International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

Dynamics of a Predator-Prey System Incorporating Fear Effects and Holling Type-IV Functional Response

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Abstract: ***The predator-prey model with Holling type IV functional response incorporates a unique form of prey-predator interaction, specifically accounting for the impact of fear on prey behaviour. In this model, the functional response of the predator is characterized by a non-linear, saturating functional response (Holling type IV), where the predation rate increases rapidly at low prey densities and then stabilizes as prey density increases. The introduction of fear effect models how prey may alter their behaviour to avoid predators, potentially reducing predation rates at higher predator densities. This modified model considers both direct and indirect interactions between predator and prey populations, emphasizing the role of fear in influencing prey distribution and vulnerability. The system of differential equations derived from this framework allows for an exploration of the dynamics between predator and prey, including the potential for oscillations, equilibrium states, and stability analyses under different parameter conditions. Lotka-Volterra [1,2] type and Holling [3] type functional response are the most commonly used functional responses to describe the average feeding rate of a predator. There are four categories of Holling type responses which are given in terms of the density*** N***, i.e.*** $f(u,v) \equiv f(u)$***:***

Holling Type-I: $f(N) = \delta N$ ***(Lotka-Volterra type); Holling Type-II:*** $f(N) = \frac{\delta N}{(\beta + \alpha N)}$
Holling Type-III: $f(N) = \frac{\delta N^2}{(\beta + \alpha N^2)}$***; Holling Type-IV:*** $f(N) = \frac{\delta N}{(\beta + N)(1 + \alpha N)}$ ***(Shen)***

It has been seen that with the increase of fear within the predator-prey system the predator population tends to decrease effecting the birth rate of both prey and predator. Here we will investigate the impact of fear with Holling type IV functional response.

Keywords: Prey-predator model, Functional response, Holling type, Lotka Volterra

1. Introduction

The Mathematical modelling of predator-prey interactions has long been a central theme in theoretical ecology. The classical framework was initiated by the pioneering works of Lotka [1] and Volterra [2], who independently developed models describing oscillatory population dynamics based on mass-action principles. Although the Lotka-Volterra model captures basic interaction mechanisms, it fails to incorporate realistic ecological processes such as predator satiation, prey refuge, and behavioural responses. To address these limitations, functional responses describing the predator's consumption rate have been extensively studied. C.S. Holling [3] introduced a sequence of functional responses (Types I – IV) to represent increasingly realistic predator-prey interactions. Among them, the Holling type IV functional response is particularly significant as it accounts for inhibitory effects at high prey densities, such as predator confusion, handling interference or prey toxicity. This non-monotonic response has been shown to induce rich dynamical behaviours, including bi-stability, limit cycles and chaos. Several studies have focused on predator-prey systems incorporating Holling type IV functional responses. These works have examined the existence and stability of equilibria, Hopf bifurcation and global dynamics, demonstrating that the inhibitory effect embedded in type IV responses can destabilize coexistence equilibria and promote complex oscillatory patterns [4 – 6]. Such dynamics are ecologically relevant in systems where excessive prey density reduces predation efficiency.

In parallel, recent ecological research emphasizes that prey populations are influenced not only by direct predation but also by non-consumptive effects, particularly fear effects. Fear of predation can alter prey behaviour, reducing foraging activity, reproduction, and growth rates. Empirical studies suggests that these indirect effects may be comparable in magnitude to direct predation mortality [7,8]. Consequently, fear effects have been increasingly incorporated into predator-prey models by modifying prey birth rates, carrying capacity or functional responses.

Mathematical investigations of predator-prey models with fear effects reveal that fear can significantly alter population dynamics [15]. Studies report that increased fear intensity may stabilize otherwise instable systems, suppress population outbreaks or even lead to predator extinction under extreme behavioural suppression of prey growth [13,14,15]. Fear has also been shown to delay or prevent Hopf bifurcation and reduce oscillation amplitudes, indicating its critical role in ecosystem stability [9 – 11].

