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Predicting the Average Annual Exchange Rate of the Sudanese Pound from 1982 to 2022 Using ARIMA Models

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Abstract: ***This study applies the Box-Jenkins methodology to model and predict the average annual exchange rate of the Sudanese pound against the US dollar, using secondary data from the Central Bank of Sudan spanning 1982 to 2022. Employing a descriptive-analytical approach, the research identifies the ARIMA (1,1,1) model as the best fit for the non-stationary time series, demonstrating high predictive accuracy. Forecasts suggest a rising exchange rate trend through 2032, offering valuable insights for Sudanese monetary policy authorities. The study underscores the utility of time series models in economic forecasting and recommends their adoption for strategic planning.***

Keywords: predict- exchange rate- ARIMA models

1. Introduction

The exchange rate is considered one of the most important economic and financial indicators that demonstrate a country's economic condition and reflect the soundness and accuracy of its monetary policy. Exchange rate stability is a positive indicator of a country's economic stability. Most countries are keen to establish clear policies to control exchange rates by controlling the factors and determinants that can affect the exchange rate. This allows them to avoid changes that could affect the economy as a result of the instability of their local currency against foreign currencies, especially the US dollar. There are a number of changing economic factors that can have a direct impact on exchange rate stability, such as the gross domestic product, the rate of trade between countries, foreign exchange reserves, inflation, and other factors. This study will address the application of the Box- Jenkins methodology to predict the average annual exchange rate in Sudan during the period 2023-2032.

2. Study Problem

The study's problem lies in the failure of the exchange rate authorities in Sudan to use modern mathematical methods that enable accurate prediction of the average annual exchange rate in the future. This affects the programs and plans developed by these authorities to control the average annual exchange rate of the pound, which in turn affects the accuracy and soundness of monetary policy. Therefore, it was necessary to understand the time series behavior of the average annual exchange rate of the pound in Sudan over the future using time series models.

3. Study Important

The importance of the study lies in developing standard models that enable forecasting the average annual exchange rate of the Sudanese pound using ARIMA time series models. This enables forecasting the average exchange rate until 2035. This enables monetary policy authorities to develop plans and programs that control the factors affecting the exchange rate, contributing to its stability and stabilization.

4. Study Objectives

- Develop a mathematical model that can predict the average annual exchange rate of the Sudanese pound.
- Test the applicability of time series models in predicting the average annual exchange rate of the Sudanese pound.
- Predict the average annual exchange rate of the Sudanese pound, using the best model developed for the period (2023-2032).

5. Study Hypotheses

- The data series for the average annual exchange rate of the Sudanese pound during the period 1982-2022 is stationary.
- A suitable mathematical model exists to predict the average annual exchange rate of the Sudanese pound.
- The data series for the average annual exchange rate of the Sudanese pound in Sudan is increasing over time.

6. Study Methodology

The study employs a descriptive and analytical approach. The descriptive approach was used to describe the time series behavior of the average annual exchange rate of the Sudanese pound over time during the period 1982-2022. The analytical approach was used to construct and examine various mathematical models, select the best model, and predict the average annual exchange rate in Sudan in the future.

7. Data Sources

The study used secondary data from the Central Bank of Sudan for 1982-2022."

8. Previous Studies

Study: Fatima Ali Abdel-Ati, Muhammad Abdel-Latif Zayed (2014): "Using time series models and neural networks to predict exchange rates in Iraq”

Estimating exchange rate is considered a key tool for economic planning and reaching economic stability. In this study, aim is to estimate and use a statistical model for predicting exchange rates of Iraqi Dinar against U.S. dollar in the period (2004-2013). For this purpose, the study adopted two methods: time-series analysis using the Box–Jenkins’s approach and Artificial Neural Networks. Forecasts obtained from the two models were compared using both mean absolute values of the errors (MAE) and the square root of the mean square error (RMSE). The ARIMA (1,1,1) model produced better forecasts compared to neural networks.

Study: Issa Abdul Rahman Abdullah (2014): "Forecasting the Sudanese Pound Exchange Rate for the Period (2014-2023) Using Time Series"

This research aims to using time series analysis method (Box and Jenkins) in the analysis (diagnosis, assessment, appropriate test model, to predict). To find the best model to predict the exchange rate of the Sudanese pound against the U.S. dollar, based on data for the period (1982-2013). The results of the data analysis show that appropriate model is integrated self-regression model of first class ARIMA (1,2,0) and based on this model was to predict the exchange rate for the ten years to come have predictive values were consistent with the values of the original series which shows the efficiency of the model.

Study: Asma Al-Tayeb Musa (2016): "Using time series models to predict the exchange rate in Sudan during the period from (2000 - 2013 AD).”

The study aimed to determine the general trend of the exchange rate in Sudan, discover the seasonal changes that occur in the exchange rate series during the period from 2000-2013 on a monthly basis, and build a mathematical model that can predict the future exchange rate in Sudan. The study relied on secondary data obtained from the Central Bank of Sudan. The study used the descriptive approach to describe the behavior of the exchange rate series over time. It also used the analytical approach to use the different mathematical models to choose the best model and predict future exchange rates. One of the most important results reached by the study is that the time series of monthly data for the exchange rate in Sudan is not stationary, and that the appropriate model for the monthly time series data during the period from 2000-2013 is (ARMA (1,1,0), and that the forecasts of the exchange rate in Sudan during the period from 2014-2016 show a general increasing trend over time. The study recommended to establish the necessary policies in the field of the exchange rate using the model (ARMA (1,1,0) that was reached in predicting the exchange rates in Sudan in the future, and to conduct further studies and research in the field of factors affecting the stability of the exchange rate in Sudan.

