An Interval Valued Double Refined Triangular Neutrosophic Framework for Image Segmentation

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Abstract: ***This paper introduces an image segmentation framework using Interval-Valued Double Refined Triangular Neutrosophic Numbers to better handle uncertainty and imprecision in image data. The proposed model extends classical neutrosophic theory by incorporating interval-valued membership functions and double refinement strategies to handle higher degrees of uncertainty, indeterminacy, and inconsistency in image processing. An algorithm is developed for segmentation of the images and implemented using MATLAB and the effectiveness of the proposed algorithm is confirmed using the appropriate metrics.***

Keywords: Neutrosophic Set, Interval Valued Neutrosophic Set, Interval Valued Double Refined Indeterminate Triangular Neutrosophic Number, Segmentation

1. Introduction

Interval valued neutrosophic set was introduced by Wang et. al; [14] as an appropriate mode to denote incomplete, uncertain, inconsistent and imprecise information. Since IVNS projects more flexibility and precision than single-valued neutrosophic sets [2], the applications of IVNS became the key concept of interest for many researchers. This made them to demonstrate the credibility of the interval valued neutrosophic set [10,11,12].

When there is a situation to provide the information in the form of intervals rather than as a single valued number, the interval numbers play a crucial role. Broumi et. al; [3] implemented a variety of operations on interval valued neutrosophic matrices using a new Matlab' package. Bhimraj and Broumi[1] proposed an interval-valued triangular neutrosophic number to solve the neutrosophic linear programming problem. In [4] [5] [6] Broumi et. al; made an overview of the Shortest Path Problem under various environments and solved Shortest Path Problem using single valued triangular and trapezoidal interval valued neutrosophic environments. Deli [7] derived aggregation operators for interval valued generalised single valued neutrosophic trapezoidal and applied in decision-making problem.

Image segmentation plays a vital role in various computer vision and image processing applications, where precise object boundary detection is crucial. Traditional segmentation techniques often struggle with uncertainty, ambiguity, and imprecise data, especially in complex or noisy images. To address these challenges, this paper proposes a segmentation framework based on Interval-Valued Double Refined Triangular Neutrosophic Numbers. The proposed model extends classical neutrosophic theory by incorporating interval-valued membership functions and double refinement strategies to handle higher degrees of uncertainty, indeterminacy, and inconsistency. Triangular representation enhances computational efficiency while preserving detailed structural information.

Interval valued double refined indeterminate triangular neutrosophic numbers is incorporated in proposing an algorithm for image processing. An algorithm is developed for deblurring the images using blind deconvolution and implemented using MATLAB and the effectiveness of the proposed algorithm is confirmed using the appropriate metrics.

2. Preliminaries

2.1 Neutrosophic Set [13]

Let E be a universe. A neutrosophic set (NS) $\tilde{A}_N$ in E are considered by a truth-membership function $T_{\tilde{A}_N}$, a indeterminacy-membership function $I_{\tilde{A}_N}$ and a falsity-membership function $F_{\tilde{A}_N}$.

It can be written as
$\tilde{A}_N$ = {< x, ($T_{\tilde{A}_N}$ (x), $I_{\tilde{A}_N}$ (x), $F_{\tilde{A}_N}$ (x)) >: x ∈ E, $T_{\tilde{A}_N}$ (x), $I_{\tilde{A}_N}$ (x), $F_{\tilde{A}_N}$ (x)∈]$^-$0, 1$^+$ [}.
so $0^- \leq T_{\tilde{A}_N}(x) + I_{\tilde{A}_N}(x) + F_{\tilde{A}_N}(x) \leq 3^+$.