Despite substantial progress in studying predator-prey models with either Holling type IV functional response or fear effects independently, their combined influence remains relatively underexplored. Preliminary results in related models suggest that fear may either enhance or counteract the destabilizing effects of Holling type IV responses, depending on parameter regimes [12,16]. Therefore, investigating predator-prey models incorporating both Holling type IV functional response and fear effects is essential for a more comprehensive understanding of ecological dynamics.

Considering the system of first order differential equation,

$$\dot{x}_1 = f_i(x_1, x_2, x_3, . . . , x_n, t)$$

If the function f_i depends on $(x_1, x_2, x_3, . . . , x_n)$ but not on t then the system is called Autonomous system. But if the function depends on t also then the system is called non-autonomous system.

Phase Plane: Now if we consider autonomous system of first order differential equation in the xy- plane, represented by,

$$\dot{x} = P(x, y) \;\; \& \;\; \dot{y} = Q(x, y)$$

Where $P(x, y)$ and $Q(x, y)$ are continuous and have first order partial derivative throughout the xy-plane. The solution $x(t), y(t)$ of the system may be represented on the xy-plane which is known as Phase Plane.

When the time t increases, $x(t), y(t)$ trace out a directed curve on the xy-plane called Phase Path or Phase Trajectory or Path of the System.

Equilibrium Point

The point (x^*, y^*) is called the **Critical Point** or the **Fixed point** or **Equilibrium Point** or **Stationary Point** when $P(x^*, y^*) = 0$ & $Q(x^*, y^*) = 0$.

In terms of the solution of a differential- equations, a function f(x) is said to be **stable** if any other solution of the equation that starts out sufficiently close to it when $x = 0$ remains close to it for succeeding values of x. If the difference between the solutions approaches zero as x increases, the solution is called **Asymptotically Stable**. If the solution does not have either of these properties, it is called **Unstable**.

2. The Mathematical Problem

When the species have interaction the populace dynamics of every species is affected. In usual there is a complete net of interacting species, occasionally referred to as a trophic web, which makes for structurally complicated communities. We think about right here structures involving two or extra species, concentrating specially on two-species systems.

Predator-Prey Models: Lotka-Volterra Systems

Volterra (1926) first proposed an easy mannequin for the predation of one species to give an explanation for the oscillatory stages of fish catching in the Adriatic during world war-I. The study was made for two species interaction. If N(t) is the prey population and P(t) that of the predator at time t then Volterra's mannequin is-

$$\frac{dN}{dt} = N(a - bP) \qquad (I)$$

$$\frac{dP}{dt} = P(cN - d) \qquad (II)$$

Where $a, b, c\ and\ d$ are positive constants.

Realistic Predator-Prey Models

The Lotka-Volterra model suggests that simple predator-prey interactions can result in periodic behaviour of populations. This is not unexpected, as if a prey population increases, it encourages growth of its predator. When less food is available, the prey population declines, allowing the predator population to increase and the whole cycle starts over again. Depending on the detailed system, oscillations can grow or decay, go into a stable limit cycle oscillation, or exhibit chaotic behaviour.

Limit cycle solutions are closed trajectories in the predator-prey space, such as the Lotka-Volterra model, where prey growth is unbounded in the absence of predation. To be more realistic, these growth rates should depend on both prey and predator densities as in-

$$\frac{dN}{dt} = N\,F(N, P), \qquad \frac{dP}{dt} = P\,G(N, P) \qquad (III)$$

Prey should have a maximum carrying capacity in the absence of predators. So, for example, more realistic prey population equation might take form –

$$\frac{dN}{dt} = N\,F(N, P), \quad F(N, P) = \left(1 - \frac{N}{K}\right) - P\,R(N) \quad (IV)$$

Model formulation

In the absence of predators and at the expense of fear, assume that the prey obey a logistic growth. Three factors can be used to analyse the logistic growth of prey: birth rate, natural death rate and density-dependent mortality, as a result of intra-species competition. Thus, the following ODE follows,

$$\frac{dx}{dt} = r_0 x - dx - ax^2 \qquad (V)$$

Where x represents the population of the prey, r_0 is the birth rate of the prey, d is the natural death rate of prey, a represents the death rate due to intra-specific competition.

