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Multiple Dependent State Sampling Plans through Probabilistic Quality Region (PQR) for Lot Acceptance based on Measurement Data

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Abstract: ***Usually Sampling Plans are designed with quality levels like AQL, LQL, and MAPD. This paper introduces a new method for designing Sampling plan based on range of quality instead of point-wise description of quality. So this method can be adopted toward elementary production process where the stipulated quality level is advisable to fix at a later stage. This paper proposes a variable multiple dependent (or deferred) state sampling plan through Probabilistic Quality Region (PQR) for the inspection of normally distributed quality characteristics. The decision about the acceptance of the lot is based on the states of the preceding lots (dependent state plan) or on the states of the forthcoming lots (deferred state plan). The lot acceptance probability is derived and the two- point approach for determining the plan parameters is described. The advantages of this new variables plan over conventional sampling plans are well discussed. Tables are constructed for the selection of parameters of this plan under the specific values of producer's and consumer's risks, indexed through acceptable quality level and limiting quality level, when for standard deviation is known or unknown.***

Keywords: Acceptable Quality Level, Limiting Quality Level, Operating Characteristic Curve, Probabilistic Quality Region, ,Quality Control, Sampling by variables.

1. Introduction

Acceptance sampling is a statistical tool used for make decisions on a lot of products, which should be sentenced for the use of consumer. Variables sampling plan constitute one of the major areas on the theory and practice for acceptance sampling. The primary advantage of the variables sampling plan is that the same operating characteristic (OC) curve can be obtained with smaller sample size than required by attributes sampling plan. When destructive testing is employed, the variables sampling is particularly useful with reduced cost of inspection than any of other attribute sampling plan.

Variables sampling plan are, of course, more difficult to apply than attributes sampling plan and the assumptions on which they are based may not be met. Collani (1990) has criticized the variables sampling plan and argued that acceptance sampling by variables is inappropriate if one is interested in the fraction nonconforming for incoming batches. But, Seidel (1997) has proved that sampling by variables is always optimal.

Some of the earlier researchers have studied the conventional variables sampling plan. Lieberman and Resnikoff (1955) have developed tables for the selection of plan parameters for various acceptable quality levels (AQL) under MIL-STD 414 scheme. Owen (1967) has developed variables sampling plans based on the normal distribution when standard deviation of the process is unknown. Hamaker (1979) has given a procedure for finding parameters with unknown sigma variables sampling plan. Bravo and Wetherill (1980) have developed a method for designing double sampling variable plan having OC curves matching with equivalent single sampling plans. Schilling (1982) has studied exclusively acceptance sampling which deals with conventional variable sampling plans. Muthuraj (1988) has given expression for finding inflection point on the OC curves for Single Sampling Variables Plans, for both cases when standard deviation known and unknown. Baillie (1992) has developed tables for variable double sampling plans when the process standard deviation is unknown. Suresh (1993) has constructed tables for designing Single Sampling Variable Plan indexed through AQL and LQL with their relative slopes as a measure for sharpness of inspection. Further Suresh (1993) has also constructed tables for designing Single Sampling Variables Plan indexed through (p_1, K_1) and (p_2, K_2) along with relative efficiency for variables plan over attributes plan considering filter and incentive effects. Balamurali et.al (2005) have proposed a procedure for designing variables repetitive group sampling plan through minimum average sample number.

The concept of multiple dependent (or deferred) state sampling (MDS) was introduced by Wortham and Baker (1976). The MDS sampling plan belongs to the group of conditional sampling procedures. In these procedures, acceptance or rejection of a lot is based not only on the sample from that lot, but also on sample results from past lots (in the case of dependent state sampling) or from future lots (in the case of deferred state sampling). The MDS plan is applicable in the case of Type B situations (that is sampling from a continuous process) where lots are submitted for inspection serially in the order of their production.

The operating procedure and characteristics of the attributes MDS sampling plan was stated in Wortham and Baker (1976) and this plan was studied further by Varest (1982), Soundararajan and Vijayaraghavan (1990), and Balamurali and Kalyanasundaram (1999). Balamurali and Jun (2007) have also studied the multiple dependent state sampling plans by variables for lot acceptance based on measurement data.

Although the MDS plan is reported to be efficient in terms of sample size required, the variables MDS plan with Probabilistic Quality Region (PQR) based on measurement data has not been studied. Suresh and Divya (2008) have studied the selection of single sampling plan through Decision Region. Further Suresh and Divya (2008) have studied the construction and selection of single sampling plan through Quality Desion Region (QDR) and Limiting Quality Level. This paper proposes a variables MDS sampling plan through Probabilistic Quality Region (PQR). The following assumptions should be valid for application of the variables MDS plan.

1) Lots are submitted for inspection serially in the order of their production from a process having a constant proportion of non- conforming.
2) The consumer has confidence in the supplier and there should be no reason to believe that a particular lot is worst than the preceding lots.
3) The quality characteristic of interest follows with normal distribution.