2.2 Double Valued Neutrosophic Set [9]

Let X be a set of points (objects) and x be a general element in X. A Double Valued Neutrosophic Set (DVNS) $\bar{\bar{A}}$ in X is characterized by a truth-membership function $T_{\bar{\bar{A}}}$(x), a indeterminacy leaning towards truth membership function $I_{T\bar{\bar{A}}}$ (x), a indeterminacy leaning towards falsity membership function $I_{F\bar{\bar{A}}}$ (x) and a falsity-membership function $F_{\bar{\bar{A}}}$ (x). For each general element x ∈ X, $T_{\bar{\bar{A}}}$(x), $I_{T\bar{\bar{A}}}$ (x)) , $I_{F\bar{\bar{A}}}$ (x) and $F_{\bar{\bar{A}}}$ (x) are the elements of [0, 1] and $0 \leq T_{\bar{\bar{A}}}(x) + I_{T\bar{\bar{A}}}(x) + I_{F\bar{\bar{A}}}(x) + F_{\bar{\bar{A}}}(x) \leq 4$.

A DVNS $\bar{\bar{A}}$ is represented as

$\bar{\bar{A}}$= {< x, ($T_{\bar{\bar{A}}}$(x), $I_{T\bar{\bar{A}}}$ (x), $I_{F\bar{\bar{A}}}$ (x) , $F_{\bar{\bar{A}}}$ (x)) >: x ∈X}.

2.3 Interval Valued Neutrosophic Set [4]

Let X be the space of certain objects and x ∈ X. An interval-valued neutrosophic set (IVNS) $\tilde{A}_{IN} \subset$ X has the form of $\tilde{A}_{IN}$=

$\{\langle x, T_{\tilde{A}_{IN}}(x), I_{\tilde{A}_{IN}}(x), F_{\tilde{A}_{IN}}(x)\rangle : x \in X\}$, where $T_{\tilde{A}_{IN}}(x) : X \to [0, 1]$, $I_{\tilde{A}_{IN}}(x): X \to [0, 1]$ and $F_{\tilde{A}_{IN}}(x) : X \to [0, 1]$ with $0 \le T_{\tilde{A}_{IN}}(x) + I_{\tilde{A}_{IN}}(x) + F_{\tilde{A}_{IN}}(x) \le 3$ or all $x \in X$.

The variables $T_{\tilde{A}_{IN}}(x)$, $I_{\tilde{A}_{IN}}(x)$ and $F_{\tilde{A}_{IN}}(x)$ define the truth membership degree function, the indeterminacy-membership degree function and the falsity-membership degree function of X to N, respectively. These functions is designated as $T_{\tilde{A}_{IN}}(x)$
$= [\, T^L_{\tilde{A}_{IN}}(x), T^U_{\tilde{A}_{IN}}(x)\,] \subseteq [0, 1]$, $I_{\tilde{A}_{IN}}(x) = [\, I^L_{\tilde{A}_{IN}}(x), I^U_{\tilde{A}_{IN}}(x)\,] \subseteq [0, 1]$, $F_{\tilde{A}_{IN}}(x)$
$= [\, F^L_{\tilde{A}_{IN}}(x), F^U_{\tilde{A}_{IN}}(x)\,] \subseteq [0, 1]$
and the sum of these functions satisfy the condition
$0 \le T^U_{\tilde{A}_{IN}}(x) + I^U_{\tilde{A}_{IN}}(x) + F^U_{\tilde{A}_{IN}}(x) \le 3$.

3. Interval Valued Double Refined Indeterminate Triangular Neutrosophic Number [8]

This section exemplifies the definition of IVDRITrNN and the segmentation metrics.

3.1 Definition

Interval Valued Double Refined Indeterminate Triangular Neutrosophic Number whose truth membership function $T_{\bar{A}_{INt}}(x)$, indeterminacy leaning towards truth membership function $I_{T\bar{A}_{INt}}(x)$, indeterminacy leaning towards falsity membership function $I_{F\bar{A}_{INt}}(x)$ and falsity membership function $F_{\bar{A}_{INt}}(x)$ are defined as follows:

$$T_{\bar{A}_{INt}}(x) = \begin{cases} \dfrac{(x-a)p_{\bar{A}_{INt}}}{b-a} & a \le x < b \\ p_{\bar{A}_{INt}} & x = b \\ \dfrac{(c-x)\,p_{\bar{A}_{INt}}}{c-b} & b < x \le c \\ 0 & Otherwise \end{cases}$$

