International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

Reliability Analysis of Some Complex Systems Based on Configuration Using Various Distributions

Dr. Veena Rani

Associate Professor, Department of Mathematics, G. N. C. Sirsa
Email: *veenaraheja.vr[at]gmail.com*

Abstract: ***This research paper deals with analysis on reliability of some complex systems using various distributions. The structural designs and configuration of different complex systems with various failure laws are analyzed. In this article, it is observed that reliability of a system can be improved by providing better configuration to components thereof. There are different types of configurations such as series, parallel, series- parallel, parallel- series, k-out-of-n and mixed configurations for a complex system, however, in this paper only two types of systems namely series system and parallel system are discussed with various failure distributions i.e. Weibull, Rayleigh and Exponential distribution. The results of the reliability for arbitrary values of parameters such as number of components, failure rates in three distributions have been obtained which may be very helpful in using various complex systems.***

Keywords: reliability, configuration, series, parallel, Weibull, Rayleigh, Exponential distributions, complex system, failure rate.

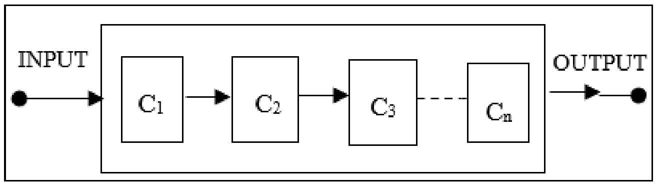
1. Introduction

In modern time when almost in each field i.e. from agriculture to industries, the use of machines is increasing comparative to the manual work, for maximum production with lowest maintenance cost, it becomes essential that working of machines must consistent and reliable for which reliability analysis plays an important role. Reliability analysis may help the management in taking timely decisions for smooth functioning of unit. Due to heavy competition and to meet ever-increasing demands of the society, industries are trying to introduce more and more automation in their industrial process and they use different types of complex systems. Basically, a system is collection of components arranged according to a specific design in order to achieve the desired functions with acceptable performance and reliability measures. Though, there are different types of configurations of a complex system such as series, parallel, series- parallel, parallel- series, k-out-of-n and mixed configurations, however, in this paper, only series configuration and parallel configuration with three types of failure distributions namely Weibull, Rayleigh and Exponential distributions are discussed. On theoretical evaluation of reliability measures of a system under different configurations of components, many scholars have written several research papers during last few decades. Kneale (1961) explain the theory of reliability of parallel systems with repair and switching. Gaver (1963) derived reliability characteristics of a parallel system subjected to the random failure and arbitrarily distributed repair times. Misra (1972) provided a simple algorithm for finding reliability optimization of series-parallel system. Balagurusamy (1984) and Srinath (1985) analysed reliability measures of a series system for Exponential Distribution. Kumar, R. and Bhatia, P. (2013) analysed performance and profit evaluations of a Stochastic Model on Centrifuge System working in Thermal Power Plant considering neglected faults. Nandal et al. (2015) evaluated the reliability and mean time to system failure (MTSF) of series and parallel systems with exponential failure laws. Renu, Bhatia, P. and Kumar, V.(2016) analysed reliability of a two-unit standby system for robotic machines. Chauhan and Malik (2017) examined the behaviour of MTSF and reliability of a parallel system with Weibull failure laws. Kumar Satish (2018) evaluated reliability & MTSF of the systems with different structural designs of the components by considering Weibull failure laws. Renu and Bhatia P. (2019) analysed reliability of a two-unit standby system for high pressure die casting Machines.

Configurations of Complex System:

i) Series configuration:

Suppose a system consists of n components $C_1, C_2, C_3 \ldots\ldots\ldots\ldots C_n$ with life time $T_1, T_2, T_3 \ldots\ldots\ldots\ldots\ldots T_n$ respectively with reliability $R_1(t)$, $R_2(t) \ldots\ldots\ldots R_n(t)$ at any time t. All the components are joint in series form. The successful operation of the system depends on the proper operation of all the components. This can be shown as:



The information at INPUT will reach at OUTPUT only if all the components function properly. In this case, the reliability of entire system is given by

$R(t) = P(T>t) = P(T_1 > t, T_2 > t, \ldots\ldots\ldots T_n > t)$

$= P(T_1 > t)\, P(T_2 > t), \ldots\ldots\ldots P(T_n > t)$

$= \prod_{i=1}^{n} P(Ti > t)$

$= \prod_{i=1}^{n} Ri(t)$

Thus, reliability of a system in series configuration is the product of reliabilities of all components joined in series.

