Bianchi Type-V Cosmological Model with Anti Stiff Fluids in Presence of Electromagnetic Field in General Relativity

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Abstract: ***In this paper, we have examined magnetized Bianchi type-V string cosmological model for anti-stiff fluid in general relativity. Anti-stiff fluid models create more interest in the study because these models are free from initial singularity. We assume that F_{12} is the only nonzero component of electromagnetic field tensor F_{ij}. For the complete determination of the model, we assume the expansion in the model is proportional to the shear. The general solution of the Einstein's field equations for the cosmological model has been obtained under the assumption of anti-stiff fluid i.e. p + ρ = 0, where ρ and p are the rest energy density and the pressure of the fluid respectively. The physical and geometrical significance of the model in the presence of magnetic field are discussed.***

Keywords: Bianchi type-V, Anti-stiff fluid, Magnetic field, General Relativity

1. Introduction

Discovering the precise physical state during the initial period of the universe's Initiation remains a highly challenging issue for us. Presently, research into cosmological constructs has become a dominant and vital instrument in the investigation of the cosmos and in particular cases. Certain universal models incorporating a magnetic field within the framework of general relativity. Bianchi type cosmological models are more beneficial and appealing for studying the development during phase transition. One of the most straightforward anisotropic universal frameworks that performs a pivotal function in comprehending the requisite characteristics of the universe is the Bianchi type-V. In the recent years, cosmic string have drawn considerable attention among researchers in various aspects such that the study of the early universe, these strings arise during phase transition after big bang explosion as temperature goes down below some critical temperature as predicted by grand unified theory studied by Kibble [1], Zel᾽dovich et al. [2], Kibble [3], it is believed that cosmic string give rise to density perturbation which leads to the formation of galaxies investigated by Zel᾽dovich [4], these cosmic string have stress energy and coupled to gravitational field, therefore it is interesting to study gravitational effect which arise from strings. The general treatment of strings was initiated by Letelier [5] and Satchel [6]. Pradhan et al. [7] endured inhomogeneous cosmological models formed by geometric strings and used these models as a source of gravitational fields.

The occurrence of magnetic fields on galactic scale is well established fact today and their importance for a variety of astrophysical phenomenon is generally acknowledge which was pointed out by Zel᾽dovich [8]. Also, Harrison [9] has suggested that magnetic field could have a cosmological origin, as a natural consequence. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological model more general than Robertson-Walker model [10], Singh et al. [11] has examined spatially homogeneous and anisotropic Bianchi type-III with massive string in presence of viscous fluid and electromagnetic field. The presence of primordial magnetic fields in the early stage of evolution of the universe has been discussed by several authors Kim et al. [12], Barrow [13], Melvin [14]. Patil et al [15]. have studied Magnetized Bianchi type-III cosmological models with wet dark fluid in general relativity, Chhajed et al. [16] investigated the Magnetized Bianchi Type-III string cosmological model for anti-stiff fluid in general relativity

Motivated by the above research work, in this paper, we have investigated magnetized Bianchi type-V string cosmological model for anti-stiff fluid in general relativity. The physical and geometrical consequences of the model in the presence of magnetic field are discussed.

2. The Metric and Field Equations

We have considered Bianchi type-V space-time,

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}B^2(dy^2 + dz^2) \tag{1}$$

in which A, B are cosmic scale functions of time t.

The Einstein field equation for the space time (1) is,

$$R_j^i - \frac{1}{2}Rg_j^i = -8\pi T_j^i \tag{2}$$

Where T_j^i is the energy momentum tensor for a cloud of strings given by Letelier (1979-1983) as

$$T_j^i = (p+\rho)v_iv^j + pg_i^j - \lambda x_ix^j + E_i^j \tag{3}$$

$$\text{With } v_iv^i = -x_ix^j = -1 \text{ and } v_ix^i = 0 \tag{4}$$

Where λ is the string tension density, ρ is the matter density, p is thermo dynamical pressure, v^i is the four velocity vector andx^i is the direction of string. The particle density associated with the configuration is given by

$$\rho_p = p - \lambda \tag{5}$$

The electromagnetic field E_j^i is defined as,

$$E_j^i = \frac{1}{4\pi}\left[-F_{jl}F^{il} + \frac{1}{4}g_j^iF_{lm}F^{lm}\right] \tag{6}$$

with the Maxwell's equation,

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0, \text{ which leads to, } F_{12} = I \quad (7)$$

