

Analysis of MHD Carreau Nanofluid Over an Inclined Stretching Cylinder and Effect of Activating Energy

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Abstract: *This study illustrates the effects of radiation, convective heating, activating energy, and second order velocity slip condition on the magnetohydrodynamic (MHD) Carreau nanofluid and heat/mass transfer. Brownian motion and thermophoresis effects are used in mathematical models of the Carreau nanofluid. A system of nonlinear coupled differential equations is created from the governing equations. The altered equations' solutions are obtained. Furthermore, a graphic representation of the effects of temperature, dimensionless velocity, and nanoparticle concentration profile is provided.*

Keywords: Carreau fluid, velocity slip, convective heating, radiation, activating energy

1. Introduction

The term "nanofluid" describes the buildup of nanoparticles in conservatively used fluids that comprise a semiliquid of particles 10–100 Nm in size. These particles are effective at increasing the rate of heat transfer and solar energy collecting. With their enormous potential to deliver better enactment properties, particularly on the areas of heat transmission, nanofluids have recently captured the attention of a global audience. In 1995, Choi [1] introduced the idea of base fluid-nanoparticle suspensions, which is well-known as nanofluid, and explained the limitations of base fluid particles. These nanofluids are made by fine-tuning common fluids including water, kerosene, ethylene glycol, and oil with distributed nanometric-sized (<10-100nm) nanoparticles. Conversely, metals, metallic oxides, alloyed nanoparticles, single and multiwalled carbon nanotubes, carbide and nitride ceramics, and other materials are used to create nanoparticles. The results of the experiment demonstrate that nanofluids are used to improve thermal conductivity as well as heat transfer efficiency. According to research by Eastman et al. [2], Das et al. [3], and Wang et al. [4], heat transfer efficiency can increase by up to 40% due to these nanofluids' improved thermal conductivity. Subsequently, Buongiorno [5] talked about how the thermophoresis effect and Brownian diffusion effects are the most important mechanisms for researching nanofluid convective transport. The free convective research of the water-driven TiO₂-based nanofluid is discussed by Wen and Ding [6]. Daungthongsuk and Wongwises [7] provide various models with low volume fractions of nanoparticles in order to investigate some additional thermophysical features of nanofluids. The effects of the inclines angle on the free convective flow of a nanofluid filled with enclosures were addressed statistically by Abu-Nada and Oztop [8]. The Cheng-Minkowycz problem pertaining to natural convection imbedded in a porous media was examined analytically over a vertical plate by Nield and Kuznetsov [9]. The laminar magnetohydrodynamic boundary layer flow of nanofluid caused by heat generation/absorption is explained by Chamkha et al. [10]. The numerical method for the boundary layer flow of a nanofluid with a natural convective enclosure

is discussed by Khan and Aziz [11]. The water-motivated aluminum oxide nanoparticles' heat transport characteristics over a tilted cylinder were investigated by Rana et al. [12]. Using an analytical technique, Turkyilmazoglu and Pop [13] examined the effects of heat radiation on a few nanofluids over a vertical panel. The thermal transport characteristics of a nanofluid across a vertical plate were examined by Loganathan et al. [14]. The rheological characteristics of nanofluid over porous media were described by Uddin et al. [15]. Brownian diffusion and thermophoretic force, as well as other nanofluidic properties, were examined in this study using the numerical quadrature approach.

The combined effects of velocity slip, convective heating, and activating energy on electrically conducting Carreau nanofluid flow along an inclined cylinder have not, as far as we are aware, been the subject of any informatic endeavor in the literature. A graphic discussion of the effects of several flow-governing physical parameters on the velocity, temperature, and nanoparticle concentration profile is shown.

2. Mathematical Formulation

An inclined stretching sheet is used to formulate the effects of second order velocity slip, thermal radiation, convective heating, and activating energy across an electrically conducting Carreau nanofluid. The flow analysis is simulated using a cylindrical coordinate system (x, r) is interpreted as being in a radial direction x , and r is regarded as normal to the cylinder. Fluid flow is subjected to a constant magnetic field of a certain magnitude. This simulation does not take into account induced magnetic fields, Hall phenomena, or fluids with higher Reynolds

numbers. There is a velocity $U_w = \frac{u_0 x}{l}$ in the region of the

stretched sheet $y \geq 0$. A heat transfer coefficient h_f is produced by the convective heating of the lower surface with temperature T_f^* . Let T_∞^* and C_∞^* stand for the

surface's constant temperature and concentration, respectively. The following are the governing equations for the Carreau model when all the constraints are used:

$$\frac{\partial(ru^*)}{\partial x} + \frac{\partial(rv^*)}{\partial y} = 0 \quad (1)$$

