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On the Topological Indices of Hexagonal Network (HX_6), Complete Bi-Partite and Complete Graphs

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Abstract: ***A graph in which any two distinct points are adjacent is called a complete graph. A complete bi-partite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset and every possible edge that could connect vertices in different subsets is part of graph and is denoted by $K_{m,n}$. In this paper some d_2-distance degree, eccentricity; sum-degree based topological indices are studied for hexagonal network with dimension six, complete bi-partite and complete graphs.***

Keywords: Complete graph, complete bi-partite graph, degree, distance-degree, eccentricity, hexagonal network, Sanskruti index, sum-degree and topological index

1. Introduction

Let G be a simple, finite, connected graph with vertex set V(G) and edge set E(G). The degree of a vertex u ∈ V(G) is denoted by d_u and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv [1]. A topological index is a numerical parameter mathematically derived from the graph structure. Topological indices based on degrees play a major role in the field of chemical graph theory among several classes of topological indices [2]. Hexagonal network is symbolized by HX_n, where n is the number of vertices in each side of hexagon. Hexagonal networks are multiprocessor interconnection network based on regular triangular tessellations [3]. The 2-dimensional hexagonal mesh HX_2 is composed of six triangles. If we add a layer of triangles around this 2-dimensional mesh, we obtain the 3-dimensional mesh HX_3 [4]. The strong metric dimension of triangular tessellations are studied in [5]. Honeycomb, hexagonal networks are known as natural architectures as they bear resemblance to atomic or molecular lattice structure [6]. Hexagonal meshes like HX_2, HX_3 were constructed and studied for topological indices in [7]. Motivated by the definitions of Gourava indices and their applications, the neighborhood Gourava indices were defined in [8-9]. The eccentric atom-bond sum connectivity index of complete graph has the lowest possible value zero [10]. Multiple atom-bond connectivity index of some grid graph were studied in [11]. In 2017, Naji et al. [12] defined first and second leap Zagreb indices by taking 2-distance degree of vertices instead of taking degree. The first and second leap hyper Zagreb indices were proposed by Kulli [13]. The first and second leap Gourava indices are defined as [14]

$$LGO_1(G)=\sum_{uv\in E(G)}[d_2(u)+d_2(v)+d_2(u)d_2(v)]. \quad (1)$$

$$LGO_2(G)=\sum_{uv\in E(G)}[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]. \quad (2)$$

The new versions of first and second leap Zagreb indices are defined as

$$LM_1^*(G)=\sum_{uv\in E(G)}[d_2(u)+d_2(v)]. \quad (3)$$

$$LM_2(G)=\sum_{uv\in E(G)}[d_2(u)\times d_2(v)], \quad (4)$$

where $d_2(v)$ is the 2-distance degree of v, defined as the number of vertices which are at distance 2 from v in G.The distance d(u,v) between two vertices u and v of a graph G is defined as the length of shortest path connecting them. For any vertex u ∈ V(G), its eccentricity ecc(u) is defined as

$$Ecc(u) = \max\{d(u,v)\forall u \in V(G)\}.$$

Some new modified versions of Zagreb indices [15-21] are expressed as follows:

$$M_1^*(G)=\sum_{uv\in E(G)}[ecc(u)+ecc(v)]. \quad (5)$$

$$M_1^{**}(G)=\sum_{v\in V(G)}ecc(v)^2. \quad (6)$$

$$M_2^*(G)=\sum_{uv\in E(G)}ecc(u)\times ecc(v). \quad (7)$$

The eccentric connectivity index [22] of molecular graph is defined as

$$ECI(G)=\sum_{u\in V(G)}d_u\times ecc(u), \quad (8)$$

where ecc (u) is the eccentricity of vertex u of a graph G.