2. Variables MDS Sampling Plans

Suppose that the quality characteristic of interest has the upper specification limit U and follows a normal distribution with unknown mean μ and known standard deviation σ. Then, the following multiple dependent state variables sampling plan is proposed.

Step 1: From each submitted lot, take a random sample of size n_σ, $(X_1, X_2, ...X_{n\sigma})$, say and compute

$$\nu = \frac{U - \overline{X}}{\sigma}, \text{ where } \overline{X} = \frac{\sum_{i=1}^{n_\sigma} X_i}{n_\sigma}.$$

Step 2: Accept the lot if $\nu \geq k_{a\sigma}$ and reject the lot if $\nu < k_{r\sigma} (k_{a\sigma} > k_{r\sigma})$. If $k_{r\sigma} \leq \nu < k_{a\sigma}$, then accept the current lot provided that the preceding m_σ lots were accepted on the condition that $\nu \geq k_{a\sigma}$ but reject the lot, otherwise.

Thus the proposed variables MDS plan is characterized with four parameters, namely $n_\sigma, m_\sigma, k_{a\sigma} \text{ and } k_{r\sigma}$.

If $k_{a\sigma} = k_{r\sigma}$, then the proposed plan reduces to basic variables single sampling plan.

When the lower specification limit L is required instead of U, the operating procedure would be same as above, except for using the statistic $\nu = \frac{\overline{X} - L}{\sigma}$ in Step1. Note that in the case of multiple deferred state sampling plan, the forthcoming m_σ lots will be considered for acceptance of the current lot, so that accept or reject decision is effectively postponed.

2.1. Case of known standard deviation

Generally, sampling plans are designed through considering two points on the OC curve, namely $(p_1, 1-\alpha)$ and (p_2, β), where p_1 is called acceptable quality level (AQL), p_2 is the limiting quality level (LQL), α is producer's risk and β is consumer's risk. A well designed sampling plan must provide at least $(1-\alpha)$ probability of acceptance for a lot when the process fraction non-conforming is at AQL level and the sampling plan must also provide not more than β probability of acceptance if the process fraction non- conforming is at the LQL level. Thus, the acceptance sampling plan must have its OC curve passing through two designated points namely $(AQL, 1-\alpha)$ and (LQL, β). The proposed variable MDS plan will also be designed so that the OC curve pass through these two points on the OC curve.

The fraction non- conforming in a lot will be expressed as

$$p = \Pr\{X > U / \mu\} = 1 - \Phi\left(\frac{U - \mu}{\sigma}\right), \quad \ldots\ldots (2.1.1)$$

Where $\Phi(.)$ is the cumulative distribution function of the standard normal variable. So, the unknown mean μ is related to the fraction non- conforming p through equation (2.1.1). Let us define the standardized quality characteristic corresponding to the fraction non-conforming p as

$$Z_p = \Phi^{-1}(1-p). \quad \ldots\ldots\ldots\ldots\ldots (2.1.2)$$

Now, the OC function for the variable MDS sampling plan, which gives the proportion of lots that are expected to be accepted for a given lot quality p, is obtained as

$$P_a(p) = \Pr\{\nu \geq k_{a\sigma} / p\} + \Pr\{k_{r\sigma} \leq \nu \leq k_{a\sigma} / p\}[\Pr\{\nu \geq k_{a\sigma} / p\}]^{m\sigma}, \quad \ldots\ldots\ldots\ldots (2.1.3)$$

Where the first term on the right hand side represents the probability of accepting a lot based on a single (current) sample and the second term is the probability of accepting a lot based on the state of preceding lots. The Probability of acceptance of the lot in equation (2.1.3) can be written as

$$P_a(p) = \Phi(w_2) + [\Phi(w_1) - \Phi(w_2)][\Phi(w_2)]^{m\sigma}, \quad \ldots\ldots (2.1.4)$$

Where

$$w_1 = (Z_p - k_{r\sigma})\sqrt{n_\sigma},$$
$$w_2 = (Z_p - k_{a\sigma})\sqrt{n_\sigma}$$

Note that the proposed plan becomes single variable sampling plan with parameters $n_\sigma, and\ k_{a\sigma}$

When m_σ goes to infinity.

2.2 Case of unknown standard deviation

Whenever the standard deviation is unknown, one may use the sample standard deviation S instead of σ. In this case, the plan operates as follows:

Step 1: From each submitted lot, take a random sample of size $n_s, (X_1, X_2, ..., X_{ns})$, and compute $v = \frac{U-\overline{X}}{S}$, where $\overline{X} = \frac{\sum_{i=1}^{n_s} X_i}{n_s}$ and $S = \sqrt{\frac{\sum (X_i - \overline{X})^2}{n_s - 1}}$.

Step 2: Accept the lot if $v \geq k_{as}$ and reject the lot if $v < k_{rs}$. If $k_{rs} \leq v < k_{as}$, then accept the current lot provided that preceding m_s lots were accepted on the condition that $v \geq k_{as}$.

