_v$ with $N_u = \{u \in V(G)|uv \in E(G)\}$. We use the following lemma for defining $d_G(e)$.

Lemma 1. Let G be a graph with u,v∈V(G) and e = uv∈E(G) then $d_G(e) = d_e = d_u + d_v - 2$.

The first and second K-Banhatti polynomials are defined as [30-31]

$$KB_1(G,x) = \sum_{ue\in E(G)} x^{(d_u+d_e)}. \quad (13)$$
$$KB_2(G,x) = \sum_{ue\in E(G)} x^{(d_u\times d_e)}. \quad (14)$$

General Zagreb polynomial is defined as

$$M_{a,b}(G,x) = \sum_{uv\in E(G)} x^{(ad_u+bd_v)}. \quad (15)$$

And modified general Zagreb polynomial is

$$M'_{a,b}(G,x) = \sum_{uv\in E(G)} x^{(d_u+a)(d_v+b)}, \quad (16)$$

where a and b are suitably chosen real number parameters.

The complement of $\overline{G}$ of a graph G is a graph whose vertex set is V(G) and two vertices of $\overline{G}$ are adjacent if and only if they are nonadjacent in G [32-36]. Therefore $\overline{G}$ has n vertices and $\binom{n}{2}$ - m edges. The degree of a vertex v in $\overline{G}$ is $d_{\overline{G}}(v) = n - 1 - d_G(v)$.

The first, second and third Zagreb polynomial of complement graph $(\overline{G})$ of G are defined as

$$M_1(\overline{G},x) = \sum_{uv\in E(G)} x^{d_{\overline{G}}(u)+d_{\overline{G}}(v)}. \quad (17)$$
$$M_2(\overline{G},x) = \sum_{uv\in E(G)} x^{d_{\overline{G}}(u)\times d_{\overline{G}}(v)}. \quad (18)$$
$$M_3(\overline{G},x) = \sum_{uv\in E(G)} x^{|d_{\overline{G}}(u)-d_{\overline{G}}(v)|}. \quad (19)$$

In this paper, Zagreb-polynomials, fifth M-Zagreb polynomials, hyper fifth M-Zagreb polynomials, first, second K-Banhatti polynomials and Zagreb polynomials of complement graph $(\overline{G})$ of silicate network are studied. Our notations are standard and mainly taken from standard books of topology [37-39].

2. Materials and methods

A molecular graph is constructed by representing each atom of a molecule by a vertex and bonds between atoms by edges. Graph polynomials are polynomials assigned to molecular graphs. The molecular graph of silicate network of dimension two is shown figure (1). Let G be the graph of silicate network. It is observed from figure that there are $15n^2+3n$ vertices and $36n^2$ edges and the vertices either of degree 3 or 6. The (d_u,d_v) partition of silicate network is represented in table (1) and sum-degree partition in table (2). To compute K-Banhatti polynomials lemma number (1) is used.

3. Results and discussion

Zagreb polynomials of silicate network

It is observed from figure (1) that there are three edges corresponding to vertices with degree 3 and 6.The degree-based Zagreb polynomials can be computed as follows.

Theorem 1.1: First Zagreb polynomial of silicate network is $6nx^6+(18n^2+6n)x^9+(18n^2-12n)x^{12}$.

Proof. By using table (1) and figure (1), we have

$M_1(G,x) = \sum_{uv\in E(G)} x^{d_u+d_v}$
$= \sum_{3,3\in E(G)} x^{3+3} + \sum_{3,6\in E(G)} x^{3+6} + \sum_{6,6\in E(G)} x^{6+6}$
$= 6nx^6+(18n^2+6n)x^9+(18n^2-12n)x^{12}$.

Theorem 1.2. Second Zagreb polynomial of silicate network is $6nx^9+(18n^2+6n)x^{18}+(18n^2-12n)x^{36}$.

Proof: By using table (1) and figure (1), we have

$M_2(G,x) = \sum_{uv\in E(G)} x^{d_u\times d_v}$
$= \sum_{3,3\in E(G)} x^{3\times 3} + \sum_{3,6\in E(G)} x^{3\times 6} + \sum_{6,6\in E(G)} x^{6\times 6}$
$= 6nx^9+(18n^2+6n)x^{18}+(18n^2-12n)x^{36}$.