The Sankruti index was introduced by Hossamani in [23] as

$$S(G)=\sum_{uv\in E(G)}\left(\frac{S_u\times S_v}{S_u+S_v-2}\right)^3. \quad (9)$$

The sum-degree based geometric-arithmetic index can be defined as [24-25]

$$GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_u\times S_v}}{S_u+S_v}, \quad (10)$$

where S_u is the sum of degrees of all adjacent vertices to a vertex u.
Gourava Sombor index [26-27] and fourth ABC index are defined as

$$GSO(G)=\sum_{uv\in E(G)}\sqrt{(d_u+d_v)^2+(d_u\times d_v)^2}. \quad (11)$$

$$ABC_4(G)=\sum_{uv\in E(G)}\frac{\sqrt{S_u+S_v-2}}{S_uS_v}, \quad (12)$$

where $S_u=\sum_{uv\in E(G)}d_v$ and $S_v=\sum_{uv\in E(G)}d_u$.

Hexagonal network is symbolized by HX_n, where n is the number of vertices in each side of hexagon. By algebraic method there are five types of edges based on the degree of the end vertices of each edge as: E_{34}, E_{36}, E_{44}, E_{46}, and E_{66}. There are $|V| = 3n^2-3n+1$ number of vertices and $|E|= 9n^2-15n+6$ edges and $\Delta=6, \delta=3$ in HX_6 [28-29]. There are 240 edges which match the HX_6 structure and twelve types of edges based on the sum-degree of vertices and the vertices are either of degree 3, 4 or 6 [30-31].

Eccentricity pairs (e_u,e_v) describe how far apart vertices are, degree pairs (d_u,d_v) describe how well connected they are. The eccentricity ecc(v) of v is the distance between v and a vertex farthest from v in G. Eccentricities of vertices for HX_6 are: corner vertices has eccentricity 3, edge vertices has eccentricity 4 and interior vertices eccentricity equal to 6.

Let $K_{m,n}$ be a complete bi-partite graph with m+n vertices as $|V_1|=m,|V_2|=n$, $V(K_{m,n})=V_1\cup V_2$.The total number of edges in $K_{m,n}$ are $|E|=m\times n$.The degrees are such that each vertex in the first set (size m) has a degree of n, each vertex in the second set (size n) has a degree of m. In $K_{m,n}$ graph 2-distane degree of a vertex in set 1 is (m-1)+n and 2-distance degree of a vertex in set 2 is (n-1)+m. All the vertices in a complete bi-partite graph have eccentricity is equal to 2 that is ecc(u) = 2∀u∈V(G).

Let K_n be a complete graph with n vertices. In this graph corner vertices has degree 3 and there are six vertices of degree 3. Every vertex of K_n is incident with n-1 edges and $|V|=n$, $d_u=n-1$, $|E|=\frac{n(n-1)}{2}$.Since K_n is symmetric, all vertices have the same sum-degree as $S_u=S_v=(n-1)^2$.Each vertex has eccentricity 1 for complete graph K_n of order $n>2$, since diameter of graph K_n =1.For any vertex v in a complete graph, $d_2(v)=0$.There is no vertex that is exactly at distance 2 from any given vertex because all vertices are directly adjacent.

All the symbols and notations used in this paper are standard and taken from standard books of graph theory [32-34]. In this paper $LGO_1(G)$, $LGO_2(G)$, $LM_1^*(G)$, $LM_2(G)$, $M_1^*(G)$, $M_1^{**}(G)$,$M_2^*(G)$,ECI(G), S(G), $GA_5(G)$,GSO(G) and $ABC_4(G)$ are studied for hexagonal network with dimension six, complete bi-partite and complete graphs.

2. Materials and Methods

A molecular graph is a graph in which the vertices correspond to the atoms and the edges to the bonds of a molecule. The molecular graphs of HX_6, complete bi-partite $K_{3,4}$ and complete K_4 graphs are given in figures (1), (2) and (3) respectively. It is observed from figure of HX_6 there are five types of edges based on (d_u,d_v) and twelve types edges for sum-degree of end vertices. Edge partition, degree, frequency, eccentricity partition and sum-degree vertex partition of HX_6 obtained from figure (1) are given in tables (1), (2) and (3).

