International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

Unveiling the Complexities of Mathematical Permutations: Innovations in Summation Techniques and Novel Approaches to Arrangement Problems

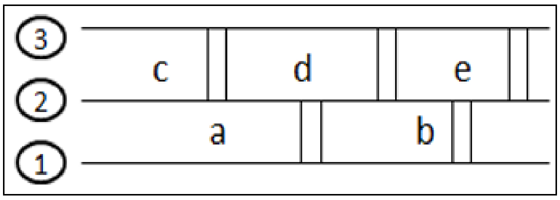
Subham Dutta

Abstract: ***Permutation is the Mathematical technique of arrangement. This paper is the study based on the summation of the numbers which is arranged some digits may be in all distinct or some of digits same. The formula provides a more efficient and structured method for computing summations in permutation, reducing complexity and improving computational efficiency. This study contributes to mathematical problem-solving by offering a new perspective on structured summation in permutation-based calculations.***

Keywords: Permutation, Arrangement, Summation, Constant

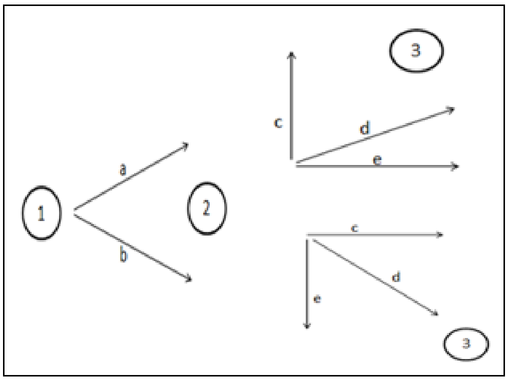
1. Introduction

Among various problems in mathematics, the challenge of arrangement is particularly significant. Often, we encounter difficulties where arranging elements in a specific order becomes constrained. For example, using the digits 1, 2, 3, and 4, we can form six unique three-digit numbers: 123, 132, 231, 213, 321, and 312. As the number of elements increases, arranging them becomes more time-consuming. The chapter on permutations in mathematics provides a systematic way to solve such problems efficiently.



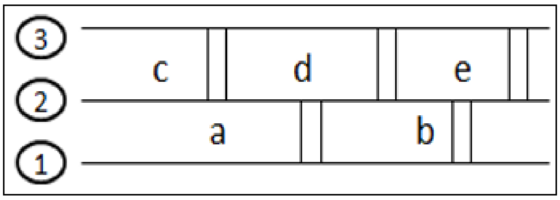
2. Discussion

Permutation: From several elements taking few of them or taking whole of them, if we make a diagram, the process of making is called -‘Permutation’.



Principle of Permutation:
If, to move from 1st floor to 2nd floor of a 3 storied building, it has two staircases - a and b and to move from 2nd floor to 3rd floor, it has 3 staircase -c, d and e. Now, if a person wants to move from first floor to 3rd floor, how many options then he will get?

To solve the above-mentioned problem, the chapter of ‘Permutation’ is used.



As for example, if, a person - X, wants to move from 1st floor to 3rd floor, X is capable to reach 3rd floor by using ac, ad , ae , bc , bd be – any one of the ways. That is to say, the total numbers of ways are = 6. Now, accordingly to the principle of Permutation, the way of reaching the 3rd floor from 1st floor is = (the way of reaching 2nd floor) × (the way of reaching from 2nd to 3rd floor) that is to say = $2 \times 3 = 6$ ways.

Examples like this can be considered as complex preliminarily, but with the help of chapter – “Permutation”, the sum can be solved spontaneously.

Symbol:
Now, to understand 1,2,3…………etc. up to n, these whole numbers there is a particular icon we use to indicate Permutation. The icon is known as ‘Factorial’. That is to say, the meaning of multiplication of , 1,2,3……………… up to 10 – is Factorial 10.

The symbol of this Factorial 10 is = 10!

Therefore, $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$

Note: There is no meaning of zero’s factorial. That’s why it is considered as 1. $0! = 1$.

Main subject of the Calculation:
Starting from my school age till having a mathematical vocation, I have come to this decision that many times, in front of students, we need to be limited to teach some mathematics. Also, diversely some solutions are complex enough by nature and some of them requires maximum intelligence to be solved. Therefore, sometimes a critical problem, regarding the solution, occurs. That's why, the concept of 'Formula' is invented. In present times, $(a + b)^2$ means $a^2+2ab+b^2$ to everyone as it's the formula of $(a + b)^2$. But, the fact is that maximum students do not know that actually the formula came from the multiplication of two (a + b). Therefore, sometimes it becomes necessary to understand the mathematics by only using the formulas. A problem like this is now we are going to discuss about.