Theorem 1.3: Addional first Zagreb polynomial of silicate network is $(18n^2+18n)x^3+(54n^2-18n)x^6$.

Proof. Silicate network with $15n^2+3n$ vertices and $36n^2$ edges, has vertices either of degree 3 or 6.By using equation (6) and figure (1), we have

$M_1^*(G,x) = \sum_{v\in V(G)} d_v x^{d_v} = \sum_{3\in V(G)} 3\times x^3 + \sum_{6\in V(G)} 6\times x^6$
$= (18n^2+18n)x^3+(54n^2-18n)x^6$.

Fifth M-Zagreb polynomials of silicate network

Theorem 2.1: Fifth M_1-Zagreb polynomial of silicate network is $6nx^{30} + 24x^{39} + 24(n-1)x^{42} + 12(n-1)x^{45} + (18n^2-30n+12)x^{48} + 12x^{51} + 6(2n-3)x^{54} + 12(n-1)x^{57} + (18n^2-36n+18)x^{60}$.

Proof: By using equation (7) and table (2), we have

$M_1G_5(G,x) = \sum_{uv\in E(G)} x^{S_u+S_v}$
$= \sum_{15,15\in E(G)} x^{15+15} + \sum_{15,24\in E(G)} x^{15+24} + \sum_{15,27\in E(G)} x^{15+27} + \sum_{18,27\in E(G)} x^{18+27} + \sum_{18,30\in E(G)} x^{18+30} + \sum_{24,27\in E(G)} x^{24+27} + \sum_{27,27\in E(G)} x^{27+27} + \sum_{27,30\in E(G)} x^{27+30} + \sum_{30,30\in E(G)} x^{30+30}$
$=6nx^{30} + 24x^{39} + 24(n-1)x^{42} + 12(n-1)x^{45} + (18n^2-30n+12)x^{48} + 12x^{51} + 6(2n-3)x^{54} + 12(n-1)x^{57} + (18n^2-36n+18)x^{60}$.

Theorem 2.2: The fifth hyper M_1-Zagreb polynomial of silicate network is $6nx^{900} + 24x^{1521} + 24(n-1)x^{1764} + 12(n-1)x^{2025} + (18n^2-30n+12)x^{2304} + 12x^{2601} + 6(2n-3)x^{2916} + 12(n-1)x^{3249} + (18n^2-36n+18)x^{3600}$.

Proof: By using equation (9) and table (2), we have

$HM_1G_5(G,x) = \sum_{uv\in E(G)} x^{(S_u+S_v)^2}$
$= \sum_{15,15\in E(G)} x^{(15+15)^2} + \sum_{15,24\in E(G)} x^{(15+24)^2} + \sum_{15,27\in E(G)} x^{(15+27)^2} + \sum_{18,27\in E(G)} x^{(18+27)^2} + \sum_{18,30\in E(G)} x^{(18+30)^2} + \sum_{24,27\in E(G)} x^{(24+27)^2} + \sum_{27,27\in E(G)} x^{(27+27)^2} + \sum_{27,30\in E(G)} x^{(27+30)^2} + \sum_{30,30\in E(G)} x^{(30+30)^2}$
$=6nx^{900} + 24x^{1521} + 24(n-1)x^{1764} + 12(n-1)x^{2025} + (18n^2-30n+12)x^{2304} + 12x^{2601} + 6(2n-3)x^{2916} + 12(n-1)x^{3249} + (18n^2-36n+18)x^{3600}$.

K-Banhatti polynomials of silicate network

Theorem 3.1: First K-Banhatti polynomial of silicate network is $6nx^{14}+ (18n^2+6n)x^{23}+(18n^2-12n)x^{32}$.

Proof: By equation (13) and using table (3), we have first K-Banhatti polynomial

$KB_1(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_e)}$
$= \sum_{3,3\in E(G)} x^{(3+4)+(3+4)} + \sum_{3,6\in E(G)} x^{(3+7)+(6+7)} + \sum_{6,6\in E(G)} x^{(6+10)+(6+10)}$
$= 6nx^{14}+ (18n^2+6n)x^{23}+(18n^2-12n)x^{32}$.