Where I is the constant

Now the nonvanishing components of E_j^i corresponding to the line element (1) are given as follows:

$$\Rightarrow E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{8\pi B^4} \quad (8)$$

In the above v^i is the flow vector satisfying

$$g_{ij}v^i v^j = -1 \quad (9)$$

And direction of string is along z-axis so that $x_1 = 0, x_2 = 0, x_3 \neq 0, x_0 = 0$

We assume the coordinates to be comoving, so that

$$v^1 = v^2 = v^3 = 0 \text{ and } v^0 = 1 \quad (10)$$

The Einstein's field equation (2) for metric (1) leads to

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -8\pi p - \frac{I^2}{B^4} \quad (11)$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{A}}{A} - \frac{1}{A^2} = -8\pi p - \frac{I^2}{B^4} \quad (12)$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{A}}{A} - \frac{1}{A^2} = -8\pi(p - \lambda) + \frac{I^2}{B^4} \quad (13)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = 8\pi\rho + \frac{I^2}{B^4} \quad (14)$$

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \quad (15)$$

Using equations (15), we have

$A = KB$, where K is integration constant, taking

$$K = 1 \quad (16)$$

$$\Rightarrow A = B \quad (17)$$

Without loss of generality the field equations (11) to (14) reduces to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = -8\pi p - \frac{I^2}{B^4} \quad (18)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = -8\pi(p - \lambda) + \frac{I^2}{B^4} \quad (19)$$

$$3\frac{\dot{B}^2}{B^2} - \frac{3}{B^2} = 8\pi\rho + \frac{I^2}{B^4} \quad (20)$$

As there are four unknowns and three equations so we take additional condition

$$p + \rho = 0 \quad (21)$$

$$\Rightarrow p = -\rho \quad (22)$$

Using equation (22) in equation (20), we get

$$3\frac{\dot{B}^2}{B^2} - \frac{3}{B^2} = -8\pi p + \frac{I^2}{B^4} \quad (23)$$

Subtracting equation (18) from equation (23), we get

$$\dot{B} = \frac{dB}{dt} = \sqrt{\frac{I^2+2L}{2B^2}} \quad (24)$$

Integrating on both sides, we get

$$\int \left(\frac{2}{I^2+2l}\right)^{\frac{1}{2}} B \, dB = t + k$$

$$\frac{B^2}{\sqrt{2I^2+4L}} = t + k \quad (25)$$

Put $t + k = T$ therefore $dt = dT$ (26)

Therefore, using above substitution equation (25) and (17) become

$$B^2 = T\sqrt{2I^2 + 4L} \quad (27)$$

$$A^2 = T\sqrt{2I^2 + 4L} \quad (28)$$

Using equations (26), (27) and equation (28) in equation (1), we have

$$ds^2 = -dT^2 + T\sqrt{2I^2 + 4L}\, dx^2 + e^{2x}T\sqrt{2I^2 + 4L}(dy^2 + dz^2) \quad (29)$$

Using equations (24), (27) in equation (23), we have

$$p = -\frac{1}{8\pi}\left[\left(\frac{5I^2+6L-T\sqrt{2I^2+4L}}{2T^2(2I^2+4L)}\right)\right] \quad (30)$$

Using equation (22) in equation (30) we have

$$\rho = \frac{1}{8\pi}\left[\left(\frac{5I^2+6L-T\sqrt{2I^2+4L}}{2T^2(2I^2+4L)}\right)\right] \quad (31)$$

Using equation (24), (27) in equation (19) we have

$$\lambda = \frac{1}{8\pi}\left[\frac{2T\sqrt{2I^2+4L}\left(\frac{I^2+2L}{2}\right)^{\frac{1}{2}}\left(2T\sqrt{2I^2+4L}-1\right)-T\sqrt{2I^2+4L}-7I^2+6L}{2T^2(2I^2+4L)}\right] \quad (32)$$