Study: Yassin Saeed Abdullah (2016): “Using time series models to predict the average monthly exchange rate in Sudan during the period (2010-2015 AD).”.

The primary objective of time series research is to develop models that can be used to describe a specific phenomenon and predict its future behavior, which is an important and fundamental topic in statistical sciences. The aim of this research is to analyze the monthly average exchange rate series of the Sudanese pound against the US dollar during the period from January 2016 to December 2015 using the Box-Jenkins methodology. The exchange rate pattern is identified to construct the best model that helps predict exchange rate values during the period from January 2016 to December 2020. This is based on secondary monthly data taken from the Central Bank of Sudan. The study followed the descriptive analytical approach. The results of the research showed that the monthly exchange rate series of the Sudanese pound against the US dollar during the period from January 2010 to December 2015 was non-stationary, and that the best model that fits the monthly exchange rate series data is ARIMA (1.1.1), which achieved high predictive ability according to the predictive ability test. The study also reached a number of recommendations, the most important of which are: The relevant authorities responsible for setting policies and making decisions to control the exchange rate of the Sudanese pound against foreign currencies should use the model that was developed in forecasting, as the forecasts indicate an increase in the average exchange rate in the future.

9. Theoretical Framework

In the autoregressive (AR) model, the current period’s variable value depends on this model (AR), the value of a variable in the current period $\boldsymbol{Z_t}$ depends on the value of the same variable in previous periods $(\boldsymbol{Z_{t-1}}, \boldsymbol{Z_{t-2}}, \ldots \boldsymbol{Z_{t-n}})$, so it is called an “autoregressive” model because the value of the variable depends on its value in previous periods (Samir Mustafa Shaarawy, (2005).

The First – Order Autoregressive Model: AR(1)
A time series data is said to be generated by a first-order autoregressive process if the current observation of the series $(\boldsymbol{Z_t})$can be expressed as a linear function of the previous observation plus a random variation $\boldsymbol{a_t}$:

$$\boldsymbol{Z_t = \emptyset Z_{t-1} + a_t} \quad \ldots\ldots\ldots(1)$$

$\emptyset$: Autoregressive parameter to be estimated
$\boldsymbol{a_t}$: Random variables (white noise) that are assumed to be independent and follow a normal distribution with a mean of zero and a constant variance of$\boldsymbol{\sigma^2}$

If the readings $\boldsymbol{Z_t}$ represents the deviation of observations from the arithmetic mean $\boldsymbol{\mu}$, we can put the equation in the following form (Saad El-Din Mohamed Saad El-Din and others, (2001): -

$$\boldsymbol{(Z_t - \mu) = \emptyset_1 (Z_{t-1} - \mu) + a_t}$$
$$\boldsymbol{Z_t = (1 - \emptyset_1)\mu + \emptyset_1 Z_{t-1} + a_t}$$

where:
$\boldsymbol{Z_t}$: Actual observations
$\boldsymbol{\mu}$**:** The mean of these observations

Stationary condition on autoregressive parameter$(\boldsymbol{\emptyset_1})$:

The value of $\boldsymbol{\emptyset_1}$ must be less than one for the series to be stationary

$$\boldsymbol{-1 < \emptyset < 1}$$

The autocorrelation coefficient ρ_k for this model (Adnan Hashem Al-Wardi, 1990):

$$\rho_k = \emptyset_1^k \quad \text{.....}(2)$$

The partial correlation coefficient ρ_{kk} is equal to the autoregressive parameter AR(1) and the other partial correlation coefficients are equal to zero.

The Second – Order Autoregressive Model: AR(2)

This model can be put in the following form (Abdul Hamid Abdul Majeed Al-Baldawi, (2004)):-

$$Z_t = \emptyset_1 Z_{t-1} + \emptyset_2 Z_{t-2} + a_t \text{ ......}(3)$$

The autocorrelation function for this model is written as follows:

$$\rho_k = \emptyset_1 \rho_{k-1} + \emptyset_2 \rho_{k-2}$$

The stationary condition of this function is: $-1 < \emptyset < 1$

As for the partial autocorrelation equation (PACF) for model AR (2), its parameters decrease after the second backset:

$$\rho_{11} \neq 0$$
$$\rho_{22} \neq 0$$

Then $\emptyset_{22} \neq 0$ at the second backset $k > 2$

Autoregressive model P AR(P):

This form is written in the following format (Khaled Zahdi Khawaja, (2000)):-

$$Z_t = \emptyset_1 Z_{t-1} + \emptyset_2 Z_{t-2} + \ldots + \emptyset_p Z_{t-p} + a_t \text{........}(4)$$

It can be written in the following formula (Nourish Abdul Muhammad Bakri, 2000):

$$Z_t = \sum_{n=1}^{p} \emptyset_n Z_{t-n} + a_t \text{............}(5)$$

Moving Average Model (MA):

It is a model that uses random changes that occurred in the past to find out whether it is possible to reach a better representation of the time series data, i.e. it uses the values of the random error a_t and does not use the values of the variable itself (Abdul Qader Muhammad, 2005).