Theorem 3.2: Second K-Banhatti polynomial of silicate network is $6nx^{24}+ (18n^2+6n)x^{63}+(18n^2-12n)x^{120}$.

Proof: By equation (14) and using table (3), we have second K-Banhatti polynomial

$KB_2(G,x) = \sum_{uv\in E(G)} x^{(d_u\times d_e)}$
$= \sum_{3,3\in E(G)} x^{(3\times 4)+(3\times 4)} + \sum_{3,6\in E(G)} x^{(3\times 7)+(6\times 7)} + \sum_{6,6\in E(G)} x^{(6\times 10)+(6\times 10)}$
$= 6nx^{24}+ (18n^2+6n)x^{63}+(18n^2-12n)x^{120}$.

General Zagreb polynomials of silicate network

Theorem 4.1: General Zagreb polynomial of silicate network is $6nx^{3a+3b} + (18n^2+6n)x^{3a+6b} + (18n^2-12n)x^{6a+6b}$.

Proof. By using equation (15) and table (1), we have

$M_{a,b}(G,x) = \sum_{uv\in E(G)} x^{(ad_u+bd_v)}$
$= \sum_{3,3\in E(G)} x^{a3+b3} + \sum_{3,6\in E(G)} x^{a3+b6} + \sum_{6,6\in E(G)} x^{a6+b6}$
$= 6nx^{3a+3b} + (18n^2+6n)x^{3a+6b} + (18n^2-12n)x^{6a+6b}$.

Theorem 4.2: Modified general Zagreb polynomial of silicate network is $6nx^{(3+a)(3+b)} + (18n^2+6n)x^{(3+a)(6+b)} + (18n^2-12n)x^{(6+a)(6+b)}$.

Proof. By using equation (16) and table (1), we have

$M'_{a,b}(G,x) = \sum_{uv\in E(G)} x^{(d_u+a)(d_v+b)}$

$= \sum_{3,3\in E(G)} x^{(3+a)(3+b)} + \sum_{3,6\in E(G)} x^{(3+a)(6+b)} + \sum_{6,6\in E(G)} x^{(6+a)(6+b)}$
$= 6nx^{(3+a)(3+b)} + (18n^2 + 6n)x^{(3+a)(6+b)} + (18n^2 - 12n)x^{(6+a)(6+b)}.$

Zagreb polynomial of complement graph $(\overline{G})$

Theorem 5.1. First Zagreb polynomial of complement graph $(\overline{G})$ of silicate network is
$6nx^{(2n-8)} + (18n^2 + 6n)x^{(2n-11)} + (18n^2 - 12n)x^{(2n-14)}.$

Proof. By using equation (17) and table (1), we have
$M_1(\overline{G},x) = \sum_{uv\in E(G)} x^{d_{\overline{G},}(u)+d_{\overline{G},}(v)}$
$= \sum_{3,3\in E(G)} x^{(n-1-3).+(n-1-3)} + \sum_{3,6\in E(G)} x^{(n-1-3)+(n-1-6)} + \sum_{6,6\in E(G)} x^{(n-1-6)+(n-1-6)}$
$= 6nx^{(2n-8)} + (18n^2 + 6n)x^{(2n-11)}$
$+ (18n^2 - 12n)x^{(2n-14)}.$

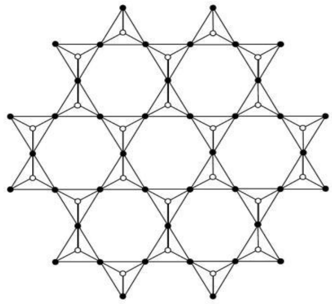

Figure 1: Silicate network of dimension two.

Table 1: (d_u, d_v) partition of silicate network

(d_u, d_v)	(3,3)	(3,6)	(6,6)
Number of edges	6n	6n(3n+1)	6n(3n-2)

Table 2: (S_u, S_v) partition of silicate network.