Results and Discussion

1) Hexagonal network HX_6
Leap degree:

Theorem 1.1: First leap Gourava index of hexagonal network HX_6 is 1546n+226.
Proof. By using definition of first leap Gourava index and d_2-distance degree from figure (1), we have

$LGO_1(HX_6)=\sum_{uv\in E(G)}[d_2(u)+d_2(v)+d_2(u)d_2(v)]$
$=|E_1|[d_2(u)+d_2(v)+d_2(u)d_2(v)]+|E_2|[d_2(u)+d_2(v)+d_2(u)d_2(v)]+|E_3|[d_2(u)+d_2(v)+d_2(u)d_2(v)]+|E_4|[d_2(u)+d_2(v)+d_2(u)d_2(v)]+|E_5|[d_2(u)+d_2(v)+d_2(u)d_2(v)]$
$=2n[3+4+3\times 4]+2n[3+6+3\times 6]+(3n+2)[4+4+4\times 4]+(11n+1)[4+6+4\times 6]+(21n+3)[6+6+6\times 6]$
=1546n+226.

Theorem 1.2: Second leap Gourava index of hexagonal network HX_6 is 12588n+1792.
Proof. By using definition of second leap Gourava index and d_2-distance degree of each vertex from figure (1), we have

$LGO_2(HX_6)=\sum_{uv\in E(G)}[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]$
$=|E_1|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]+|E_2|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]+|E_3|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]+|E_4|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]+|E_5|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]$
$=2n[(3+4)\times 3\times 4]+2n[(3+6)\times 3\times 6]+(3n+2)[(4+4)\times 4\times 4]+(11n+1)[(4+6)\times 4\times 6]+(21n+3)[(6+6)\times 6\times 6]$
=12588n+1792.

Eccentricity:

Theorem 1.3: $M_1^*(HX_6)$ of hexagonal network HX_6 is 2148.
Proof. By using equation (5) and table (2), we have

$M_1^*(HX_6)=\sum_{uv\in E(G)}[ecc(u)+ecc(v)]$
$=|E_1|[ecc(u)+ecc(v)]+|E_2|[ecc(u)+ecc(v)]+|E_3|[ecc(u)+ecc(v)]+|E_4|[ecc(u)+ecc(v)]+|E_5|[ecc(u)+ecc(v)]$
$=36[3+4]+72[3+6]+48[4+4]+72[4+6\}+12[6+6]=2148$.

Theorem 1.4: Eccentric connectivity index of hexagonal network HX_6 is 2514.
Proof. By using equation (8) and tables (1) and (2), we have

$ECI(HX_6)=\sum_{u\in V(G)} d_u\times ecc(u)$
$=|E_1|[d_u\times ecc(u)]+|E_2|[d_u\times ecc(u)]+|E_3|[d_u\times ecc(u)]$
$=6\times 3\times 3+30\times 4\times 4+55\times 6\times 6$
= 2514.

Theorem 1.5: New modified version second Zagreb index of hexagonal network HX_6 is 4656.
Proof. By using equation (7) and table (2), we have

$M_2^*(HX_6)=\sum_{uv\in E(G)}[ecc(u)\times ecc(v)]$
$=|E_1|[ecc(u)\times ecc(v)]+|E_2|[ecc(u)\times ecc(v)]+|E_3|[ecc(u)\times ecc(v)]+|E_4|[ecc(u)\times ecc(v)]+|E_5|[ecc(u)\times ecc(v)]$
$=36(3\times 4)+72(3\times 6)+48(4\times 4)+72(4\times 6)+12(6\times 6)=4656$.