The expansion (θ) and shear scalar (σ) are given as

$$\theta = 3\frac{1}{2T} \quad (33)$$

$$\sigma = \sqrt{\frac{7}{2}\frac{1}{2T}} \quad (34)$$

and the generalized mean Hubble parameter H is given by,

$H = \frac{1}{3}\theta$ Therefore

$$H = \frac{1}{2T} \quad (35)$$

The deceleration parameter is defined as,

$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right)$ on solving, we get

$$q = -\frac{1}{2T^2} - 1 \quad (36)$$

The ratio of shear scalar (σ) and expansion (θ)

$$\frac{\sigma}{\theta} = \frac{1}{3}\sqrt{\frac{7}{2}} \quad (37)$$

3. Some Special Cases

Case I: $I = 1$ **and** $L = 1$

From equations (27), (28), (29) and (30)-(32) are yields,

$$B^2 = T\sqrt{6} \quad (38)$$

$$A^2 = T\sqrt{6} \quad (39)$$

$$ds^2 = -dT^2 + T\sqrt{6}\, dx^2 + e^{2x}T\sqrt{6}(dy^2 + dz^2) \quad (40)$$

$$p = -\frac{1}{8\pi}\left[\left(\frac{11-T\sqrt{6}}{12T^2}\right)\right] \quad (41)$$

$$\rho = \frac{1}{8\pi}\left[\left(\frac{11-T\sqrt{6}}{12T^2}\right)\right] \quad (42)$$

$$\lambda = \frac{1}{8\pi}\left(\frac{2T\sqrt{6}\left(\frac{3}{2}\right)^{\frac{1}{2}}\left(2T\sqrt{6}-1\right)-T\sqrt{6}-1}{8T^2}\right) \quad (43)$$

Graphical Representation: For $I = 1$ **and** $L = 1$

$$p = -\frac{1}{8\pi}\left[\left(\frac{11-T\sqrt{6}}{12T^2}\right)\right]$$

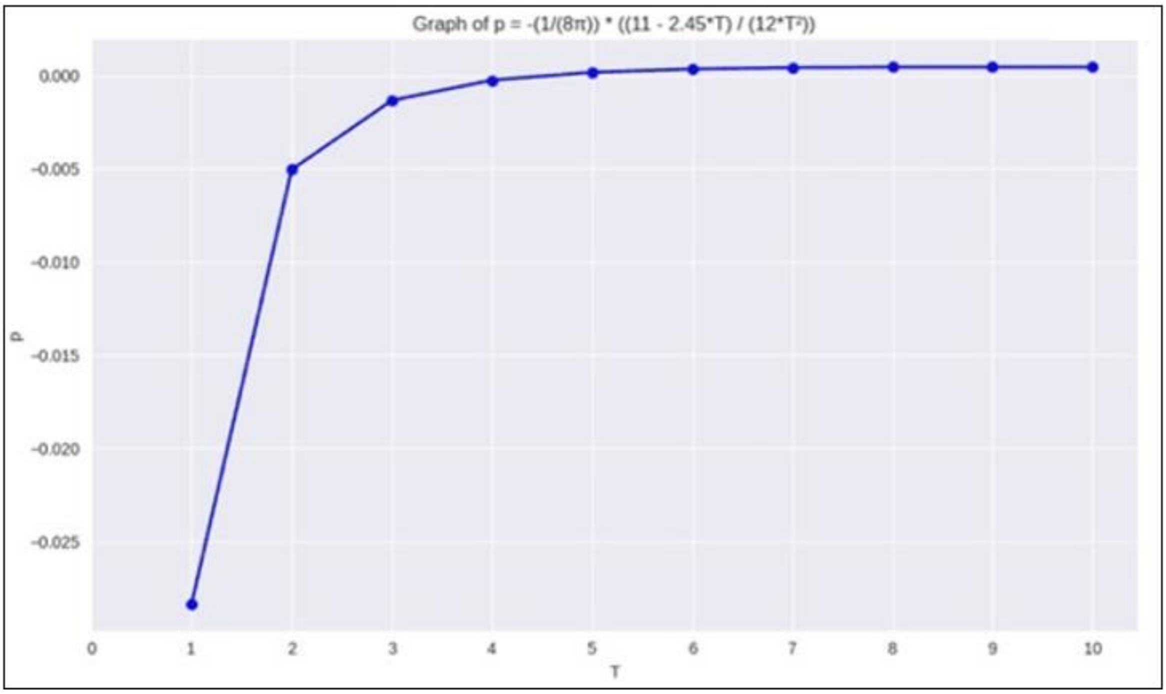

Figure 1: Graph for p vs T

Fig. 1

From fig.-1, it clears that as time T increases, pressure p is undefined at negative for small, crosses zero around and then approaches zero from the positive side as it increases.