(S_u, S_v)	(15,15)	(15,24)	(15,27)	(18,27)	(18,30)	(24,27)	(27,27)	(27,30)	(30,30)
Number of edges	6n	24	24(n-1)	12(n-1)	$18n^2$-30n+12	12	6(2n-3)	12(n-1)	$18n^2$-36n+18

Table 3: Edge partition of silicate network for K-Banhatti polynomial.

(d_u,d_v)	(3,3)	(3,6)	(6,6)
$d_G(e)$	4	7	10
Number of edges	6n	$18n^2$+6n	$18n^2$-12n

Table 4: $M_3(G,x)$, $M_4(G,x)$, $M_5(G,x)$, $M_0(G,x)$, $M_2G_5(G,x)$, $HM_2G_5(G,x)$, $M_1^{\alpha}G_5(G,x)$, $M_2^{\alpha}G_5(G,x)$, $M_2(\overline{G},x)$ and $M_3(\overline{G},x)$ polynomials of silicate network

Topological polynomial	Polynomial for silicate network
$M_3(G,x)$	$(18n^2-6n)+(18n^2+6n)x^3$
$M_4(G,x)$	$6nx^{18}+(18n^2+6n)x^{27}+(18n^2-12n)x^{72}$
$M_5(G,x)$	$6nx^{18}+(18n^2+6n)x^{54}+(18n^2-12n)x^{72}$
$M_0(G,x)$	$(6n^2+6n)x^3+(9n^2-3n)x^6$
$M_2G_5(G,x)$	$6nx^{225} + 24x^{360} + 24(n-1)x^{405} + 12(n-1)x^{486} + (18n^2 - 30n + 12)x^{540} + 12x^{648} + 6(2n-3)x^{729} + 12(n-1)x^{810} + (18n^2 - 36n + 18)x^{900}$
$HM_2G_5(G,x)$	$6nx^{50625} + 24x^{129600} + 24(n-1)x^{164025} + 12(n-1)x^{236196} + (18n^2 - 30n + 12)x^{291600} + 12x^{419904} + 6(2n-3)x^{531441} + 12(n-1)x^{656100} + (18n^2 - 36n + 18)x^{810000}$
$M_1^{\alpha}G_5(G,x)$	$6nx^{(30)^{\alpha}} + 24x^{(39)^{\alpha}} + 24(n-1)x^{(42)^{\alpha}} + 12(n-1)x^{(45)^{\alpha}} + (18n^2 - 30n + 12)x^{(48)^{\alpha}} + 12x^{(51)^{\alpha}} + 6(2n-3)x^{(54)^{\alpha}} + 12(n-1)x^{(57)^{\alpha}} + (18n^2 - 36n + 18)x^{(60)^{\alpha}}$
$M_2^{\alpha}G_5(G,x)$	$6nx^{(225)^{\alpha}} + 24x^{(360)^{\alpha}} + 24(n-1)x^{(405)^{\alpha}} + 12(n-1)x^{(486)^{\alpha}} + (18n^2 - 30n + 12)x^{(540)^{\alpha}} + 12x^{(648)^{\alpha}} + 6(2n-3)x^{(729)^{\alpha}} + 12(n-1)x^{(810)^{\alpha}} + (18n^2 - 36n + 18)x^{(900)^{\alpha}}$
$M_2(\overline{G},x)$	$6nx^{(n-4)^2} + (18n^2 + 6n)x^{(n^2-11n+28)} + (18n^2 - 12n)x^{(n-7)^2}$
$M_3(\overline{G},x)$	$(18n^2 - 6n) + (18n^2 + 6n)x^3$

4. Conclusion

The degree-based Zagreb polynomials, fifth M-Zagreb polynomials, K-Banhatti polynomials, general Zagreb polynomials, Zagreb polynomials of complement graph $(\overline{G})$ and additional two polynomials related to first Zagreb index are obtained for silicate network. Some topological polynomials: $M_3(G,x)$, $M_4(G,x)$, $M_5(G,x)$, $M_0(G,x)$, $M_2G_5(G,x)$, $HM_2G_5(G,x)$, $M_1^{\alpha}G_5(G,x)$, $M_2^{\alpha}G_5(G,x)$, $M_2(\overline{G},x)$ and $M_3(\overline{G},x)$ are computed. Third Zagreb polynomial of molecular graph (G) and complement graph $(\overline{G})$ have the same value for silicate network.