Thus, the proposed unknown sigma variables MDS plan is characterized through four parameters, namely n_s, m_s, k_{as} and k_{rs}. If $k_{as} = k_{rs}$, then the proposed plan reduces to single variables sampling plan with unknown standard deviation. The determination of parameters of the unknown sigma plan is slightly different from the known sigma case. It is known that $\overline{X} \pm k_{as} S$ for a large sample size approximately follows (see Hamaker, 1979; Duncan, 1986):

$$\overline{X} \pm k_{as} S \quad \square \quad N\left(\mu + k_{as}\sigma, \frac{\sigma^2}{n} + k_{as}^2 \frac{\sigma^2}{2n}\right). \qquad \ldots\ldots (2.2.1)$$

Therefore, the probability of accepting a lot based on a single sample is given approximately as

$$P\{v \geq k_{as} / p\} = P\{\overline{X} \leq U - k_{as} S / p\} = \Phi\left(\frac{U - k_{as}\sigma - \mu}{(\sigma/\sqrt{n_s})\sqrt{1 + k_{as}^2/2}}\right) = \Phi\left((z_p - k_{as})\sqrt{\frac{n_s}{1 + k_{as}^2/2}}\right). \qquad \ldots\ldots\ldots\ldots (2.2.2)$$

Analogously to (2.1.4), the lot acceptance probability for the sigma unknown case is given as

$$P_a(p) = \Phi(y_2) + [\Phi(y_1) - \Phi(y_2)][\Phi(y_2)]^{ms} \qquad \ldots (2.2.3)$$

where

$$y_2 = (z_p - k_{as})\sqrt{\frac{n_s}{1 + k_{as}^2/2}},$$

$$y_1 = (z_p - k_{rs})\sqrt{\frac{n_s}{1 + k_{rs}^2/2}}.$$

3. Single Sampling Attribute Plan with Probabilistic Quality Region (PQR)

It is an interval of quality $(p_1 < p < p_2)$ in which product is accepted with a minimum probability 0.10 and maximum probability 0.95.

Probabilistic Quality Region denoted as $d_2 = (p_2 - p_1)$ is derived using the probability of acceptance

$$P_a(p_1 < p < p_2) = \sum_{r=0}^{c} \frac{e^{-np}(np)^r}{r!} \quad for \; p_1 < p < p_2 \qquad \ldots. (3.1)$$

Where $p < 0.1$ and the number of defects assumed to follow Poisson distribution.

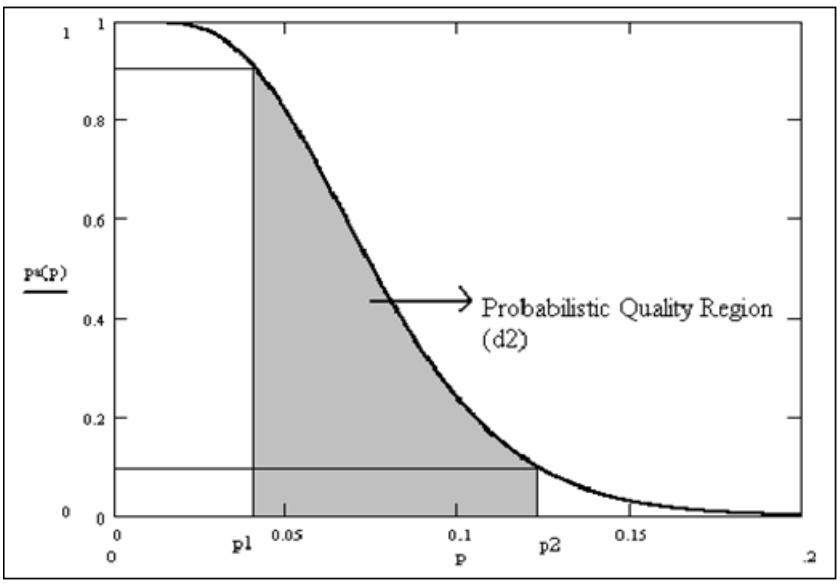

Figure 1: OC Curve of the SSP (75,5) with Probabilistic Quality Region (d_2)

Figure 1 represents the OC curve indicating the Probabilistic Quality Region (PQR). From Figure 1 d_2 represent the Probabilistic Quality Region (PQR). A general result is that $p_1 < p_* < p_2$ for any Single Sampling Attribute plan. When p_* approaching p_1, then np_* also approaching to np_1 . Then the quality region is small. Therefore, straighter OC curve can be preferred. When p_* approaching p_2, then np_* approaching np_2 . That is when p_* approaching p_2 , at lower quality level probability of acceptance is more. After MAPD, at lower quality level if acceptance is more then the product is not at all useful. Therefore When np_* approaching np_2, OC will be relatively less discriminating the quality. Thus Probabilistic Quality Region (PQR), d_2 is a good measure for defining quality of the lot.