$$\rho = \frac{1}{8\pi}\left[\left(\frac{11-T\sqrt{6}}{12T^2}\right)\right]$$

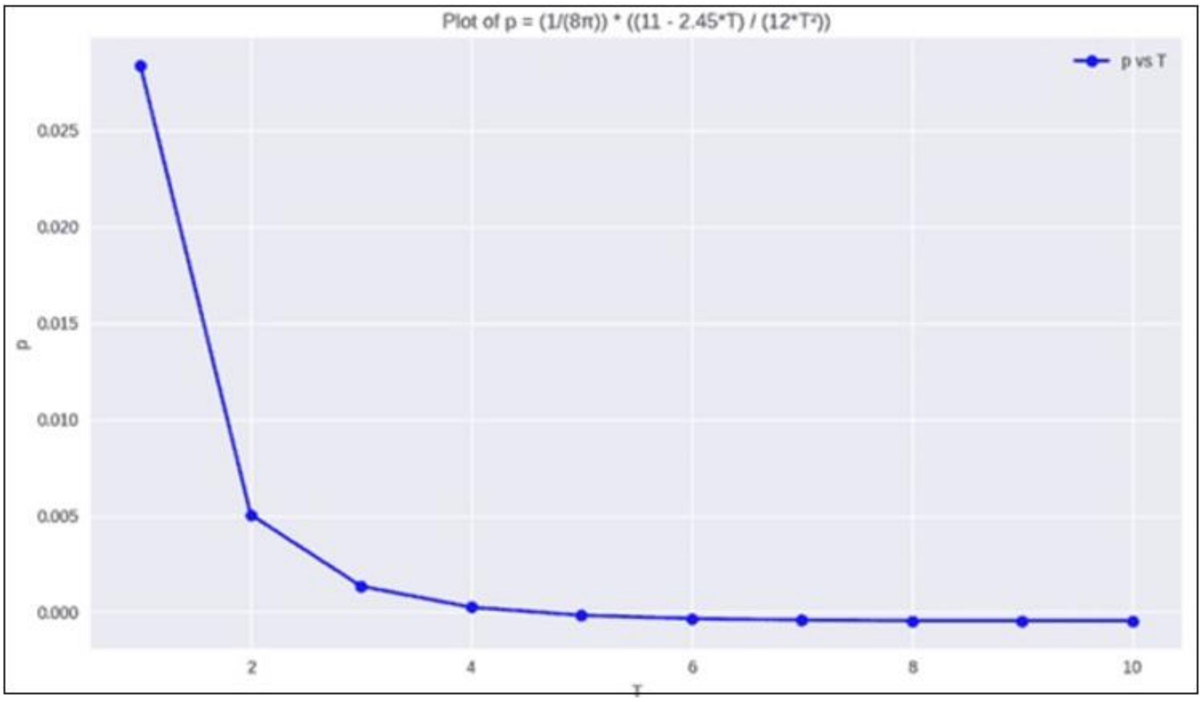

Figure 2: Graph for ρ vs T

From fig.-2, The graph for density ρ starts undefined at time T=0, shows positive values for small T, crosses zero near T=4.5, and then approaches zero from the negative side as T increases.

$$\lambda = \frac{1}{8\pi}\left(\frac{2T\sqrt{6}\left(\frac{3}{2}\right)^{\frac{1}{2}}(2T\sqrt{6}-1)-T\sqrt{6}-1}{8T^2}\right)$$