$$I_{F\bar{A}_{INt}}(x) = \begin{cases} \dfrac{b-x+r_{\bar{A}_{INt}}(x-a)}{b-a} & a \le x < b \\ r_{\bar{A}_{INt}} & x = b \\ \dfrac{x-b+r_{\bar{A}_{INt}}(c-x)}{c-b} & b < x \le c \\ 1 & Otherwise \end{cases}$$

$$I_{T\bar{A}_{INt}}(x) = \begin{cases} \dfrac{(x-a)q_{\bar{A}_{INt}}}{b-a} & a \le x < b \\ q_{\bar{A}_{INt}} & x = b \\ \dfrac{(c-x)\,q_{\bar{A}_{INt}}}{c-b} & b < x \le c \\ 0 & Otherwise \end{cases}$$

$$F_{\bar{A}_{INt}}(x) = \begin{cases} \dfrac{b-x+s_{\bar{A}_{INt}}(x-a)}{b-a} & a \le x < b \\ s_{\bar{A}_{INt}} & x = b \\ \dfrac{x-b+s_{\bar{A}_{INt}}(c-x)}{c-b} & b < x \le c \\ 1 & Otherwise \end{cases}$$

An interval Valued Double Refined Indeterminate Triangular Neutrosophic number can be represented by
$\bar{\bar{A}}_{INt} = \langle (a, b, c): p_{\bar{A}_{INt}}, q_{\bar{A}_{INt}}, r_{\bar{A}_{INt}}, s_{\bar{A}_{INt}} \rangle$
Where $p_{\bar{A}_{INt}} = \left[T^L_{\bar{A}_{INt}}, T^U_{\bar{A}_{INt}}\right]$, $q_{\bar{A}_{INt}} = \left[I_T{}^L_{\bar{A}_{INt}}, I_T{}^U_{\bar{A}_{INt}}\right]$, $r_{\bar{A}_{INt}} = \left[I_F{}^L_{\bar{A}_{INt}}, I_F{}^U_{\bar{A}_{INt}}\right]$,
$s_{\bar{A}_{INt}} = \left[F^L_{\bar{A}_{INt}}, F^U_{\bar{A}_{INt}}\right]$.
$T^L_{\bar{A}_{INt}}, I_T{}^L_{\bar{A}_{INt}}, I_F{}^L_{\bar{A}_{INt}}, F^L_{\bar{A}_{INt}}$ being the lower bounds and $T^U_{\bar{A}_{INt}}, I_T{}^U_{\bar{A}_{INt}}, I_F{}^U_{\bar{A}_{INt}}, F^U_{\bar{A}_{INt}}$, the upper bounds for the truth, indeterminacy leaning towards truth membership, indeterminacy leaning towards falsity membership and falsity membership functions.

3.2 Metrics of Segmentation

The efficacy of the segmentation algorithm is evaluated with the following metrics.

3.2.1 Jaccard Index (Intersection-Over-Union IoU)

Jaccard Index is one of the most commonly used metrics in segmentation. It is a very straightforward metric and also extremely effective. The Jaccard Index is also called as the Intersection-Over-Union (IoU). The IoU is the area of intersection between the predicted segmentation and the ground truth divided by the area of union between the predicted segmentation and the ground truth (Fig 4.5.8). This metric ranges from 0–1 (0–100%) with 0 signifying no overlap and 1 signifying perfectly overlapping segmentation.

$$\text{IoU} = \frac{target \cap prediction}{target \cup prediction}$$

3.2.2 Precision

To estimate the collection of predicted masks, each of the predicted masks is compared with the available target marks for a given input image.