3.1. Variables MDS sampling plan with Probabilistic Quality Region (PQR) - case of known standard deviation

Probabilistic Quality Region (PQR) is an interval of quality $(p_1 < p < p_2)$ in which product is accepted with a minimum probability 0.10 and maximum probability 0.95.

Probabilistic Quality Region (PQR) is denoted as $d_{2\sigma} = n_\sigma (p_2 - p_1)$ and is derived from probability of acceptance

$$P_a(p_1 < p < p_2) = \Pr\{\nu \geq k_{a\sigma} / p\} + \Pr\{k_{r\sigma} \leq \nu \leq k_{a\sigma} / p\}[\Pr\{\nu \geq k_{a\sigma} / p\}]^{m\sigma} \text{ for } p_1 < p < p_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1.1)$$

Where p is the fraction non- conforming in a lot.

Values of the parameters $(n_\sigma, k_{a\sigma}, k_{r\sigma})$ and the Probabilistic Quality Region (PQR), when $m_\sigma = 1$ and when sigma is known, are tabulated in Table1.1; this is carried for several combinations of (AQL, LQL) with $\alpha = 0.05 \; and \; \beta = 0.10$. These combinations are chosen so that the ratio $\left(\frac{LQL}{AQL}\right)$ ranges between 2 and 10 in most cases. Those values when $m_\sigma = 2$ and $m_\sigma = 3$ were reported in Tables 1.2 and 1.3, respectively.

3.2 Variables MDS sampling plan with Probabilistic Quality Region (PQR) - case of unknown standard deviation:

Probabilistic Quality Region (PQR) is an interval of quality $(p_1 < p < p_2)$ in which product is accepted with a minimum probability 0.10 and maximum probability 0.95.

Probabilistic Quality Region (PQR) is denoted as $d_{2s} = n_s (p_2 - p_1)$ and is derived from probability of acceptance

$$P_a(p_1 < p < p_2) = \Phi(y_2) + [\Phi(y_1) - \Phi(y_2)][\Phi(y_2)]^{ms}, \; p_1 < p < p_2 \quad \ldots\ldots\ldots\ldots (3.2.1)$$

Where p is the fraction non- conforming in a lot.

Values of the parameters (n_s, k_{as}, k_{rs}) and the Probabilistic Quality Region (PQR), for the unknown sigma variables MDS plan when $m_s = 1$ are tabulated in Table 2.1 according to several combinations of (AQL, LQL) under $\alpha = 0.05 \; and \; \beta = 0.10$ . . Those values when $m_s = 2$ and $m_s = 3$ were reported in Tables 1.2 and 1.3, respectively.

4. Justification for the Use of Probabilistic Quality Region (PQR)

1) Probabilistic Quality Region (PQR) is based on range of quality instead of point-wise description of basic quality levels. So, this method can be adopted in the elementary production process where the stipulated quality level is advisable to fix at a later stage.
2) In point-wise description of quality up to that fixed quality level only, which can tell what variations happened to quality. After that particular point one cannot tell anything about the variations of quality. But in Probabilistic Quality Region (PQR) method one can predict how much variations are there in between the quality levels. Point – wise method is a definite quality level. But in PQR method, one considers two probabilistic quality levels. So when a modern product is launched in industry, then PQR method is quite reasonable for selection of plan parameters.
3) PQR indexed plans give higher probability of acceptance compared with (AQL, LQL) indexed plans. Therefore, indexing a plan with PQR provides a more desirable OC curves, compared with other method.

Considering the advantages, the PQR has major practical advantages in acceptance sampling compared to other methods.

5. Comparison of Sampling Plans

For given values of AQL=2%, $\alpha = 5\%$, LQL=5%, $\beta = 10\%$, one can find from Tables 1.1 the following values of parameters of variables MDS sampling plans (known sigma) are:

$n_\sigma = 22, \; k_{r\sigma} = 1.531, \; k_{a\sigma} = 1.878, \; PQR(d_{2\sigma}) = 0.8800$.

For the same values of the AQL, LQL, α and β, one can find from Table 4, the following parameters of the single sampling plan (for known sigma) are:
$n_\sigma = 35,\ k_\sigma = 1.770, , PQR(d_{2\sigma}) = 1.400$.

It is observed that, in this case, variables MDS plans yield a reduction in sample size than the single variables sampling plan at the same AQL and LQL conditions. It indicates that the variables MDS plan can achieve the same operating characteristics with smaller sample size as compared with single variables sampling plan.