The First – Order Moving Average Model MA (1):

It is a linear function in terms of the current random change (error) a_t and the previous random change a_{t-1} and its formula is as follows:

$$Z_t = a_t - \theta_1 a_{t-1} \text{ .....}(6)$$

Where:

θ_1: is the parameter of the moving changes model.

The behavior of the autocorrelation function and the partial autocorrelation function in the MA (1) model is the same as their behavior in the AR (1) model.

The second-order moving average model MA (2)

The mathematical formula for this model is as follows (1.A.H. studenmd and henry,1991):

$$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \text{.........}(7)$$

The ACF equation for the MA (2) model is written as follows:

$$\rho_k = \frac{-\theta^2}{(1+\theta_1^2+\theta_2^2)} \text{ ... ...}(8)$$

when:

$k \leq 2$

$\rho_k = 0$, $k > 2$

The PACF equation can be written in the following form: (Mahmoud Hussein Mohammed Rashid, (2003)):

$$\rho_{kk} = \frac{-\theta_1^k(1-\theta_1^2)}{\left(1+\theta_1^{2(k+1)}\right)} \text{ ... .} \quad (9)$$

q-order moving average (q) MA model

The mathematical formula for this model is as follows (Tariq Muhammad Al-Rashid and others, 2010):

$$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \text{...}(10)$$

When moving average parameters are added to the ACF equation, its equation becomes as follows ():

$$\rho_k = \frac{(-\theta_k + \theta_1\theta_{k+1} + \ldots + \theta_{q-k}\theta q)}{(1+\theta_1^2 + \ldots + \theta_1^2)} \text{ .....}(11)$$

when:

$K \leq q$

$P_k = 0$,$K > q$

The partial autocorrelation function (PACF) never has any parameters that go to zero (it is exponentially inconsistent).

Autoregressive and Mixed Moving Average Models (ARMA):

ARMA (1,1) models:

The mathematical formula for this model is as follows:

$$Z_t = \emptyset_1 Z_{t-1} + a_t - \theta_1 a_{t-1} \text{ ...}(12)$$

This model is a combination of models AR (1) & MA (1).

The autocorrelation function of this model takes the following form:

$$\rho_k = \frac{(1-\emptyset_1)(\emptyset_1-\theta_1)}{1+\theta_1^2-2\emptyset_1\theta_1} \text{ ...} \quad (13)$$

The formula for the partial correlation function is as follows (Adnan Majid Abdul Rahman Bri, 2002):

$$P_{kk} = \emptyset_1 P_{k-1} \quad \text{....}(14)$$

Whereas: $k \geq 1$

Higher-order ARMA models:

The ARMA (1,1) model can be expanded using a larger number of autoregressive parameters and moving averages. In general, the ARMA (p, q) model is formulated as follows:

$$Z_t = \emptyset_1 Z_{t-1} + \emptyset_2 Z_{t-2} + \ldots + \emptyset_P Z_{t-P} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \text{.....}(15)$$

Integrate Autoregressive and Moving Average Models

Most time series are non-stationary, and as we mentioned, non-stationary time series can be transformed into stationary ones using the difference transformation. When the difference coefficient d is introduced into the (p,q) ARMA model, the model is transformed into the (p,d,q) ARIMA model.

Where:

p: autoregressive order

d: rank of differences

q: rank of moving averages

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The mathematical formula for the (p,d,q) ARMA models is as follows:

$$\boldsymbol{Z_t} = \emptyset_1 \boldsymbol{Z_{t-1}} + \emptyset_2 \boldsymbol{Z_{t-2}} + \ldots + \emptyset_p \boldsymbol{Z_{t-p}} + \boldsymbol{a_t} - \emptyset_1 \boldsymbol{a_{t-1}} + \ldots + \boldsymbol{\theta_q a_{t-q}} \ldots .. (\mathbf{16})$$

Stages of applying Box-Jenkins models in forecasting (Khaled bin Saad Al-Jad'i, 2006)

Identification & Diagnostic stage:

Model definition here means specifying the ranks of p, d, and q for the ARIMA model so that it can be estimated by observing both the autocorrelation functions (ACF) and partial autocorrelation functions (PACF). If the shape of the ACF falls within the 95% confidence interval from the beginning, the autocorrelation coefficient $\boldsymbol{\rho_k}$does not differ significantly from zero. This means that the data series is stationary and integrated of order zero. We perform the analyses on the original values of the variable $\boldsymbol{Z_t}$ without making any transformations. However, if the spread shape of the ACF does not fall within the 95% confidence interval for a number of time lags, the autocorrelation coefficient $\boldsymbol{\rho_k}$ differs significantly from zero. This means that the time series is not stationary, and its differences must be taken to make it a stationary series. When we take the first difference, d = 1, and if we take the second difference, d = 2 in the (p, d, q) ARIMA model.