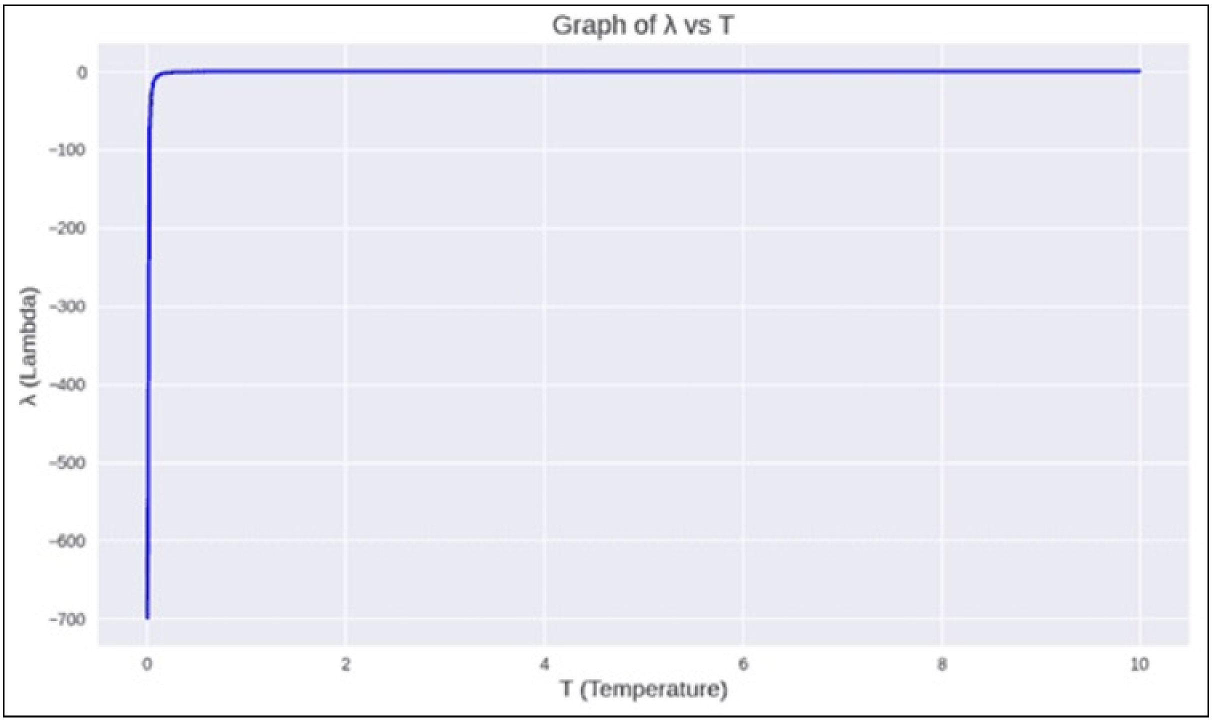

Figure 3: Graph for λ vs T

From fig.-3, The graph for string tension density λ Starts with a relatively high value near T = 0, dips to a minimum around T ≈ 3, Gradually increases and levels off as T increases, the shape of the curve is smooth and continuous, showing a clear dip and recovery.

Case II: $I = 1$ And $L = 0$

From equations (27), (28), (29) and (30)-(32) are yields,

$$B^2 = T\sqrt{2} \quad (44)$$

$$A^2 = T\sqrt{2} \quad (45)$$

$$ds^2 = -dT^2 + T\sqrt{2}\,dx^2 + e^{2x}T\sqrt{2}(dy^2 + dz^2) \quad (46)$$

$$p = -\frac{1}{8\pi}\left[\left(\frac{5 - T\sqrt{2}}{4T^2}\right)\right] \quad (47)$$

$$\rho = \frac{1}{8\pi}\left[\left(\frac{5 - T\sqrt{2}}{4T^2}\right)\right] \quad (48)$$

$$\lambda = \frac{1}{8\pi}\left[\frac{2T\sqrt{2}\left(\frac{1}{2}\right)^{\frac{1}{2}}(2T\sqrt{2} - 1) - T\sqrt{2} - 7}{4T^2}\right] \quad (49)$$