Problem:
Using 1,2,3,4 – these 4 digits create as many numbers are possible and adding them, calculate their total value.

Solution:
There are 4 digits.

That's why using above mentioned 4 digits no, we can create 4! = 24 numbers

Now, every numbers of 1^{st} place will come 24/4 = 6 times. Therefore, 24 numbers' first place' addition number will be = 6(1+2+3+4) = 60.

Just like the same, 10^{th} places', 100^{th} places', 1000^{th} places' numbers' addition will be also = 60

Therefore, the total addition will be = $60 \times 10^0 + 60 \times 10^1 + 60 \times 10^2 + 60 \times 10^3 = 60 + 600 + 6000 + 60000 = 66660$.

At last we are succeeded to solve the problem but as we can see, the solve is little bit lengthy and based on intelligence. I was really searching the solution of this kind of problem. After this I am go to tell my invented results that is related to this problem. At first lets' get the first decision, then we will find the easy ways to promote the last formula.

First stage of Research :
When the topic firstly came as a topic of my rearch, I granted some constant numbers for having a solution. It is different for different numbers digit. I delivered some conditions in the first stage of my research.

Condition No.: 1
The numbers should be different from each other

Condition No.: 2
Once a number is used, it cannot be counted again.

Fulfilling the above-mentioned conditions, we can create a table of constant numbers like this:

Numbers	Constant
1	1
2	11
3	222
4	6666
5	266664
6	13333320
7	799999920
8	55999999440
9	4479999995520
10	403199999959680

Using these constant numbers, solution of any problem gets easy. The numbers of before mentioned problem's solution were = 4

That is to say, the constant number will be = 6666

Now, to calculate the adding number – we need to calculate that constant numbers' adding result with that constant number's multiplication and the result is equal to that 4 numbered digits, which we got earlier.

Therefore, the addition is = 6666 x (1+2+3+4) = 6666 x 10 = 66660.

To prove this result, we can go through over the previous rule-based solution.

We can use this constant number for any field.

Therefore, my experiments first stage is –

If every number is different and if every number is used more than one time, then, the created numbers addition from the early mentioned numbers' will be –

CONSTANT NUMBERS (from the table) x [the addition of the digits]

Second stage of the Research:
After inventing the first stage of my research, the thought that came in my mind that is –

If the given digit is not different, rather be same, then what will be the solution?

The solution of the specific problem took not much time, because a little portion of the solution is already present in Higher Mathematics.

Problem:
1, 1, 2, 2 – from these how many four digits numbers can be created?

Solution:
As, the digit - 1 is used twice and digit – 2 is also used twice , that's why from 1 , 1 ,2 ,2 these 4- numbered digits created numbers will be = $\frac{4!}{2! \; X \; 2!} = 6$

As we've already got the solution of above-mentioned problem, just like that, if we want to calculate the adding

Paper ID: SR25401114501 DOI: https://dx.doi.org/10.21275/SR25401114501

number, then using the previous formula , the result is to be divided the number, which is used more than one time.

Therefore, the adding result of the numbers, that we've got from 1,1,2,2 – these 4 – numbered digits, will be = $\frac{6666\times(1+1+2+2)}{2!\times 2!} = 9999$

Therefore, I solved the critical issue of my second stage experiment, as I have shown the formula – how to solve the problem when one specific number comes more than one time.

The formula of my 2nd stage research now is to be changed a little bit. Now, the formula will be –

The amount of how many – numbered digits are to be created and how many time they are used, if we know these two factors, then the addition made by those numbers will be –

$$\frac{Constant\ numbers\ (from\ the\ table)}{(how\ many\ time\ thaq\ same\ number\ is\ used, that\ number's factorial)} \times (the\ adding\ result\ of\ those\ number)$$

Again, showing as a solution of this problem, we are going to prove the formula –

Problem:
Using the numbers 3 and 4 – create as many numbers and calculate the adding result of them. [3 can be used 3 times and 4 can be used one time.]