Sum-degree:

Theorem 1.6: Sankruti index of hexagonal network HX_6 is 1017953.
Proof. By using equation (9) and table (3), we have

$S(HX_6)=\sum_{uv\in E(G)}\left(\frac{S_u\times S_v}{S_u+S_v-2}\right)^3$
$=12\left(\frac{14\times 19}{14+19-2}\right)^3+12\left(\frac{19\times 20}{19+20-2}\right)^3+6(n-5)\left(\frac{20\times 20}{20+20-2}\right)^3+12\left(\frac{19\times 29}{19+29-2}\right)^3+12\left(\frac{19\times 32}{19+32-2}\right)^3+12(n-4)\left(\frac{20\times 32}{20+32-2}\right)^3+6\left(\frac{14\times 29}{14+29-2}\right)^3+12\left(\frac{29\times 32}{29+32-2}\right)^3+6\left(\frac{29\times 36}{29+36-2}\right)^3+$

$12(n-3)(\frac{32\times36}{32+36-2})^3 + 6(n-4)(\frac{32\times32}{32+32-2})^3 + (9n^2-51n+72)(\frac{36\times36}{36+36-2})^3$
$=1017953$.

Theorem 1.7: Gourava Sombor index of hexagonal network HX_6 is 7778.
Proof. GSO(G) of hexagonal network HX_6
$GSO(HX_6) = \sum_{uv\in E(G)} \sqrt{(d_u+d_v)^2+(d_u\times d_v)^2}$
$=|E_1|\sqrt{(3+4)^2+(3\times4)^2} + |E_2|\sqrt{(3+6)^2+(3\times6)^2} + |E_3|\sqrt{(4+4)^2+(4\times4)^2} + |E_4|\sqrt{(4+6)^2+(4\times6)^2} + |E_5|\sqrt{(6+6)^2+(6\times6)^2}$
$=12\sqrt{(3+4)^2+(3\times4)^2} + 6\sqrt{(3+6)^2+(3\times6)^2} + (6n-18)\sqrt{(4+4)^2+(4\times4)^2} + (12n-24)\sqrt{(4+6)^2+(4\times6)^2} + (9n^2-33n+30)|\sqrt{(6+6)^2+(6\times6)^2} = 7778$.

2) Complete bi-partite graph ($K_{m,n}$)
Leap degree:

Theorem 2.1: First leap Gourava index of complete bi-partite graph is $mn(m^2+n^2+2mn-1)$.
Proof. By using equation (1) and d_2-distance degree of each vertex from figure (2), we have
$LGO_1(K_{m,n})= \sum_{uv\in E(G)}[d_2(u)+d_2(v)+d_2(u)d_2(v)]=|E|[d_2(u)+d_2(v)+d_2(u)d_2(v)]$
$=mn[(m+n-1)+(m+n-1)+(m+n-1)(m+n-1)]$
$= mn(m^2+n^2+2mn-1)$.

Theorem 2.2: Second leap Gourava index of complete bi-partite graph is $2mn(m+n-1)^3$.
Proof. By using definition of second leap Gourava index (2) and d_2-distance degree of each vertex from figure (2), we have
$LGO_2(K_{m,n})= \sum_{uv\in E(G)}[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]=|E|[(d_2(u)+d_2(v))\times d_2(u)d_2(v)]$
$=mn[((m+n-1)+(m+n-1))\times(m+n-1)(m+n-1)]$
$= 2mn(m+n-1)^3$.

Eccentricity:

Theorem 2.3: New modified version of first Zagreb index of complete bi-partite graph is 4mn.
Proof. By using equation (5) and figure (2), we have
$M_1^*(K_{m,n}) = \sum_{uv\in E(G)} [ecc(u)+ecc(v)]$
$= |E|[ecc(u)+ecc(v)]$
$=mn(2+2)=4mn$.

Theorem 2.4: Eccentric connectivity index of complete bi-partite graph is 4mn.
Proof. By using equation (8) and figure (2), we have
$ECI\ (K_{m,n}) = \sum_{\mathbf{u}\in\mathbf{V(G)}} d_u\times ecc(u)$
$= \sum_{v\in V_1(G)} n\times2 + \sum_{v\in V_2(G)} m\times2$
$= (m\times n\times2)+(n\times m\times2)= 4mn$.