The Probabilistic Quality Region (PQR) obtained in Variable MDS plan is less compared to variable single sampling plan. If the quality region is small, straighter OC can be performed. Therefore, it can be seen that the sampling plan with a smaller value of PQR (d_2) seems to be closer to the ideal OC curve. So compared to variable single sampling plan, the variable MDS sampling plan seems to be closer to the ideal OC curve.
Variable MDS sampling plan through Probabilistic Quality Region (PQR) having $m_\sigma = 1\,(or\ m_s = 1)$ indexed through AQL and LQL for $\alpha = 5\%,$ and $\beta = 10\%$ are given in Table 1 Variables MDS sampling plan through Probabilistic Quality Region (PQR) having $m_\sigma = 2\,(or\ m_s = 2)$ indexed through AQL and LQL for specified $\alpha = 5\%,$ and $\beta = 10\%$ are given in Table 2. Table3 shows the Variable MDS sampling plan through Probabilistic Quality Region (PQR) having $m_\sigma = 3\,(or\ m_s = 3)$ indexed through AQL and LQL for specified $\alpha = 5\%,$ and $\beta = 10\%$ .Tables 4 and 5 shows MDS (0,1) and MDS (0,2) attribute sampling plans through Probabilistic Quality Region (PQR) for various 'm' values indexed through AQL and LQL for specified $\alpha = 5\%,$ and $\beta = 10\%$. In general, a sampling plan having smaller average sample number (ASN) would be more desirable. Table 6 shows the ASNs for single variables sampling plan, double variables plan along with the variables MDS plan with $m_\sigma = 1\,(or\ m_s = 1)$ for certain selected combinations of AQL and LQL having the ratio of $\frac{LQL}{AQL} = 3$ at $\alpha = 0.05\ and\ \beta = 0.10.$ The ASN for single and double variables sampling plans can be found in Sommers (1981). It is seen that the variable MDS through Probabilistic Quality region (PQR) is economically superior to single and double sampling plans in terms of ASN. Variables single sampling plan through Probabilistic Quality Region (PQR) indexed through AQL and LQL are given in Table 7.

6. Conclusions

The purpose of this paper is to develop a new procedure on multiple dependent (or deferred) state sampling plan through Probabilistic Quality Region (PQR) for accepting lot of products whose quality characteristic follows a normal distribution. The proposed variable MDS plan through Probabilistic Quality Region (PQR) provides better protection than conventional single and double variable sampling plans with smaller sample size. This plan is based on a simple statistic and it is easier to apply than double and multiple sampling plans. It is observed that the sampling plan with a smaller value of PQR (d_2) seems to be closer to the ideal shape of OC curve. So compared to variables single sampling plan through PQR, the variable MDS sampling plan through PQR seems to be closer to ideal shape of OC curve. It is seen that the variable MDS through Probabilistic Quality region (PQR) is economically superior to single and double sampling plans in terms of ASN.

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Table 1: Parameters for Variables MDS sampling plans through Probabilistic Quality Region (PQR) having $m_\sigma \ (or\ m_s) = 1$ indexed with AQL and LQL for $\alpha = 5\%$ *and* $\beta = 10\%$.