The definition of p and q is done by looking at the spread shape of the autocorrelation function and the partial autocorrelation function. When the autocorrelations decline exponentially to zero, this means the presence of an autoregressive AR model whose rank is determined by the number of partial autocorrelations that differ significantly from zero. If the partial autocorrelations decline exponentially to zero, the model is an MA model whose rank is determined by the number of statistically significant autocorrelations. If the partial autocorrelations both decline exponentially to zero, this is an ARMA model. The rank of both AR and MA is determined by observing the spread of the autocorrelation function and the partial autocorrelation function. If the autocorrelation function does not decline quickly with increasing degrees of slowing, this means that the time series is unstable and needs to take the differences. The diagnosis process is summarized through the following table:

Table 1: Shows the diagnosis of the ranks of ARIMA models

Model	PACF	ACF
AR(1)	Zero after p_{kk1}	Geometrically descend from p_1
AR(2)	Zero after p_{kk2}	Geometrically descend from p_2
AR(P)	Zero after p_{kkp}	Geometrically descend from p_2
MA(1)	Give up after p_{kk1}	Zero after ρ_1
MA(2)	Give up after p_{kk2}	Zero after ρ_2
MA(q)	Give up after p_{kkp}	Zero after ρ_2
ARMA(1,1)	Give up after p_{kk1}	Geometrically descend from p_1
ARIMA(p,d,q)	Zero after p_{kkp}	Geometrically descend from p_q

Source: Saad El-Din Mohamed Saad El-Din and others (2001)

Whereas:

ρ : Autocorrelation function coefficient

$\boldsymbol{\rho_{kk}}$: partial autocorrelation function coefficient

Stages of Estimation & Testing

After identifying the model or set of models that might fit the data series, the parameters of these models are estimated, which are ($\emptyset_{\boldsymbol{i}}$, $\boldsymbol{i} = \mathbf{1}$, $\mathbf{2}$, . . . , $\boldsymbol{p}$)in the AR(P) model, $\emptyset_{\boldsymbol{i}}$, $\boldsymbol{i} = \mathbf{1}$, $\mathbf{2}$, . . . , q) in the MA(q) model, and $\emptyset_{\boldsymbol{i}}$ and $\boldsymbol{\theta_i}$ in the ARMA(p, q) model. After the model is characterized and its parameters are estimated, the model's quality and efficiency must be tested, a stage called the inspection and testing stage. If it passes this test, it is suitable for use.

After the model is diagnosed and its features are assessed, the quality and efficiency of the model must be tested. This is the stage called the inspection and testing stage. If it passes this test, it is suitable for use.

Stages of Forecasting:

After determining the model ranks p, d, and q, estimating and identifying the appropriate model, it is used for prediction. This is done by substituting the current and past values of the dependent variable $\boldsymbol{Z_t}$ and the residuals $\boldsymbol{a_t}$ as estimated values for the error term to obtain the first predicted future value $\boldsymbol{Z_{t+1}}$, which is called single-period forecasting. The second future value $\boldsymbol{Z_{t+2}}$can also be obtained by substituting the first future value $\boldsymbol{Z_{t+1}}$, which was obtained in the first step of prediction, into the prediction equation, assuming the out-of-sample error term for the function equals zero, thus arriving at the desired period. The prediction equation for the first period is (Ali Al-Alauna et al., 2005):

$$\boldsymbol{Z_{t+1}} = \emptyset \boldsymbol{Z_t} + \emptyset_1 \boldsymbol{Z_{t-1}} + \boldsymbol{a_t} - \boldsymbol{\theta_1 a_{t-1}}$$

The forecast equation for the second period is:

$$\boldsymbol{Z_{t+2}} = \emptyset_1 \boldsymbol{Z_{t+1}} + \boldsymbol{a_t} - \boldsymbol{\theta_2 Z_t}$$

10. Data analysis and discussion of results

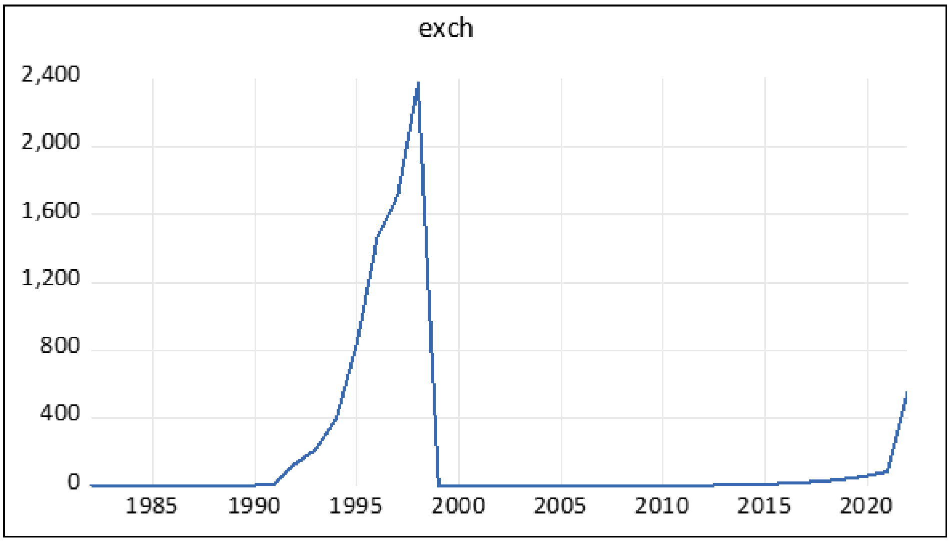

Figure 1: Graphical presentation of the average annual exchange rate during the period 1982-2022 AD

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

From the above graph of the average annual exchange rate series, we notice that the series is relatively stable, as it reached its highest level during the period 1990-2000. It can be noted that the time series is not stationary, and to confirm this, a number of tests were conducted to check the stationarity of the series.