$$u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial r} = \frac{v}{r} \frac{\partial u^*}{\partial r} \left[1 + \Gamma^2 \left(\frac{\partial u^*}{\partial r} \right)^2 \right]^{\frac{n-1}{2}} + \left[1 + \Gamma^2 \left(\frac{\partial u^*}{\partial r} \right)^2 \right]^{\frac{n-3}{2}} - \frac{\sigma_{nf} B_0^2 u^*}{\rho_f} + g \beta_c (C^* - C_\infty^*) \cos \varphi + g \beta_T (T^* - T_\infty^*) \cos \varphi \quad (2)$$

$$u^* \frac{\partial T^*}{\partial x} + v^* \frac{\partial T^*}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T^*}{\partial r} \right) + \tau D_B \left(\frac{\partial C^*}{\partial r} \frac{\partial T^*}{\partial r} \right) + \tau \frac{D_T}{T_\infty} \left(\frac{\partial T^*}{\partial r} \right)^2 - \frac{1}{(\rho C_p)_f} \frac{\partial q_r}{\partial r} \quad (3)$$

$$u^* \frac{\partial C^*}{\partial x} + v^* \frac{\partial C^*}{\partial r} = D_B \frac{\partial}{\partial r} \left(r \frac{\partial C^*}{\partial r} \right) + \frac{1}{r} \frac{D_T}{T_\infty} \frac{\partial}{\partial r} \left(r \frac{\partial T^*}{\partial r} \right) - K_r^2 \left(\frac{T^*}{T_\infty} \right) e^{-\frac{E_a}{K_1 T^*}} (C^* - C_\infty^*) \quad (4)$$

With the following boundary conditions:

$$\left. \begin{aligned} u^* &= u_w + U_{slip}^* \\ v^* &= 0, -K \frac{\partial T^*}{\partial r} = h_f (T_w^* - T_\infty^*), C^* = C_w^* \text{ at } r = R \\ u^* &\rightarrow 0, T^* \rightarrow T_\infty^*, C^* \rightarrow C_\infty^* \text{ as } r \rightarrow \infty \end{aligned} \right\} \quad (5)$$

Where the velocity components are u^* and v^* , ρ_{nf} is the density of fluid, ν denotes kinematics viscosity, c_p is the heat at constant temperature, D_T is the diffusion coefficients, D_B be the Brownian diffusion coefficient, $\tau = \frac{\rho_p c_p}{\rho_f c_f}$ is the ratio of nanoparticle heat capacity and base fluid heat capacity, T_w^* and y are the temperature of the fluid at the wall and ambient temperature when $y \rightarrow \infty$, C_w^* and C_∞^* are the nano volume fraction of the fluid at the wall and ambient nano volume fractions when $y \rightarrow \infty$, g represent gravity, β_T and β_c are the coefficients of thermal development and solutal growth respectively, φ is the inclination angle, Γ be the Carreau fluid parameter, $K_r^2 = K_0^2 x$ is the stretching sheet rate, $\left(\frac{T}{T_\infty} \right)^m e^{-E_a/K_1 T}$ be the Arrhenius function where m denotes exponent rate from -1 to 1.

Where U_{slip} represent second order momentum slip condition given by

$$U_{slip}^* = \frac{2}{3} \left(\frac{3-\alpha \rho^3}{\alpha} - \frac{2(1-\rho^2)}{3 K_n} \right) \lambda \frac{\partial u^*}{\partial y} - \frac{1}{4} \left(\rho^4 + \frac{2}{K_n} (1 - \rho^2) \right) \lambda^2 \frac{\partial^2 u^*}{\partial y^2} = A \frac{\partial u^*}{\partial y} + B \frac{\partial^2 u^*}{\partial y^2} \quad (6)$$

Here K_n denotes Knudsen number famously known as the ratio of molecular malicious free conduit λ to a physical extent for the flow.

approximation for radiation is express as

$$q_r = -\frac{4\sigma^* \partial T^{*4}}{3K^* \partial y} \quad (7)$$

For a thick medium the temperature difference are small enough, expanding T^{*4} as a Taylor series expansion and neglecting the higher order, we have

$$T^{*4} = 4T_\infty^{*3} T^* - 3T_\infty^{*4} \quad (8)$$

Here σ^* is the Stefan Boltzman coefficients.

Introducing the following similarity transformation

$$\psi = \sqrt{u^* v^* x} R f(\eta), \quad \eta = \left(\frac{r^2 - R^2}{2R} \right) \sqrt{\frac{u^*}{v^* x}} \quad (9)$$

$$\theta(\eta) = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad \phi(\eta) = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}$$

$$\begin{aligned} (1 + 2\gamma\eta)(1 + W_e^2(f'')^2)^{\frac{n-1}{2}} f''' + 2\gamma(1 + nW_e^2(f'')^2)^{\frac{n-1}{2}} f'' + (1 + 2\gamma\eta)(1 + nW_e^2(f'')^2)^{\frac{n-3}{2}} (f'')^3 (n-1)W_e^2 + 2\gamma\eta(1 + nW_e^2(f'')^2)^{\frac{n-3}{2}} (f'')^2 W_e^2 + f f'' - f' - M f'(Gc\phi + Gr\theta) \cos \varphi = 0 \end{aligned} \quad (10)$$