- A **true positive** is ensured when a prediction-target mask pair has an IoU score greater than some predefined threshold.
- A **false positive** specifies a predicted object mask has no related ground truth object mask.
- A **false negative** indicates a ground truth object mask has no associated predicted object mask.
- Precision effectually defines the purity of our positive detections relative to the ground truth.

$$\text{Precision} = \frac{TP}{TP+FP}$$

3.3.3 Dice Coefficient (F1 Score)

The Dice Coefficient is 2 * the Area of Overlap divided by the total number of pixels in both images

4. Algorithm for Segmentation using Interval Valued Double Refined Indeterminate Triangular Neutrosophic Number

In this section an Interval Valued Double Refined Indeterminate Neutrosophic algorithm is utilised to process

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the images. The Interval Valued Double Refined Indeterminate Neutrosophic set split the membership into upper bound and lower bound for increase the veracity of data. The upper and lower bound values must be in between the interval (0, 1). Now the truth membership, falsity membership, indeterminate towards falsity and indeterminate towards truth membership with upper and lower bounds are obtained. The upper and lower bound values are obtained using the definition, and those values are used for the procedure of image processing.

Images that require processing are imported into system memory and then transformed into M-N-3 matrices. On the imported image, Interval Valued Double Refined Indeterminate Neutrosophic Sets are applied.

As for Image segmentation process the indeterminate members in upper and lower bound pixel were used to find the edges of the image. Once the edges of the image are determined, the K-means is applied to segment the image.

4.1 Steps involved in Segmentation:

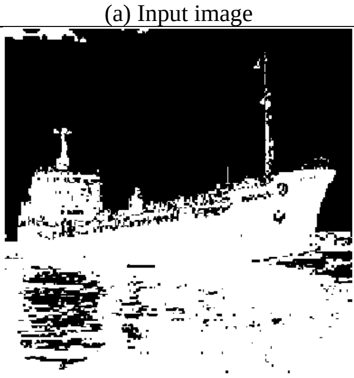
1) Importing image into memory.
2) Rescaling and matrix conversion.
3) Applying Interval Valued Double Refined Indeterminate Neutrosophic set in image matrix.
4) Identifying edges using indeterminate membership values.
5) Applying K-means in true members TU, TL of image.
6) Combine edges and K-means results to segmentation image.
7) Export image into local memory.

4.2 Results and Discussion

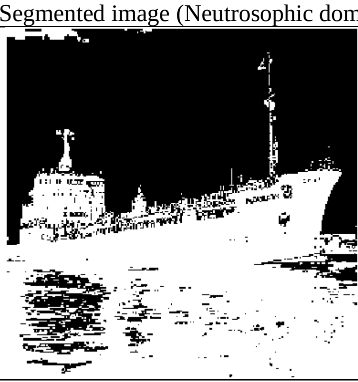
The performance of the algorithm is evaluated by implementing the above algorithm to the different images. The blurred images are given as the input for performing the process. The input images are processed using algorithms in three different domains namely Neutrsophic domain, DRITrN domain and IVDRITrN domain and the outputs are recorded. A sample image is given below.

(a) Input image

(b) Segmented image (Neutrosophic domain)



(c) Segmented image (Double Refined Neutrosophic domain)



(d) Segmented image (IVDRITrN domain)

Table 4.2.1: Segmentation – Evaluation Metric Values

Domain	Metrics		
	JAC	PRE	DICE
Neutrosophic domain	0.7385	0.7328	0.7729
Double Refined Neutrosophic domain	0.8072	0.8473	0.7976
IVDRTrN domain	0.9191	0.9345	0.9213

The input for the process is shown in Fig (a). The resulting images of three different algorithms of neutrosophic domain, Double refined neutrosophic domain and IVDRITrN domain are shown in Fig (b), Fig (c) and Fig (d) respectively. The evaluation metric values are registered in table 4.2.1. It obviously shows that the jaccard index (0.9191), precision value (0.9345) and dice coefficient (0.9213) is relatively higher for IVDRITrN method. The algorithm which gives higher values in terms of these metric seems to be an effective one. From the analysis of the output images and the values of the metrics in the tables, it is found that the IVDRITrN method gives higher values for all the metrics.

5. Conclusion

This work presents an advanced image segmentation approach using Interval-Valued Double Refined Triangular Neutrosophic Numbers, offering a powerful way to model uncertainty and indeterminacy in complex visual data. The findings highlight the potential of advanced neutrosophic models in enhancing image analysis tasks. Its flexible and refined neutrosophic structure opens new avenues for intelligent image analysis in future segmentation tasks.

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