Graphical Representation: For $I = 1$ and $L = 0$

$$p = -\frac{1}{8\pi}\left[\left(\frac{11 - T\sqrt{6}}{12T^2}\right)\right]$$

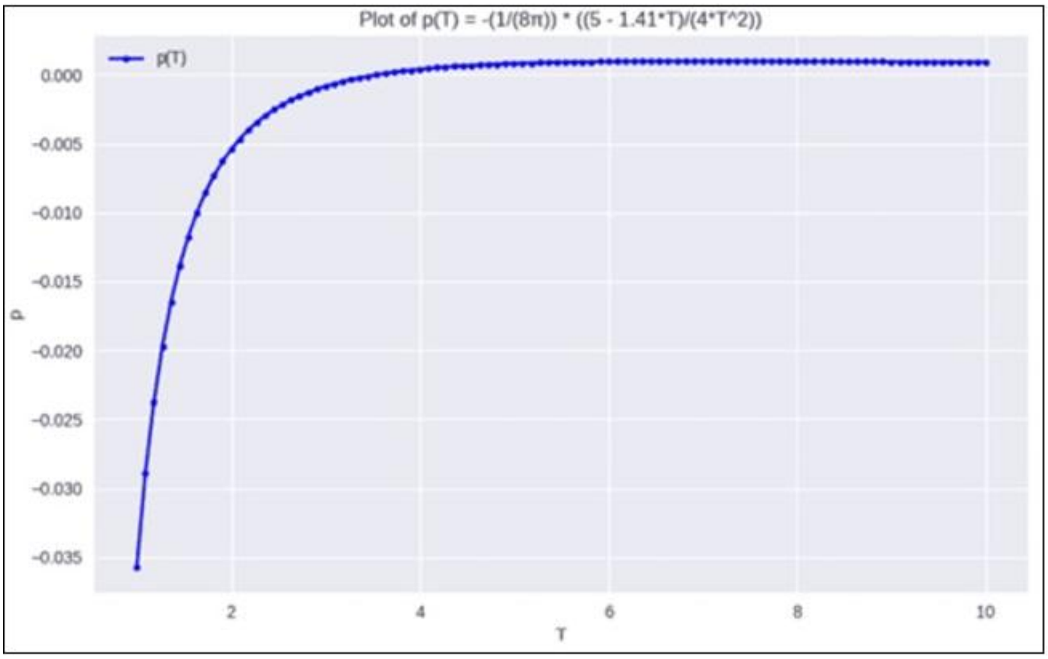

Figure 1: Graph for p vs T

From Fig.1, As time T increases the graph for pressure p starts negative, crosses zero, then becomes positive but diminishes.