$$(1 + 2\gamma\eta) \left(1 + \frac{4}{3} R d \right) \theta'' + 2\gamma\theta' + Pr(f\theta') + (1 + 2\gamma\eta)(N_b\theta'\phi' + N_t\theta'^2) = 0 \quad (11)$$

$$(1 + 2\gamma\eta)\phi'' + (1 + 2\gamma\eta) \frac{N_t}{N_b} \theta + 2\gamma\phi' + 2r \frac{N_t}{N_b} \theta' +$$

$$Sc(f\phi' - f'\phi) - Sc(1 + A_r\theta)^m e^{-\frac{E}{(1+A_r\theta)}} = 0 \quad (12)$$

The associated boundary conditions are as follows:

$$\begin{aligned} f(0) &= 0, f'(0) = 1 + \delta f' + \lambda f'', \theta'(0) = B_i(1 - \theta(0)), \phi(0) = 1, \\ f'(\infty) &\rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \end{aligned} \quad (13)$$

where $\gamma = \frac{1}{R} \sqrt{\frac{\nu l}{u_0}}$ is the curvature parameter

$W_e = \frac{r^2 r^2 v_0^3 x^2}{\rho^3 v^*}$ is the Carreau fluid parameter

$M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter

$Pr = \frac{\nu}{\alpha}$ is the Prandtl number

$G_c = \frac{g \beta_c (C_w^* - C_\infty^*) l^2}{u_0^2 x^2}$ is the soluble Grashof number,

$G_r = \frac{g \beta_T (T_w^* - T_\infty^*) l^2}{u_0^2 x^2}$ is the thermal Grashof number

$Le = \frac{\alpha}{D_B}$ is the Lewis number

$\sigma = \frac{2lK_0^2}{u_0}$ is the constant chemical spices rate

$E = \frac{E_0}{K_1 T_\infty^*}$ is the activation energy

φ is the inclination angle,

$N_t = \frac{\tau D_T (T_w^* - T_\infty^*)}{v T_\infty^*}$, $N_b = \frac{\tau D_B (\tau_w - \tau_\infty)}{v}$, are the

Thermophoresis and Brownian motion coefficients

$R_d = \frac{16\sigma^* T_\infty^{*3}}{3KK^*}$ is the radiation parameter

$\delta = a \sqrt{\frac{u_0}{v_l}}$ is the first velocity slip parameter

And $\lambda = \frac{b u_0}{v_l}$ is the second velocity slip parameter

For the engineering point of view, one is usually more interested is the value of skin friction and the Nusselt number, are given as follows,

$$C_f = \frac{(\tau_{rx})_{r=R}}{\frac{1}{2} \rho u^*{}^2}, \quad N_u = \frac{x(q_w)_{r=R}}{K(T_w^* - T_\infty^*)} \quad (14)$$

$$\text{Where } \tau_{rx} = \left(\mu \left(\frac{\partial u^*}{\partial r} \right) \left[1 + \tau^2 \left(\frac{\partial u^*}{\partial r} \right)^2 \right]^{\frac{n-1}{2}} \right), \quad q_w =$$

$$-K \left(\frac{\partial T}{\partial r} \right)_{r=R} \quad (15)$$

By using the similarity transformation

$$\frac{C_f \sqrt{Re_x}}{2} = f''(0)(1 + W_e^2 f''(0)^2)^{\frac{n-1}{2}}, \quad \frac{N_u}{\sqrt{Re_x}} = -\theta'(0) \left(1 + \frac{4}{3} R_d \right) \quad (16)$$

Numerical value of the skin friction coefficient for various parameters

We	M	n	δ	λ	ϕ	$Re_x^{\frac{1}{2}} C_f$
0.1	0.2	1.0	0.5	0.5	0	-1.364782
0.2						-1.345650
0.3						-1.342428
0.4	0.5					-1.437346
	1.0					-1.483794
	2.0					-1.507856
	3.0	2.0				-1.093485
		3.0				-1.167896
		4.0				-1.256322
		1.5	0.05			-0.876952
			0.10			-0.759813
			0.15			-0.712348
				0.1		-0.841223
				0.3		-0.691234
				0.4		-0.661713
					$\pi/6$	-0.917237
					$\pi/4$	-0.867135
					$\pi/3$	-0.831775

Heat transfer and Carreau nanoliquid flow over an inclined cylinder using magnetohydrodynamics (MHD). We investigated the values of skin friction coefficients for a number of fixed parameters, and the table discusses the differences between the present and laid-back results.

3. Conclusions

Here, we covered the phenomena of activating energy, first and second order slip, thermophoresis, and Brownian motion in relation to the magnetohydrodynamic two-dimensional Carreau nanofluid flow caused by a stretched cylinder with heat and mass transfer. Analytical clarification is provided for the transmuted equations of velocities, temperature, and nano-profile.

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