Series stationary tests: There are several criteria in series stationary testing, including:

Autocorrelation and partial autocorrelation function:

Table 1: Autocorrelation and partial correlation function for the average annual price series during the period 1982-2022 AD:

<table>
<tr><td colspan="7">Date: 03/22/25 Time: 23:13</td></tr>
<tr><td colspan="7">Sample: 1982 2022</td></tr>
<tr><td colspan="7">Included observations: 41</td></tr>
<tr><td>Autocorrelation</td><td>Partial Correlation</td><td></td><td>AC</td><td>PAC</td><td>Q-Stat</td><td>Prob</td></tr>
<tr><td>. |***** |</td><td>. |***** |</td><td>1</td><td>0.655</td><td>0.655</td><td>18.927</td><td>0.000</td></tr>
<tr><td>. |*** |</td><td>. | . |</td><td>2</td><td>0.409</td><td>-0.035</td><td>26.496</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>.*| . |</td><td>3</td><td>0.154</td><td>-0.177</td><td>27.594</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>4</td><td>-0.002</td><td>-0.044</td><td>27.594</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>5</td><td>-0.079</td><td>-0.004</td><td>27.898</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>6</td><td>-0.123</td><td>-0.051</td><td>28.655</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>7</td><td>-0.155</td><td>-0.074</td><td>29.902</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>8</td><td>-0.162</td><td>-0.032</td><td>31.296</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>9</td><td>-0.166</td><td>-0.048</td><td>32.805</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>10</td><td>-0.169</td><td>-0.066</td><td>34.439</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>11</td><td>-0.171</td><td>-0.059</td><td>36.153</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>12</td><td>-0.170</td><td>-0.058</td><td>37.920</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>13</td><td>-0.167</td><td>-0.061</td><td>39.670</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>14</td><td>-0.155</td><td>-0.053</td><td>41.230</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>15</td><td>-0.130</td><td>-0.037</td><td>42.378</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>16</td><td>-0.101</td><td>-0.037</td><td>43.092</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>17</td><td>-0.058</td><td>-0.015</td><td>43.341</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>18</td><td>-0.060</td><td>-0.094</td><td>43.614</td><td>0.001</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>19</td><td>-0.060</td><td>-0.062</td><td>43.907</td><td>0.001</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>20</td><td>-0.060</td><td>-0.042</td><td>44.209</td><td>0.001</td></tr>
</table>

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

From the above table of the autocorrelation function and partial autocorrelation that most of the coefficients of the autocorrelation function fall outside the confidence interval (95%) for a long period, which indicates that the data series of the average annual exchange rate in Sudan during the period 1982 - 2022 AD is not stationary.

Unit root test for stationary:

Table 2: ADF test for the average annual exchange rate series in Sudan during the period 1982-2022 AD with a constant and a time trend

Null Hypothesis: EXCH has a unit root			
Exogenous: Constant, Linear Trend			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.717008	0.2356
Test critical values:	1% level	-4.205004	
	5% level	-3.526609	
	10% level	-3.194611	
*MacKinnon (1996) one-sided p-values.			

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 3: ADF test for the series of the average annual exchange rate in Sudan during the period 1982-2022 AD with constant

Null Hypothesis: EXCH has a unit root			
Exogenous: Constant			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.743609	0.0757
Test critical values:	1% level	-3.605593	
	5% level	-2.936942	
	10% level	-2.606857	
*MacKinnon (1996) one-sided p-values.			

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 4: ADF test for the average annual exchange rate series in Sudan during the period 1982-2022 AD, without a constant or time trend

Null Hypothesis: EXCH has a unit root			
Exogenous: None			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.482245	0.0641
Test critical values:	1% level	-2.624057	
	5% level	-1.949319	
	10% level	-1.611711	
*MacKinnon (1996) one-sided p-values.			

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Through the data of the previous tables above for the ADF tests, we find that most of the absolute values of the critical values are less than the table values, which indicates the lack of stationarity and stability of the time series, meaning that the series of data for the annual average of the exchange rate in Sudan during the period 1982 - 2022 AD is not stationary.

Through the previous review, we find that the time series of the average annual exchange rate in Sudan during the period 1982-2022 AD is not stationary. Therefore, the first difference was taken and the unit root was retested to confirm stationarity as follows:

Unit root test after taking the first difference:

Table 5: ADF test for the average annual exchange rate series in Sudan during the period 1982-2022 AD with a constant and a time trend

Null Hypothesis: DFECCH has a unit root			
Exogenous: Constant, Linear Trend			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.688515	0.0000
Test critical values:	1% level	-4.211868	
	5% level	-3.529758	
	10% level	-3.196411	

*MacKinnon (1996) one-sided p-values.