AQL	LQL	Known sigma				Unknown sigma			
		n_σ	$k_{r\sigma}$	$k_{a\sigma}$	$PQR\ (d_{2\sigma})$	n_s	k_{rs}	k_{as}	$PQR\ (d_{2s})$
0.0010	0.0020	118	2.868	3.016	**0.1180**	647	2.877	3.015	**0.6470**
	0.0025	67	2.794	2.991	**0.1005**	357	2.808	2.990	**0.5355**
	0.0030	46	2.731	2.970	**0.0920**	241	2.575	2.968	**0.4820**
	0.0040	28	2.631	2.936	**0.0840**	144	2.665	2.934	**0.4320**
	0.0050	21	2.551	2.909	**0.0840**	103	2.597	2.906	**0.4120**
	0.0060	16	2.485	2.887	**0.0800**	80	2.542	2.883	**0.4000**
	0.0070	14	2.427	2.868	**0.0840**	66	2.495	2.863	**0.3960**
	0.0080	12	2.376	2.851	**0.0840**	56	2.454	2.845	**0.3920**
	0.0090	11	2.331	2.836	**0.0880**	49	2.417	2.829	**0.3920**
	0.1000	10	2.290	2.822	**0.9900**	44	2.385	2.815	**4.3560**
	0.0120	8	2.217	2.797	**0.0880**	36	2.329	2.789	**0.3960**
	0.0150	7	2.126	2.767	**0.0980**	29	2.259	2.757	**0.4060**
0.0025	0.0050	100	2.565	2.726	**0.2500**	462	2.575	2.725	**1.1550**
	0.0060	61	2.498	2.703	**0.2135**	280	2.515	2.515	**0.9800**
	0.0080	38	2.414	2.675	**0.1900**	170	2.441	2.441	**0.8500**
	0.0100	23	2.303	2.638	**0.1725**	101	2.346	2.346	**0.7575**
	0.0120	18	2.231	2.614	**0.1710**	76	2.286	2.286	**0.7220**
	0.0150	14	2.140	2.583	**0.1750**	55	2.211	2.211	**0.6875**
	0.0200	10	2.018	2.542	**0.1750**	38	2.115	2.115	**0.6650**
	0.0250	8	1.919	2.509	**0.1800**	30	2.039	2.039	**0.6750**
	0.0300	7	1.836	2.481	**0.1925**	24	1.977	1.977	**0.6600**
	0.0350	6	1.764	2.457	**0.1950**	21	1.923	1.923	**0.6825**
0.0050	0.0100	86	2.314	2.488	**0.4300**	345	2.327	2.487	**1.7250**
	0.0120	53	2.242	2.464	**0.3710**	207	2.262	2.462	**1.4490**
	0.0150	33	2.151	2.433	**0.3300**	125	2.183	2.431	**1.2500**
	0.2000	20	2.029	2.392	**3.9000**	73	2.080	2.389	**14.2350**
	0.0250	14	1.931	2.359	**0.2800**	52	2.000	2.354	**1.0400**
	0.0300	11	1.848	2.332	**0.2750**	40	1.934	2.325	**1.0000**
	0.0350	10	1.775	2.307	**0.3000**	32	1.878	2.300	**0.9600**
	0.0400	8	1.711	2.286	**0.2800**	27	1.829	2.277	**0.9450**
0.0100	0.0200	72	2.041	2.231	**0.7200**	245	2.057	2.229	**2.4500**
	0.0250	40	1.942	2.198	**0.6000**	132	1.970	2.196	**1.9800**
	0.0300	27	1.860	2.170	**0.5400**	88	1.900	2.167	**1.7600**
	0.0350	21	1.787	2.146	**0.5250**	65	1.839	2.142	**1.6250**
	0.0400	16	1.723	2.124	**0.4800**	51	1.787	2.119	**1.5300**
	0.0450	14	1.665	2.105	**0.4900**	42	1.741	2.099	**1.4700**
	0.0500	12	1.612	2.087	**0.4800**	35	1.699	2.080	**1.4000**
	0.0600	9	1.518	2.055	**0.4500**	27	1.626	2.047	**1.3500**
0.0200	0.0400	58	1.736	1.947	**1.1600**	165	1.757	1.946	**3.3000**
	0.0450	42	1.678	1.928	**1.0500**	116	1.706	1.926	**2.9000**
	0.0500	32	1.625	1.910	**0.9600**	88	1.661	1.907	**2.6400**
	0.0600	22	1.531	1.878	**0.8800**	58	1.582	1.875	**2.3200**
	0.0700	16	1.448	1.851	**0.8000**	42	1.515	1.846	**2.1000**
	0.0800	13	1.374	1.826	**0.7800**	33	1.456	1.820	**1.9800**
	0.0900	11	1.307	1.803	**0.7700**	27	1.403	1.796	**1.8900**
	0.1000	9	1.245	1.782	**0.7200**	22	1.356	1.774	**1.7600**

Table 1: (continued)

AQL	LQL	Known sigma				Unknown sigma			
		n_σ	$k_{r\sigma}$	$k_{a\sigma}$	$PQR\ (d_{2\sigma})$	n_s	k_{rs}	k_{as}	$PQR\ (d_{2s})$
0.0400	0.080	45	1.389	1.629	**1.8000**	102	1.415	1.627	**4.0800**
	0.090	32	1.321	1.607	**1.6000**	71	1.358	1.604	**3.5500**
	0.100	25	1.259	1.586	**1.5000**	53	1.306	1.582	**3.1800**
	0.110	20	1.201	1.566	**1.4000**	42	1.258	1.562	**2.9400**
	0.120	16	1.147	1.548	**1.2800**	34	1.215	1.543	**2.7200**
	0.130	14	1.097	1.531	**1.2600**	29	1.174	1.525	**2.6100**
	0.140	12	1.048	1.515	**1.2000**	25	1.136	1.508	**2.500**
	0.150	11	1.002	1.500	**1.2100**	22	1.100	1.492	**2.4200**

Table 2: Variable MDS sampling plan through Probabilistic Quality Region (PQR) having $m_\sigma\ (or\ m_s) = 2$ indexed through AQL and LQL for $\alpha = 5\%\quad and\ \beta = 10\%$.