References

[1] G. H. Fath-Tabar, Zagreb polynomial and Pi indices of some nanostructures, Digest Journal of Nanomaterials and Biostructures, 4(1 (2009 189-191.

[2] M. R. R. Kanna, S. Roopa and H. L. Parshivamurthy, Topological indices of Vitamin D_3, International Journal of Engineering and Technology, 7(4 (2018 6276-6284.

[3] A. R. Bindusree, I. N. Cangul, V. Lokesha and A. S. Cevic, Zagreb polynomials of three operators, Filomat, 30(7 (2016 1979-1986.

[4] M. Ajmal, W. Nazeer, M. Munir, S. M. Kang and Y. C. Kwun, Some Algebraic Polynomials and Topological Indices of Generalized Prism and Toroidal

Polyhex Networks, Symmetry, 9,5; (2017 doi:10. 3390/sym9010005.
[5] M. R. Farahani, Hosoya, Schultz, Modified Schultz polynomials and their indices of PAHs, International Journal of Theoretical Chemistry, 1(2 (2013 09-16.
[6] M. R. Farahani, Zagreb indices and Zagreb polynomials of polycyclic aromatic hydrocarbons, Journal of Chemica Acta, 2(2013 70-72.
[7] M. K. Siddiqi, M. Imran and A. Ahmad, On Zagreb indices, Zagreb polynomials of some nanostar dendrimers, Appl. Math. Comput., 280(2016 132–136.
[8] Y. C. Kwun, M. A Zahid, W. Nazeer, A. Ali, M. Ahmad and S. M. Kang, On the Zagreb polynomials of benzenoid systems, De Gruyter, Open Phys., 16(2018 734-740.
[9] N. K. Raut, The Zagreb group indices and polynomials, International Journal of Modern Engineering Research, 6(10 (2016 84-87.
[10] S. M. Kang, M. Yousaf, M. A. Zahid,M. Younas and W. Nazeer, Zagreb polynomials and redefined Zagreb indices of nanostar dendrimers, De Gruyter, Open Physics, 17(2019 31-40.
[11] Z. Shao, M. K. Siddiqui and M. H. Muhammad, Computing Zagreb indices and Zagreb polynomials for symmetrical nanotubes, Symmetry, 10 (7 (2018 244.
[12] B. Basavanagoud, E. Chitra, Zagreb polynomials of graph operations, International Journal of Applied Engineering Research, 15(3 (2020 287-293.
[13] C. P. Li, C. Zhonglin, M. Munir,K. Yasmin,and J. B. Liu, M-Zagreb polynomials and topological indices of linear chains of benzene, naphthalene and anthracene, Mathematical Biosciences and Engineering, 17(3 (2020 2384-2398.
[14] M. Munir, W. Nazeer, S. Rafique and S. M. Kang, M-Zagreb polynomials and degree-based topological indices of polyhex nanotubes, Symmetry, MDPI, 8,149, (2016 01-08.
[15] E. Deutsch, O. S. Klavzar, M- polynomials and degree-based topological indices, arXiv, 1407. 1592v1 [math. CO], (2014 01-10.
[16] W. Gao,M. Younas,A. Farooq, A. Mahboob and W. Nazeer, M-polynomials and degree-based topological indices of the crystallographic structure of molecules, Biomolecules, MDPI, (2018 01-18.
[17] V. R. Kulli, On Augmented Revan Index and its Polynomial of Certain Families of Benzenoid System, Int. J. Math. And Appl., 6(4 (2018 43-50.
[18] V. R. Kulli, Revan polynomials of Chloroquine, Hydroxychloroquine, Remdesivir: Research for the treatment of COVID-19, SSRG International Journal of Applied Chemistry, 7(2 (2020 06-12.
[19] V. R. Kulli, Hyper-Revan Indices and their Polynomials of Silicate networks, International Journal of Current Research in Science and Technology, 4(3 (2018 17-21.
[20] V. R. Kulli, Geometric–Arithmetic reverse and Sum connectivity reverse indices of silicate and hexagonal networks, International Journal of Current Research in Sciences and Technology, 3(10 (2017 29-33.