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 6: ADF test for the series of the average annual exchange rate in Sudan during the period 1982-2022 AD with constant

Null Hypothesis: DFECCH has a unit root			
Exogenous: Constant			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.786218	0.0000
Test critical values:	1% level	-3.610453	
	5% level	-2.938987	
	10% level	-2.607932	

*MacKinnon (1996) one-sided p-values.

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 7: ADF test for the average annual exchange rate series in Sudan during the period 1982-2022 AD, without a constant or time trend

Null Hypothesis: DFECCH has a unit root			
Exogenous: None			
Lag Length: 0 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.872038	0.0000
Test critical values:	1% level	-2.625606	
	5% level	-1.949609	
	10% level	-1.611593	

*MacKinnon (1996) one-sided p-values.
Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Through the data of the previous tables above for the ADF tests, we find that most of the absolute values of the critical values are more than the table values, which indicates the lack of stationarity and stability of the time series, meaning that the series of data for the annual average of the exchange rate in Sudan during the period 1982 - 2022 AD is stationary.

Steps for building and checking the models and predictions according to the Box- Jenkins methodology:
dentification: In this stage, the ranks p.q of the ARIMA model are determined so that it can be estimated. This is determined by the autocorrelation and partial autocorrelation functions shown.

Table 8: Autocorrelation function and partial autocorrelation for the first difference series of the average annual exchange rate in Sudan during the period 1982-2022

Date: 03/22/25 Time: 23:16						
Sample (adjusted): 1983 2022						
Included observations: 40 after adjustments						
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.*| . |	.*| . |	1	-0.121	-0.121	0.6346	0.426
. | . |	. | . |	2	0.015	0	0.6445	0.725
.*| . |	.*| . |	3	-0.148	-0.148	1.6337	0.652
.*| . |	.*| . |	4	-0.117	-0.158	2.2708	0.686
. | . |	.*| . |	5	-0.049	-0.091	2.387	0.793
. | . |	.*| . |	6	-0.016	-0.066	2.4003	0.879
. | . |	.*| . |	7	-0.039	-0.105	2.4771	0.929
. | . |	.*| . |	8	-0.004	-0.077	2.4779	0.963
. | . |	. | . |	9	0	-0.056	2.4779	0.981
. | . |	. | . |	10	-0.001	-0.06	2.4779	0.991

. | . |	. | . |	11	0	-0.058	2.4779	0.996
. | . |	. | . |	12	0	-0.049	2.4779	0.998
. | . |	. | . |	13	0.001	-0.042	2.478	0.999
. | . |	. | . |	14	0.002	-0.04	2.4782	1
. | . |	. | . |	15	0.003	-0.035	2.4786	1
. | . |	. | . |	16	0.004	-0.028	2.4795	1
. | . |	. | . |	17	-0.001	-0.03	2.4796	1
. | . |	. | . |	18	-0.002	-0.028	2.4799	1
. | . |	. | . |	19	-0.002	-0.026	2.4803	1
. | . |	. | . |	20	-0.003	-0.025	2.4811	1

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

From the above table of the autocorrelation and partial data functions, we can identify the appropriate model by determining the MA and AR rank to which the stationary time series is subject, by observing the outcome of the departure from the confidence interval for the autocorrelation and partial functions, where we notice that all the autocorrelation coefficients are within the confidence interval and are significantly equal to zero after the first time gap, which means that the series is subject to the MA(1) model. On the other hand, by looking at the partial autocorrelation function, we notice that the coefficients are equal to zero and are significantly equal to zero after the first-time gap, thus the series is also subject to the AR (1) model.

To estimate the appropriate model, a comparison will be made between three models: a model before the proposed model and a model after to arrive at the best model as follows:

Table 9: ARIMA (1,1,1) model

Dependent Variable: DFEXCH				
Method: ARMA Maximum Likelihood (OPG - BHHH)				
Date: 03/22/25 Time: 23:18				
Sample: 1983 2022				
Included observations: 40				
Failure to improve objective (non-zero gradients) after 87 iterations				
Coefficient covariance computed using outer product of gradients				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.772977	44.21568	0.017482	0.9861
AR(1)	0.691721	0.421908	6.639506	0.0000
MA(1)	-1.000000	5476.610	-5.000183	0.0042
SIGMASQ	146025.0	17735824	0.008233	0.9935
R-squared	0.724012	Mean dependent var		14.12750
Adjusted R-squared	0.714054	S.D. dependent var		422.4438
S.E. of regression	402.8027	Akaike info criterion		14.98228
Sum squared resid	5840999.	Schwarz criterion		15.15117
Log likelihood	-295.6457	Hannan-Quinn criter.		15.04335
F-statistic	4.298701	Durbin-Watson stat		1.971316
Prob(F-statistic)	0.004512			