If $h_i(t)$ is instantaneous failure rate (hazard rate) of i^{th} component then

h(t), the failure rate of the system is given by

$h(t) = \sum_{i=1}^{n} hi(t)$

Weibull Distribution: Suppose failure rate of all components follow Weibull distribution i.e. $h_i(t) = \lambda_i t^{\beta i}$

The component reliability is given by

$R_i(t) = e^{-\int_0^t \lambda_i u^{\beta_i} du} = e^{-\frac{\lambda_i t^{\beta_i+1}}{\beta_i+1}}$

And system reliability is given by $R(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n}[e^{-\frac{\lambda_i t^{\beta_i+1}}{\beta_i+1}}] = e^{-\sum_{i=1}^{n}\frac{\lambda_i t^{\beta_i+1}}{\beta_i+1}}$

Rayleigh Distribution: If the failure rate of all components follows Rayleigh distribution then $h_i(t) = \lambda_i t$ the component reliability is given by $R_i(t) = e^{-\int_0^t \lambda_i u du} = e^{-\frac{\lambda_i t^2}{2}}$ and system reliability is given by $R(t) = \prod_{i=1}^{n} Ri(t) = \prod_{i=1}^{n}[e^{-\frac{\lambda_i t^2}{2}}] = e^{-\sum_{i=1}^{n}\frac{\lambda_i t^2}{2}}$.

Exponential Distribution: In the case of failure rate following Exponential distribution we have $h_i(t) = \lambda_i$, the component reliability in this case is given by

$R_i(t) = e^{-\int_0^t \lambda_i du} = e^{-\lambda_i t}$ and system reliability is given by

$R(t) = \prod_{i=1}^{n} Ri(t) = \prod_{i=1}^{n}[e^{-\lambda_i t}] = e^{-\sum_{i=1}^{n}\lambda_i t}$

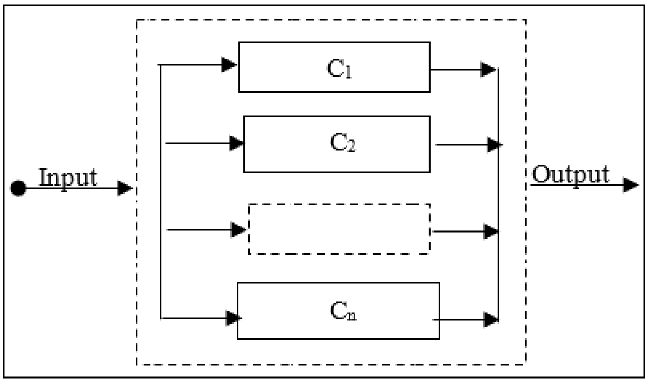
Now we discuss the case of five identical elements i.e. having same failure rate and, therefore, same reliability. It is to be noted that Rayleigh and Exponential Distributions are particular cases of Weibull Distribution for β =1 and β =0 respectively. We will calculate reliability of the above referred five elements for aforesaid three distributions in tabular form as under:

n	β = 0 λ = 0.01 t = 10	β = 0.1 λ = 0.01 t = 10	β = 0.2 λ = 0.01 t = 10	β = 1 λ = 0.01 t = 10
1	0.9048	0.8918	0.8763	0.6065
2	0.8187	0.7954	0.7678	0.3679
3	0.7408	0.7094	0.6728	0.2231
4	0.6703	0.6327	0.5896	0.1353
5	0.6065	0.5643	0.5166	0.0821

In above table the reliability of system with arbitrary values of failure rate λ and operating time (t) associated with no. of components (n) is shown. We observed that the reliability of a series system of five identical components keeps on decreasing with the increase of number of components. However, in case of Rayleigh distribution (β =1), the effect of no. of components upon reliability is much more than that of Exponential (β =0) & Weibull distribution (β =0.1,0.2). Also it is clear from above table that as the value of β is increasing from 0.1 to 0.2, the reliability is decreasing correspondingly. Therefore, reliability also depends on shape parameter (β). Consequently, we conclude that least no. of components should be used in a series system for better performance and also the performance of such systems can be improved by using the components which follow Exponential law.

Parallel Configuration:

In this type of system, all the n components $C_1, C_2, C_3 .... C_n$ are joined in parallel format to form the entire system. In this case, the system will fail if all the components of the system fail. This can be shown as:



Again, using the same notations as above, we get reliability of the entire system given by:

$R(t) = P(T>t) = 1-P(T_i \le t)$

$= 1-P\{Max.(T_1, T_2, ...T_n) \le t\}$

$= 1-\prod_{i=1}^{n} P(Ti \le t)$

$= 1-\prod_{i=1}^{n}\{1-P(Ti > t)\}$

$= 1-\{\prod_{i=1}^{n} 1-Ri(t)\}$

Thus, unreliability of system in parallel configuration is the product of unreliability of all components joined in parallel form.

Weibull Distribution: Suppose failure rate of all components follow Weibull distribution i.e. $h_i(t) = \lambda_i t^{\beta_i}$

The component reliability is given by

$R_i(t) = e^{-\int_0^t \lambda_i u^{\beta_i} du} = e^{-\frac{\lambda_i t^{\beta_i+1}}{\beta_i+1}}$

And system reliability is given by $R(t) = 1-\prod_{i=1}^{n}[1-Ri(t)]$

$= 1-\prod_{i=1}^{n}[1-e^{-\frac{\lambda_i t^{\beta_i+1}}{\beta_i+1}}]$

Rayleigh Distribution: If the failure rate of all components follows Rayleigh distribution then

$h_i(t) = \lambda_i t$ the component reliability is given by $R_i(t) = e^{-\int_0^t \lambda_i u du} = e^{-\frac{\lambda_i t^2}{2}}$ and system reliability is given by R(t)

$= 1-\prod_{i=1}^{n}[1-Ri(t)] = 1-\prod_{i=1}^{n}[1-e^{-\frac{\lambda_i t^2}{2}}]$.