AQL	LQL	Known sigma				Unknown sigma			
		n_σ	$k_{r\sigma}$	$k_{a\sigma}$	$PQR\ (d_{2\sigma})$	n_s	k_{rs}	k_{as}	$PQR\ (d_{2s})$
0.001	0.002	118	2.797	3.000	**0.1180**	644	2.815	3.000	**0.6440**
	0.005	20	2.378	2.871	**0.0800**	102	2.474	2.871	**0.4080**
	0.010	10	2.033	2.765	**0.0900**	44	2.224	2.764	**0.3960**
	0.020	5	1.656	2.649	**0.0950**	22	1.977	2.646	**0.4180**
	0.030	4	1.417	2.575	**0.1160**	16	1.830	2.572	**0.4640**
0.005	0.010	85	2.231	2.47	**0.4250**	343	2.258	2.469	**1.7150**
	0.025	14	1.724	2.314	**0.2800**	51	1.864	2.313	**1.0200**
	0.050	7	1.288	2.179	**0.3150**	21	1.571	2.177	**0.9450**
0.01	0.020	71	1.949	2.210	**0.7100**	244	1.983	2.210	**2.4400**
	0.050	12	1.383	2.036	**0.4800**	35	1.556	2.035	**1.4000**
	0.100	5	0.881	1.882	**0.4500**	14	1.228	1.878	**1.2600**
0.02	0.040	58	1.634	1.925	**1.1600**	164	1.677	1.924	**3.2800**
	0.100	9	0.985	1.725	**0.7200**	22	1.201	1.723	**1.7600**
	0.200	4	0.377	1.538	**0.7200**	8	0.815	1.534	**1.4400**
0.04	0.080	45	1.272	1.604	**1.8000**	101	1.677	1.924	**4.0400**
	0.200	7	0.493	1.364	**1.1200**	13	1.201	1.723	**2.0800**
	0.400	3	0.001	1.106	**1.0800**	4	0.815	1.534	**1.4400**
0.05	0.100	40	1.142	1.49	**2.0000**	84	1.202	1.490	**4.2000**
	0.250	6	0.302	1.232	**1.2000**	10	0.602	1.229	**2.0000**
	0.500	3	0.001	0.92	**1.3500**	3	0.001	0.935	**1.3500**

Table 3: Variables MDS sampling plan through Probabilistic Quality Region (PQR) having $m_\sigma\ (or\ m_s) = 3$ indexed with AQL and LQL for $\alpha = 5\%\quad and\ \beta = 10\%$.

AQL	**LQL**	**Known sigma**				**Unknown sigma**			
		n_σ	$k_{r\sigma}$	$k_{a\sigma}$	$PQR\ (d_{2\sigma})$	n_s	k_{rs}	k_{as}	$PQR\ (d_{2s})$
0.001	0.002	126	2.341	2.993	**0.1260**	687	2.763	2.993	**0.6870**
	0.005	22	2.223	2.855	**0.0880**	108	2.375	2.855	**0.4320**
	0.010	10	1.803	2.740	**0.0900**	46	2.100	2.740	**0.4140**
	0.020	6	1.344	2.616	**0.1140**	24	1.832	2.615	**0.4560**
	0.030	4	1.052	2.536	**0.1160**	17	1.675	2.536	**0.4930**
0.005	0.010	91	2.155	2.462	**0.4550**	365	2.200	2.462	**1.8250**
	0.025	15	1.538	2.294	**0.3000**	54	1.758	2.294	**1.0800**
	0.050	7	1.007	2.150	**0.3150**	22	1.437	2.149	**0.9900**
0.01	0.020	76	1.867	2.202	**0.7600**	260	1.922	2.202	**2.6000**
	0.050	13	1.178	2.014	**0.5200**	37	1.444	2.014	**1.4800**
	0.100	6	0.566	1.848	**0.5400**	14	1.086	1.848	**1.2600**
0.02	0.040	62	1.543	1.915	**1.2400**	174	1.611	1.915	**3.4800**
	0.100	10	0.753	1.700	**0.8000**	24	1.080	1.700	**1.9200**
	0.200	4	0.011	1.499	**0.7200**	9	0.650	1.498	**1.6200**
0.04	0.080	48	1.168	1.592	**1.9200**	107	1.256	1.592	**4.2800**
	0.200	7	0.219	1.334	**1.1200**	13	0.627	1.334	**2.0800**

	0.400	3	0.001	1.061	**1.0800**	4	0.008	1.064	**1.4400**
0.05	0.100	43	1.0330	1.479	**2.1500**	90	1.128	1.478	**4.5000**
	0.250	6	0.010	1.200	**1.2000**	11	0.445	1.200	**2.2000**
	0.500	3	0.001	0.877	**1.3500**	3	0.001	0.887	**1.3500**

Table 4: MDS (0, 1) attribute sampling plan through Probabilistic Quality Region (PQR) for various 'm' values indexed with AQL and LQL for $\alpha = 5\%$ *and* $\beta = 10\%$.

m	np_1	np_2	PQR(d_2)
1	0.30781	2.52796	2.22015
2	0.26768	2.32537	2.05769
3	0.23448	2.30489	2.07041
4	0.20734	2.30282	2.09548
5	0.18525	2.30261	2.11736
6	0.16724	2.30259	2.13535
7	0.15248	2.30259	2.15011
8	0.14028	2.30259	2.16231
9	0.13010	2.30259	2.17249
10	0.12154	2.30259	2.18105
11	0.11426	2.30259	2.18833
12	0.10803	2.30259	2.19456
13	0.10264	2.30259	2.19995
14	0.09796	2.30259	2.20463
15	0.93860	2.30259	1.36399
∞	0.05129	2.30259	2.25130