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 10: ARIMA (1,1,2) model

Dependent Variable: DFEXCH				
Method: ARMA Maximum Likelihood (OPG - BHHH)				
Date: 03/22/25 Time: 23:19				
Sample: 1983 2022				
Included observations: 40				
Convergence not achieved after 500 iterations				
Coefficient covariance computed using outer product of gradients				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.633989	45.41833	0.013959	0.9889
AR(1)	0.676883	0.624490	1.083896	0.2858
MA(1)	-0.972544	23.73251	-0.040979	0.9675
MA(2)	-0.026528	0.816272	-0.032499	0.9743
SIGMASQ	145960.7	3398003.	0.042955	0.9660
R-squared	0.161132	Mean dependent var		14.12750
Adjusted R-squared	0.065262	S.D. dependent var		422.4438
S.E. of regression	408.4266	Akaike info criterion		15.03180

Sum squared resid	5838430.	Schwarz criterion	15.24291
Log likelihood	-295.6361	Hannan-Quinn criter.	15.10813
F-statistic	1.680725	Durbin-Watson stat	1.971282
Prob(F-statistic)	0.176473		

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Table 11: ARIMA (1,1,0) model

Dependent Variable: DFEXCH				
Method: ARMA Maximum Likelihood (OPG - BHHH)				
Date: 03/22/25 Time: 23:20				
Sample: 1983 2022				
Included observations: 40				
Convergence achieved after 24 iterations				
Coefficient covariance computed using outer product of gradients				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	12.88441	111.8028	0.115242	0.9089
AR(1)	-0.122259	0.205906	-0.593758	0.5563
SIGMASQ	171350.3	24202.62	7.079824	0
R-squared	0.015213	Mean dependent var		14.1275
Adjusted R-squared	-0.038019	S.D. dependent var		422.4438
S.E. of regression	430.3993	Akaike info criterion		15.03972
Sum squared resid	6854011	Schwarz criterion		15.16638
Log likelihood	-297.7944	Hannan-Quinn criter.		15.08552
F-statistic	0.285788	Durbin-Watson stat		1.973924
Prob(F-statistic)	0.753065			

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

Choosing the appropriate model and estimating the parameters of the selected model: There are several criteria for selecting the best model. We used two of them: Akaika (AIC) and Schwarz (SC).

Table 12: Shows the criteria for selecting the best model for ARIMA models

MODEL	SC	AIC	HQ
ARIMA (1,1,1)	15.15	14.98	15.04
ARIMA (1,1,2)	15.24	15.03	15.11
ARIMA (1,1,0)	15.17	15.04	15.09

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

By comparing the indicators and standards that are used to judge the quality of the model, we find that the ARIMA (1,1,1) model has the lowest value for both the AIC and SC criteria, and thus it is considered the best model through which future values of the average annual exchange rate in Sudan can be predicted.

Diagnostic testing of the model: We will test the model fit, i.e., ensure that the model is suitable for the prediction process, by comparing the original and estimated series:

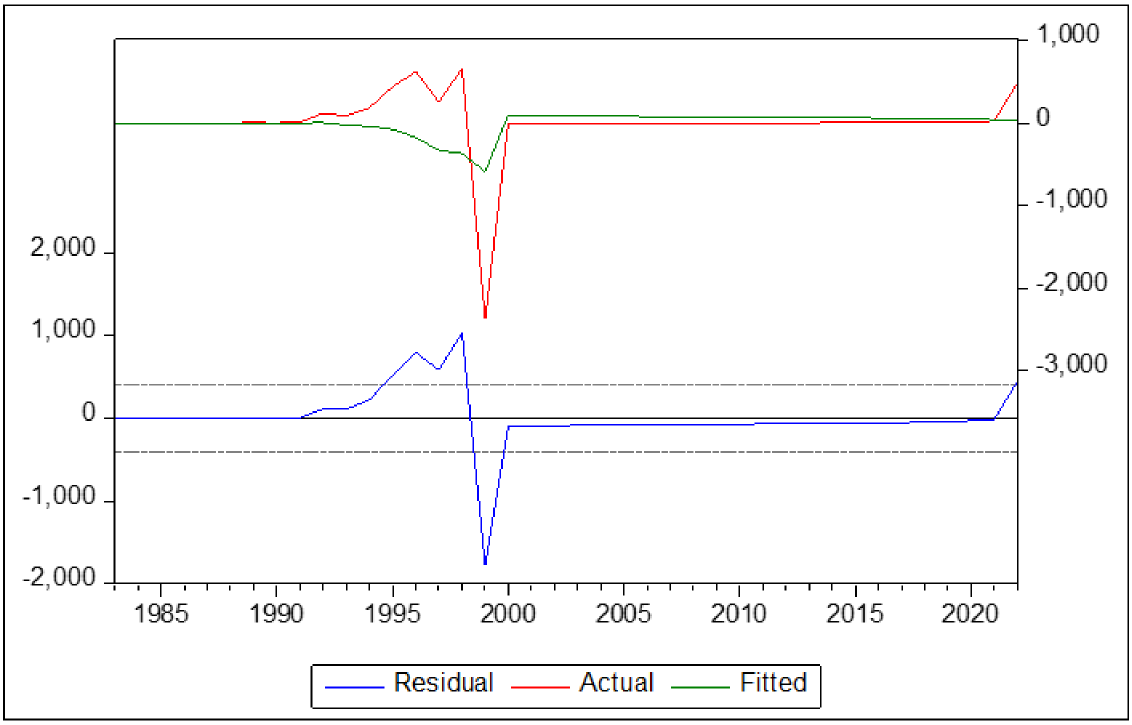

Figure 2: Comparison chart between the original and estimated series

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

From the figure above, we find that there is a near match between the two curves: the actual series curve and the fitted series curve, which confirms that the model that was reached has a good predictive ability to represent data in future periods. As for the estimated residual series curve, it randomly wraps around the interval axis, and this gives us an initial indication of the absence of self-correlation between the errors.