Exponential Distribution: In the case of failure rate following Exponential distribution we have $h_i(t) = \lambda_i$ the component reliability in this case is given by

$$R_i(t) = e^{-\int_0^t \lambda_i du} = e^{-\lambda_i t}$$ and system reliability is given by

$$R(t) = 1 - \prod_{i=1}^{n}[1 - Ri(t)] = 1 - \prod_{i=1}^{n}[1 - e^{-\lambda_i t}].$$

Like Series Configuration's case, now we discuss the case of five identical elements i.e. having same failure rate and, therefore, same reliability in Parallel configuration. It is to be noted that Rayleigh and Exponential Distributions are particular cases of Weibull Distribution for β =1 and β =0 respectively. We will calculate reliability of the above referred five elements for aforesaid three distributions in tabular form as under:

n	β = 0 λ = 0.01 t = 10	β = 0.1 λ = 0.01 t = 10	β = 0.2 λ = 0.01 t = 10	β = 1 λ = 0.01 t = 10
1	0.9512	0.8918	0.8763	0.6065
2	0.9976	0.9883	0.9847	0.8452
3	0.9999	0.9987	0.9981	0.9391
4	0.99999	0.9998	0.9998	0.9760
5	0.999997	0.99998	0.9997	0.9906

A parallel system of five identical components with different failure distributions has been analyzed to see the effect of number of components (n), failure law (λ) and shape parameter (β) on reliability. It is found that the reliability keeps on increasing with the increase of number of components while it decreases with the increase of shape parameter. Also, it can be observed that system is more reliable when components' failure rate follows Exponential distribution than that of Weibull or Rayleigh distribution. However, the effect of no. of components on reliability is much more when components follow Rayleigh failure laws than that of Exponential and Weibull failure laws.

2. Conclusion

After going through the results derived above for arbitrary values of the parameter from the aforesaid analysis of reliability of complex system with Series configuration as well as Parallel configuration by applying three failure laws, it can be concluded that parallel configuration of a system is more helpful in improving system reliability than that of series configuration. In both the configurations, it can also be observed that system is more reliable when components' failure rate follows Exponential distribution than that of Weibull or Rayleigh distribution.

References

[1] Weibull, W. (1951): A Statistical Function of Wide Applicability, Journal of Applied Mechanics, Vol. 18, pp.293-297.
[2] Kneale, S.G. (1961): Reliability of Parallel Systems with Repair and Switching, Proc.Seventh National Symposium on Reliability and Quality Control, pp. 129-133.
[3] Gaver, D.P. (1963): Time to Failure and Availability of Paralleled Systems with Repair, IEEE Transactions on Reliability, Vol. 12(2), pp. 30-38.
[4] Misra, K.B. (1972): Reliability Optimization of a Series-Parallel System, IEE Transactions on Reliability, Vol. R-21(4), pp.230-238.
[5] Balagurusamy, E. (1984): Reliability Engineering, Tata McGraw Hill Publishing Co.Ltd., India.
[6] Srinath, L.S. (1985): Concept in Reliability Engineering, Affiliated East-West Press (P) Ltd.
[7] R.Kumar and P.Bhatia (2011): "Reliability and Cost Analysis of One Unit Centrifuge System with Singh Repairman and Inspection", Pure and Applied Mathematika Sciences, 74(1-2), 113-121.
[8] Kumar, R. and Bhatia, P. (2013): "Performance and Profit Evaluations of a Stochastic Model on Centrifuge System working in Thermal Power Plant considering neglected faults. International Journal of Scientific and Statistical Computing, Malasia, ISSN:2180-1339, Vol.4(1), pp.10-18,2013
[9] Nandal, J., Chauhan, S.K. & Malik, S.C. (2015): Reliability and MTSF of a Series and Parallel systems, International Journal of Statistics and Reliability Engineering, Vol. 2(1), pp. 74-80.
[10] Renu, Bhatia P. and Kumar, V. (2016): "Reliability analysis of a two unit standby system for robotic machines", Journal of Mathematics and System Science Vol.12, pp.93-100.
[11] Chauhan, S.K. and Malik, S.C. (2017): Evaluation of Reliability and MTSF of a Parallel System with Weibull Failure Laws, Journal of Reliability and Statistical Studies, Vol. 10 (1), pp.137-148.
[12] Kumar Satish (2018) a thesis on, "Evaluation of Reliability Measures of Systems Under Different Configurations" Submitted to Maharshi Dayanand University, Rohtak.
[13] Renu, Bhatia P. (2019): "Reliability Analysis of a two-unit standby system for high pressure die casting Machines", Aryabhatta Journal of Mathematics and Informatics, Vol.11, issue.1, pp. 18-25, ISSN:0975-7139.