$$\rho = \frac{1}{8\pi}\left[\left(\frac{5 - T\sqrt{2}}{4T^2}\right)\right]$$

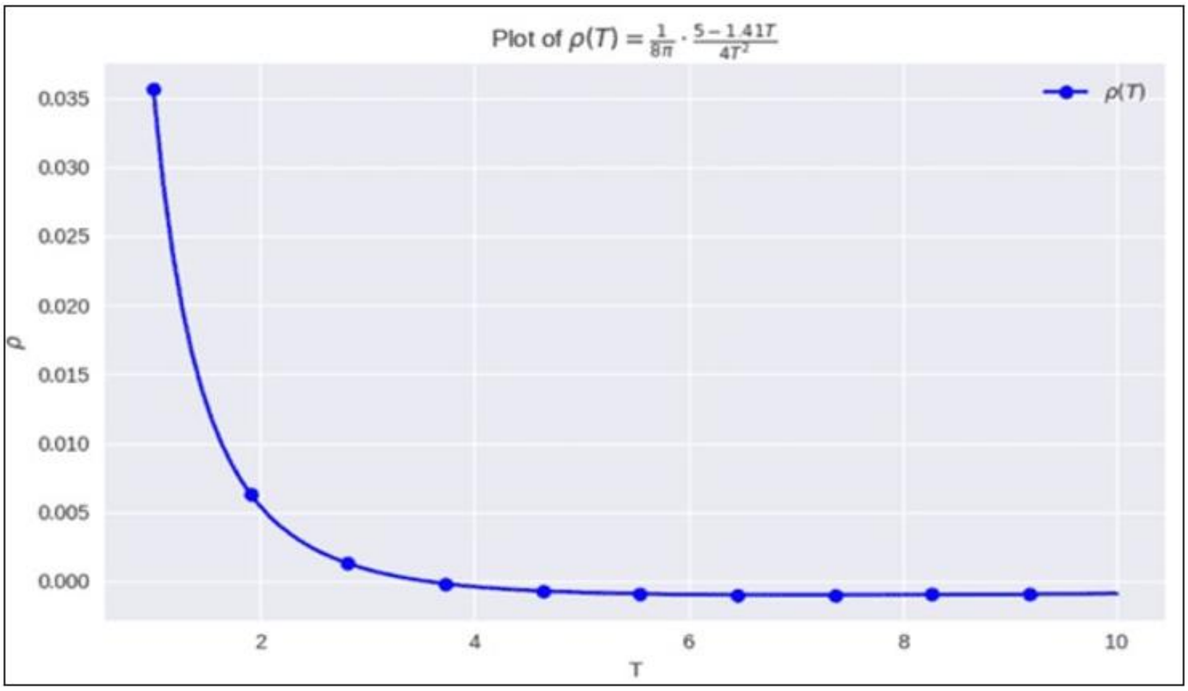

Figure 2: Graph for ρ vs T

From Fig.2, As time T increases the graph for density ρ gradually decreases and then approaches to zero.

$$\lambda = \frac{1}{8\pi}\left[\frac{2T\sqrt{2}\left(\frac{1}{2}\right)^{\frac{1}{2}}(2T\sqrt{2}-1)-T\sqrt{2}-7}{4T^2}\right]$$

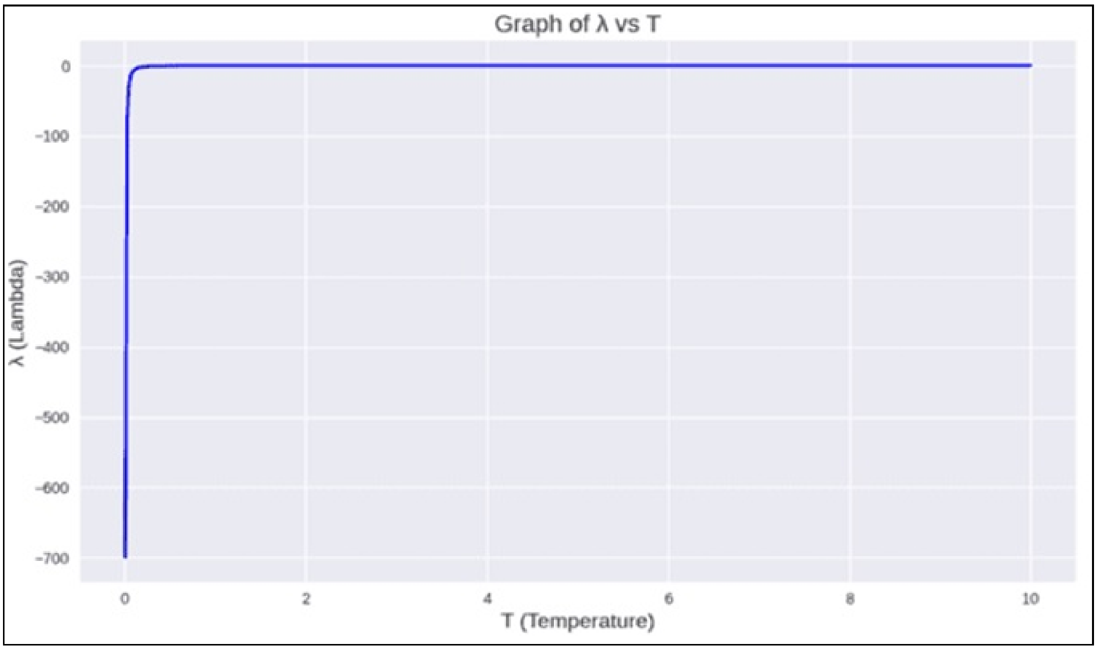

Figure 3: Graph for λ vs T

From fig.-3, The graph for string tension density λ is dominated by a singularity at T=0 and is blowing up near zero, while for values closer to ±1 it remains relatively stable and small.

4. Conclusion

In this chapter, we have investigated the nature of magnetized Bianchi type-V string cosmological model in presence of electromagnetic fluid for anti-stiff fluid in general relativity. The solution has been obtained in quadrature form. The geometrical and physical properties of model have been discussed in details. The model get shrink in presence of magnetic field and expand in its absence respectively. As t tends to zero, it tends to zero and as t increases or decreases respectively model expand or shrink, correspondingly, as t increases, the scalar of expansion and shear scalar σ

decreases. Also special cases for model such as power law and exponential form have been discussed here.

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