Autocorrelation function and partial autocorrelation function of a series of residuals

Table 13: Autocorrelation and partial correlation functions for the series of residuals of the ARIMA (1,1,1) model

Date: 03/23/25 Time: 11:38						
Sample (adjusted): 1983 2022						
Q-statistic probabilities adjusted for 2 ARMA terms						
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. | . |	. | . |	1	0.012	0.012	0.0063	
. |*. |	. |*. |	2	0.085	0.084	0.3223	
.*| . |	.*| . |	3	-0.091	-0.094	0.7005	0.403
.*| . |	.*| . |	4	-0.092	-0.097	1.0913	0.579
. | . |	. | . |	5	-0.048	-0.031	1.2031	0.752
. | . |	. | . |	6	-0.028	-0.019	1.2418	0.871
. | . |	. | . |	7	-0.052	-0.063	1.3785	0.927
. | . |	. | . |	8	-0.024	-0.036	1.4084	0.965
. | . |	. | . |	9	-0.021	-0.023	1.432	0.985
. | . |	. | . |	10	-0.022	-0.034	1.4591	0.993
. | . |	. | . |	11	-0.021	-0.037	1.4858	0.997
. | . |	. | . |	12	-0.022	-0.033	1.5137	0.999
. | . |	. | . |	13	-0.021	-0.032	1.5405	1
. | . |	. | . |	14	-0.02	-0.035	1.5661	1
. | . |	. | . |	15	-0.019	-0.034	1.5896	1
. | . |	. | . |	16	-0.017	-0.033	1.6097	1
. | . |	. | . |	17	-0.022	-0.04	1.6461	1
. | . |	. | . |	18	-0.023	-0.042	1.6879	1
. | . |	. | . |	19	-0.025	-0.043	1.7363	1
. | . |	. | . |	20	-0.026	-0.047	1.7918	1

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

from the previous table of the autocorrelation function and the partial autocorrelation function of the residual series that all the autocorrelation coefficients fall within the confidence intervals (95%) for long periods, which indicates the stationarity of the residual series.

Du**rbin-Watson statistic: This statistic is used to detect autocorrelation of residuals by testing the following hypotheses:**
Null hypothesis: "There is no autocorrelation"
Alternative hypothesis: "There is autocorrelation"

Referring to Table (9) above for the estimation of the ARIMA (1,1,1) model, the calculated D.W was approximately equal to (1.97) which is equal to (2). Therefore, we accept the null hypothesis that there is no correlation between the values of the random error term. Variance stability test for the model:

Table 13: Shows the ARCH test for the ARIMA (1,1,1) model.

Heteroskedasticity Test: ARCH			
F-statistic	0.9812	Prob. F(1,38)	0.4521
Obs*R-squared	1.0721	Prob. Chi-Square (1)	0.3145

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

From the previous table above, we find that the probability value of the F-statistic and the probability value of the chi-square test are greater than 0.05%, which indicates the stability of the variance of the model.

Forecasting: After ensuring the validity of the model, it is used to predict future observations of the phenomenon. This stage cannot be moved to until the proposed model passes the model validity tests. If this does not happen, we must return again to the first stage, which is determining the model. The process is repeated until we obtain a model that passes all validity tests. By using the appropriate model ARIMA (1,1,1) for the data using the E-views program, Predictions for 2023 to 2035 were obtained for the next 5 years in the following table:

Table 14: Predictive values for the average annual exchange rate during the period 2023-2035 AD

Year	Estimated value of the average exchange rate
2023	934
2024	1541
2025	2543
2026	2975
2027	3213
2028	3341
2029	3442
2030	3510
2031	3545
2032	3581

Source: Prepared by the researcher based on the annual average exchange rate series during the period 1982-2022 using the Eviews_9 program.

11. Results and Recommendations Results

11.1 Results

1) The data series for the average annual exchange rate in Sudan for the period 1982-2022 is non-stationary.
2) The appropriate model for predicting future values for the average annual exchange rate in Sudan for the period 1982-2022 is ARIMA (1,1,1).
3) The predictive values for the average annual exchange rate in Sudan for the period 2023-2032 showed a general increasing trend over time.

11.2 Recommendations

1) The authorities responsible for the annual exchange rate in Sudan should use the ARIMA model (1.1.1) to forecast the average exchange rate.
2) The authorities responsible for the annual exchange rate in Sudan should develop plans and programs that contribute to controlling it during the coming period (2023-2032), as forecasts during this period have shown a general upward trend over time.
3) Conduct further studies and research using time series models and other statistical methods on other factors that enable prediction of the exchange rate in Sudan.

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