Table 5: MDS (0, 2) attribute sampling plans with Probabilistic Quality Region (PQR) for various 'm' values indexed through AQL and LQL for $\alpha = 5\%$ *and* $\beta = 10\%$

m	np_1	np_2	PQR (d_2)
1	0.2480	2.6610	2.4130
2	0.1790	2.2480	2.0690
3	0.1490	2.3030	2.1540
4	0.1310	2.3030	2.1720
5	0.1180	2.3030	2.1850
6	0.1090	2.3030	2.1940
7	0.1030	2.3030	2.2000
8	0.0970	2.3030	2.2060
9	0.0930	2.3030	2.2100
10	0.0890	2.3030	2.2140

Table 6: Comparison of ASN for variables single, double and MDS sampling plans

AQL	LQL	Average sample number					
		Known sigma			Unknown sigma		
		Single	Double	MDS	Single	Double	MDS
0.0010	0.0030	74	59.4	46	381	302.4	241
0.0025	0.0075	62	50.1	38	267	214.2	170
0.0050	0.0150	53	43.0	33	196	157.6	125
0.0100	0.0300	44	35.0	27	137	107.5	88
0.0200	0.0600	35	28.3	22	89	70.4	58

Table 7: Variable Single sampling plans with Probabilistic Quality Region (PQR) indexed through AQL and LQL

AQL	LQL	Known sigma			unknown sigma		
		n_σ	k_σ	$PQR\ (d_{2\sigma})$	n_s	k_s	$PQR\ (d_{2s})$
0.0010	0.0020	191	2.97	**0.1910**	1032	2.97	**1.0320**
	0.0025	107	2.93	**0.1605**	567	2.93	**0.8505**
	0.0030	74	2.90	**0.1480**	381	2.90	**0.7620**
	0.0040	45	2.84	**0.1350**	226	2.84	**0.6780**
	0.0050	33	2.80	**0.1320**	160	2.80	**0.6400**
	0.0060	26	2.77	**0.1300**	124	2.77	**0.6200**
	0.0070	22	2.73	**0.1320**	102	2.73	**0.6120**
	0.0080	19	2.71	**0.1330**	87	2.71	**0.6090**
	0.0090	17	2.68	**0.1360**	76	2.68	**0.6080**
	0.0100	15	2.66	**0.1350**	67	2.66	**0.6030**
	0.0120	13	2.62	**0.1430**	55	2.62	**0.6050**

	0.0150	11	2.57	**0.1540**	44	2.57	**0.6160**
0.0025	0.0050	161	2.68	**0.4025**	736	2.68	**1.8400**
	0.0060	99	2.64	**0.3465**	443	2.64	**1.5505**
	0.0075	62	2.60	**0.3100**	267	2.60	**1.3350**
	0.0100	38	2.54	**0.2850**	157	2.54	**1.1775**
	0.0120	29	2.50	**0.2755**	117	2.50	**1.1115**
	0.0150	22	2.45	**0.2750**	85	2.45	**1.0625**
	0.0200	16	2.38	**0.2800**	59	2.38	**1.0325**
	0.0250	12	2.33	**0.2700**	45	2.33	**1.0125**
	0.0300	10	2.29	**0.2750**	37	2.29	**1.0175**
	0.0350	9	2.25	**0.2925**	31	2.25	**1.0075**
0.0050	0.0100	138	2.44	**0.6900**	547	2.44	**2.7350**
	0.0120	85	2.40	**0.5950**	327	2.40	**2.2890**
	0.0150	53	2.35	**0.5300**	196	2.35	**1.9600**
	0.0200	32	2.28	**0.4800**	114	2.28	**1.7100**
	0.0250	23	2.23	**0.4600**	79	2.23	**1.5800**
	0.0300	18	2.19	**0.4500**	61	2.19	**1.5250**
	0.0350	15	2.15	**0.4500**	49	2.15	**1.4700**
	0.0400	13	2.11	**0.4550**	41	2.11	**1.4350**
0.0100	0.0200	116	2.17	**1.1600**	388	2.17	**3.8800**
	0.0250	64	2.12	**0.9600**	208	2.12	**3.1200**
	0.0300	44	2.08	**0.8800**	137	2.08	**2.7400**
	0.0350	33	2.04	**0.8250**	100	2.04	**2.5000**
	0.0400	26	2.00	**0.7800**	78	2.00	**2.3400**
	0.0450	22	1.97	**0.7700**	64	1.97	**2.2400**
	0.0500	19	1.94	**0.7600**	54	1.94	**2.1600**
	0.0600	15	1.89	**0.7500**	41	1.89	**2.0500**
0.0200	0.0400	94	1.88	**1.8800**	259	1.88	**5.1800**
	0.0450	67	1.85	**1.6750**	182	1.85	**4.5500**
	0.0500	52	1.82	**1.5600**	137	1.82	**4.1100**
	0.0600	35	1.77	**1.4000**	89	1.77	**3.5600**
	0.0700	26	1.73	**1.3000**	64	1.73	**3.2000**
	0.0800	21	1.69	**1.2600**	50	1.69	**3.0000**
	0.0900	17	1.65	**1.1900**	40	1.65	**2.8000**
	0.1000	15	1.62	**1.2000**	34	1.62	**2.7200**