Solution:
3 can be used 3 times and 4 can be used one time.
Therefore, the numbers are = 3,3,3,4. The adding result is = $\frac{6666\times(3+3+3+4)}{3!} = 14443$
[Here, the division is done by 3! because the number 3 is used three times.]

Third stage of the Research :
After establishing a better form of the formula in my second stage research, after that a thought came in my mind that, may be because of my formula this process has become easier, but one has to remember the exact, constant numbers and that is quite difficult to make it simple I felt the need to invent one more formula. Though I tried hard, but I was unable to create any relation between the constant numbers. After someday I finally got success to create the relation between these constant numbers.

Rules:
Suppose, I know the 5-numbered constant number but I cannot remember the 6-numbered constant number. Now, the number which comes most of the time in the 5 -numbered constant number, that digit is to be written once more and that is to be multiplied. [As it is 5 -numbered constant numbers]

Therefore, 5 -numbered constant number is – 266664.

Here, the digit 6 has come most of the time. So, 6 is to be written once again.

Therefore, 2666664

Now, the mentioned digit is to be multiplied by 5 and we will get the 6 -numbered constant number.

So, the constant number will be = 266666 X 5 = 13333320

If we go through the table, we can easily understand that 6-numbered constant number has come correct.

That's how, step-by-step I invented the formula to remember any constant number.

Lets' see some more example:
Suppose, we know the 3 – numbered digits constant number, that is = 222
∴Now, to calculate 4 – numbered digit's constant number 2222 is to be multiplied by 3.
∴Therefore, 4 numbered digits' constant number is = 2222X3 = 6666
This is also can be proved correct if we follow the table.
Now, 4 – numbered digit's constant number will be = 6666X4 = 266664.
∴ That's how, using any new formula I finally got success to establish a relation between constant numbers.

3. Final Stage of the Research

The thought of fourth stage's research came into mind recently and I got success too, regarding this matter. The thought was about _how to break the bonding of those step-by-step formula.

After a long time, I finally invented a formula to solve this complex problem.

The problem is –

If, using n- numbered digits we create n- numbered digits, and to calculate the adding result, the result will be =

$$\frac{(\mathbf{1111 ....}\ \boldsymbol{n\ times})\times(\boldsymbol{n}-\mathbf{1})!}{(\boldsymbol{m_1})!\times(\boldsymbol{m_2})!\times.........(\boldsymbol{m_r})!} \times (\boldsymbol{the\ adding\ result\ of\ the\ numbers})$$

[Note : the n – numbered digits which are used many times, they are represented one by one as $m_1, m_2, \dots \dots \dots\ m_r$]

Prove:
Suppose, 3,2,6,8 using these 4 – numbered digits create as many numbers you can and calculate their adding result. [every digit has to be used for only one time]

Research's Result:
Every digit has to be used once. $m_1 = m_2 = m_3 = m_4 = 1, n = 4$

Therefore, the adding result will be = $1111 \times 3! \times 1! \times 1! \times 1! \times 1! \times (3 + 2 + 6 + 8) = 126654$

After creating the numbers, their adding result will be =

3268	2368	6238	8236
3628	2386	6283	8263
3286	2863	6382	8632
3682	2836	6328	8623
3826	2638	6823	8326
3862	2683	6832	8362

$= 21552 + 15774 + 38886 + 50442 = 126654$

It's clear that the result is equal to the research's result.

Problem:

Suppose, if we create a permutation of 12232 -this number, then how many numbers we will get , calculate their adding result.

Solution:

Research's result:

The above-mentioned number has number1-one time, number 2-two times and number 3-three times.

Suppose: If we create permutation of 12332-this number, the total number will be $= \frac{5!}{2!\times 2!} = 30$s.

$\therefore Therefore, n = 5, m_1 = 1, m_2 = 2, m_3 = 3$

$\therefore Therefore, the\ adding\ result\ will\ be$

$$= \frac{11111 \times 4!}{1! \times 2! \times 2!} \times (1 + 2 + 2 + 3 + 3)$$
$$= 73326$$

After creating the numbers their adding result will be =

12332	23231	32182	21233	33221
12323	23213	32123	31232	33212
12233	23321	32321	31223	33122
13223	23312	32312	31322	22331
23132	32231	21332	13232	22313
23123	32213	21323	13322	22133

= 96366 + 157521 + 171543 + 141564 + 166332 = 733326

It's clear that the result is equal to the research's result.