Theorem 2.5: New modified version of second Zagreb index of complete bi-partite graph is 4mn.
Proof. By using (7) and figure(2), we have new modified version of second Zagreb index
$M_2^*(K_{m,n}) = \sum_{uv\in E(G)} ecc(u)\times ecc(v)$
$= |E|\ [ecc(u)\times ecc(v)]$
$= mn(2\times2)=4mn$.

Sum-degree:
Theorem 2.6: Sankruti index complete bi-partite graph is $mn(\frac{m^2n^2}{2(mn-1)})^3$.
Proof. It is observed from figure (2), $S_u= \sum_{uv\in E(G)} d_v =mn$ and $S_v=\sum_{uv\in E(G)} d_u=mn$, so the Sankruti index of complete bi-partite graph is
$S(K_{m,n}) = \sum_{uv\in E(G)} (\frac{S_u\times S_v}{S_u+S_v-2})^3$
$= |E|(\frac{mn\times mn}{mn+mn-2})^3$
$= mn(\frac{m^2n^2}{2(mn-1)})^3$.

Theorem 2.7: Gourava Sombor index of complete bi-partite graph is $mn\sqrt{(m+n)^2+(mn)^2}$.
Proof. Each vertex in the first set (size m) has degree of n and each vertex in the second set (size n) has degree of m, then Gourava Sombor index of complete bi-partite graph is
$GSO(K_{m,n}) = \sum_{uv\in E(G)} \sqrt{(d_u+d_v)^2+(d_u\times d_v)^2}$
$= |E|\sqrt{(d_u+d_v)^2+(d_u\times d_v)^2}$
$= mn\sqrt{(m+n)^2+(mn)^2}$.

3) Complete graph
Leap degree:

Theorem 3.1: First leap Gourava index of complete graph is zero.
Proof. For a complete graph K_n of order n, the diameter is 1.So for any vertex v in a complete graph, the $d_2(v)$ = 0.There is no vertex that is exactly at distance 2 from any given vertex because all vertices are directly adjacent.
$LGO_1(K_n)= 0$.

Theorem 3.2: Second leap Gourava index of complete graph is zero.
Proof. For a complete graph K_n of order n, the diameter is 1.So for any vertex v in a complete graph, $d_2(v) = 0$.There is no vertex that is exactly at distance 2 from any given vertex because all vertices are directly adjacent.
$LGO_2(K_n) = 0$.

Eccentricity:

Theorem 3.3: New modified version of first Zagreb index of complete graph is $n(n-1)$.
Proof. By using equation (5) and figure (3), we have
$M_1^*(K_n) = \sum_{uv\in E(G)} ecc(u)+ecc(v)$
$= |E|[ecc(u)+ecc(v)]$
$= \frac{n(n-1)}{2}(1+1) = n(n-1)$.

Theorem 3.4: Eccentric connectivity index complete graph is $\frac{n(n-1)^2}{2}$.
Proof. By using (8) and figure (3), we have
$ECI\ (K_n) = \sum_{u\in V(G)} d_u\times ecc(u)$
$= \sum_{v\in V(G)} (n-1)\times1$

$= \frac{n(n-1)^2}{2}$.

Theorem 3.5: New modified version of second Zagreb index of complete graph is $\frac{n(n-1)}{2}$.

Proof. By using equation (7) and figure (3), we have

$M_2^*(K_n) = \sum_{uv\in E(G)} ecc(u) \times ecc(v)$

$= |E|[ecc(u) \times ecc(v)]$

$= \frac{n(n-1)}{2}(1 \times 1) = \frac{n(n-1)}{2}$.

Sum-degree:

Theorem 3.6: Sankruti index of complete graph is $\frac{n(n-1)^{13}}{16(n^2-2n)^3}$.

Proof. By using equation (9) and figure (3), we have sum-degree $S_u = S_v = (n-1)^2$. Then Sankruti index of complete graph is

$S(K_n) = \sum_{uv\in E(G)} \left(\frac{S_u \times S_v}{S_u + S_v - 2}\right)^3$

$= |E|\left(\frac{(n-1)^2 \times (n-1)^2}{(n-1)^2 + (n-1)^2 - 2}\right)^3$

$= \frac{n(n-1)^{13}}{16(n^2-2n)^3}$.