[21] S. Wang, J. B. Liu,C. Wang and S. Hayat, Further results on computation of topological indices of certain networks, arXiv:1605. 00253v2[math. CO]5 May (2016 01-18.
[22] V. R. Kulli, I. Gutman, Computation of Sombor indices of certain networks, SSRG International Journal of Applied Chemistry, 8(1 (2021 01-05.
[23] S. Hayat, M. Imran, Computation of topological indices of certain networks, Journal of Applied Mathematics and Computation, 240(2014 213-228.
[24] M. U. Ghani, M. Inc, S. F. Sultan, M. Cancan and A. Houwe, Computation of Zagreb polynomial and indices of silicate network and silicate chain network, Hindawi, Journal of Mathematics, Volume 2023, Article Id 9722878, 9-pages
[25] B. Basavanagoud, P. Jakkannavar, On the Zagreb polynomials of transformation graphs, International Journal of Scientific Research in Mathematical and Statistical Sciences, 5(6 (2018 328-335.
[26] A. U. Rehman,W. Khalid, Zagreb polynomials and redefined Zagreb indices of line graph $HAC_5C_6C_7[p,q]$ nanotube, Open Journal of Chemistry, 1(2 (2018 26-35.
[27] P. Sarkar, A. Pal, General fifth M-Zagreb polynomials of Benzene ring implanted in the P-type surface in 2D network, Biointerface Research in Applied Chemistry, 10(6 (2020 6881-6892.
[28] F. B. Farooq, General fifth M-Zagreb indices and general fifth M-Zagreb polynomials of Dyck-56 networks, Annals of Biostatistics and Biometric Applications, 4(4 (2021 01-05.
[29] N. N. Swamy, Sangeetha T. I. and B. Sooryanarayana, General fifth M-Zagreb polynomials of the $TUC_4C_8[p,q]$ 2D-lattice and its derived graphs, Letters in Applied NanoScience Platinum Open access Journal, 10(1 (2021 1738-1747.
[30] V. R. Kulli, ABC Banhatti and Augmented Banhatti indices of chemical networks, Journal of Chemistry and Chemical Sciences, 8(8 (2018 1018-1025.
[31] S. Jawame, M. Ghods, Analysis of K-Banhatti polynomials and calculation of some degree based indices using (a,b -Nirmala index in molecular graph and line graph of $TUC_4C_8[S]$ nanotube, Chemical Methodologies, 7(2023 237-247.
[32] N. De, Sk. Md. Abu Nayeem and A. Pal, F-coindex of some graph operations, Spring Plus, S: 221(2016 01-13.
[33] D. Maji, G. Ghorai, Computing F-index, co-index and Zagreb polynomials of the kth generalized transformation graphs, Science Direct, Heliyon, 6(12 (2020 e05781,01-10.
[34] M. Alsharafi, Y. Zeren and A. Alameri, The second hyper-Zagreb index of complement graphs and its applications of some nanostructures, Asian Journal of Probability and Statistics, 15(4 (2021 54-75.
[35] B. Furtula,I. Gutman, Z. K. Vukisevic,G. LEkishvili and G. Popivoda, On old/new degree-based topological index, Bull. Acad. Serb. Sci. Arts. (Cl. Sci. Math. Natur., 149(2015 19-31.
[36] K. Kiruthika, Zagreb indices and Zagreb co-indices of some graph operations, International Journal of Advanced Research in Engineering and Technology, 7(3 (2016 25-41.
[37] Narsing Deo, Graph Theory, Prentice-Hall of India, New Delhi (2007.

[38] N. Trinajstić, Chemical Graph Theory, CRC Press, Boca Raton, FL, 1992.
[39] R. Todeschini, and V. Consonni, Handbook of Molecular Descriptors, Wiley-VCH, 2000.