Theorem 3.7: Gourava Sombor index of complete graph is $\frac{n(n-1)^2}{2}\sqrt{2 + (n-1)^2}$.

Proof. By using equation (11) and figure (3), we have

$GSO(K_n) = \sum_{uv\in E(G)} \sqrt{(d_u + d_v)^2 + (d_u \times d_v)^2} = |E|\sqrt{(d_u + d_v)^2 + (d_u \times d_v)^2}$

$= \frac{n(n-1)}{2}\sqrt{[(n-1) + (n-1)]^2 + [(n-1) \times (n-1)]^2}$

$= \frac{n(n-1)^2}{2}\sqrt{2 + (n-1)^2}$.

The computed values of $LM_1^*(G)$, $LM_2(G)$, $M_1^{**}(G)$, $GA_5(G)$ and $ABC_4(G)$ topological indices of HX_6, complete bi-partite and complete graphs are represented in table (4).

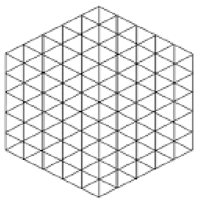

Fig.1.

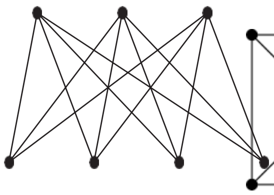

fig.2.

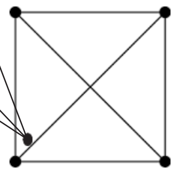

fig.3.

Figure1.Hexagonal network of dimension 6, figure 2. Complete bipartite graph $K_{3,4}$,
Figure 3: Complete graph K_4.

Table 1: Edge partition of HX_n.

(d_u, d_v)	(3, 4)	(3, 6)	(4, 4)	(4, 6)	(6, 6)
No. of edges	12	6	6n-18	12n-24	$9n^2$-33n+30

Table 2.Degree, frequency, eccentricity partition of HX_6.

Position of vertex	d_u	Eccentricity	Count
corner vertices	3	3	6
edge vertices	4	4	30
interior vertices	6	6	55

Table 3: Sum-degree partition of HX_6.

(S_u, S_v)	(14, 19)	(19,20)	(20,20)	(19, 29)	(19,32)	(20,32)	(14,29)	(29,32)	(29,36)	(32,36)	(32,32)	(36,36)
No. of edges	12	12	6(n-5)	12	12	12(n-4)	6	12	6	12(n-3)	6(n-4)	$9n^2$-51n+72

Table 4: Topologocal indices of HX_6, complete bi-partite and complete graphs.

Topological index →	$LM_1^*(G)$	$LM_2(G)$	$M_1^{**}(G)$	$GA_5(G)$	$ABC_4(G)$
HX_6	418n+62	1128n+164	2514	239	2.455
complete bi-partite graph	2mn(m+n-1)	$mn(m+n-1)^2$	4mn	mn	$\frac{\sqrt{2mn-2}}{mn}$
complete graph	0	0	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n\sqrt{2(n-1)^2-2}}{2(n-1)^3}$

3. Conclusion

The topological indices $LGO_1(G)$, $LGO_2(G)$, $LM_1^*(G)$, $LM_2(G)$, $M_1^*(G)$, $M_1^{**}(G)$, $M_2^*(G)$, ECI (G), S(G), $GA_5(G)$, GSO(G) and $ABC_4(G)$ are studied for hexagonal network with dimension six, complete bi-partite and complete graphs. $M_1^*(G)$, $M_2^*(G)$, $M_1^{**}(G)$ and ECI(G) has the same value 4mn for complete bi-partite graph. $LGO_1(G)$, $LGO_2(G)$, $LM_1^*(G)$ and $LM_2(G)$ topological indices has zero value for complete graph (K_n).

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