Let y stand for the predator's population. We change (V) by multiplying the production term by a factor $f(k, y)$ that compensates for the cost of anti-predator defence due to anxiety, since field trials demonstrate that the fear impact would reduce the production,

$$\frac{dx}{dt} = [f(k, y) r_0] x - dx - ax^2 \qquad (VI)$$

Here, the parameter k reflects the level of fear which drives anti-predator behaviours of the prey. By the biological meanings of k, y and $f(k, y)$, it is reasonable to assume that

$$f(0, y) = 1, \qquad f(k, 0) = 0, \qquad \lim_{k \to \infty} f(k, y) = 0$$

$$\frac{\partial f(k, y)}{\partial k} < 0, \quad \frac{\partial f(k, y)}{\partial y} < 0 \qquad (VII)$$

Although there are theories and arguments (e.g., Clinchy et al. 2013) suggesting that fear may affect adult survival rates due to physiological effects on children, there are by far no actual experimental data supporting such a claim. Because of this, we do not represent this factor in our study, so d and a are treated as consonants.

Next, we add a predation term $g(x)y$ to (VI) to create the generic prey-predator model shown below, which accounts for the cost of fear.

$$\frac{dx}{dt} = x r_0 f(k, y) - dx - ax^2 - g(x)y \;\; \& \;\; \frac{dy}{dt} = y\big(-m + cg(x)\big) \quad (VIII)$$

Here, $g: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$, the functional response of predators, y for predator density, c for the rate at which prey is converted into predator's biomass, and m for the mortality rate of predators. The most common version of $g(x)$ is $xp(x)$ with $p: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$.

When $p(x) = p$ is a constant, the functional response is linear; however, when $p(x) = \frac{p}{1+qx^2}$, $g(x)$ represents the Holling type IV functional response.

One can readily demonstrate that (VIII) has a unique solution for any initial value $(u_0, v_0) \in \mathbb{R}_+{}^2$ using the basic theory of ODE systems, and with the form $g(x) = p(x)x$, it is obvious that the answer is still positive and bounded, thus it must exist everywhere.

Consider the following differential equation to determine the shape of the phase portrait with respect to different set of parameter with non-negative values:

$$\frac{dx}{dt} = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{a+x^2}$$
$$\frac{dy}{dt} = \frac{emxy}{a+x^2} - d_2y - h_2y^2 \qquad (1)$$

$\Rightarrow$

$$\frac{dx}{dt} = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{a+x^2} = f_1(x,y),$$
$$\frac{dy}{dt} = \frac{emxy}{a+x^2} - d_2y - h_2y^2 = f_2(x,y) \qquad (2)$$

Table 1: Parameter

S. No.	Parameters	Meaning of Parameter
1	r	Intrinsic growth rate of prey
2	k	Fear coefficient
3	d_1	Natural death rate of prey
4	h_1	Coefficient of intra-specific competition of prey
5	m	Predation rate
6	e	Conversion factor
7	a	Half saturation constant
8	d_2	Natural death rate of predator
9	h_2	Coefficient of intra-specific competition of predator

For critical points, $\frac{dx}{dt} = 0 = \frac{dy}{dt}$

Therefore, $\frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{a+x^2} = 0$

$$\& \quad \frac{emxy}{a+x^2} - d_2y - h_2y^2 = 0 \qquad (3)$$

Therefore, the critical points are, $E_0(0,0), E_1(x_1,0), E_2(0,y_2), E_3(x_3,y_3)$.

For E_1, $\frac{r}{1+ky} - d_1 - h_1x - \frac{my}{a+x^2} = 0 \qquad (4)$

For E_1, $y = 0$, $r - d_1 - h_1x = 0$

$$\Rightarrow \quad x = \frac{r-d_1}{h_1}$$

E_1 will be feasible if $r > d_1$.

For E_2, $\frac{emx}{a+x^2} - d_2 - h_2y = 0 \qquad (5)$

For E_2, $x = 0$, $\quad -d_2 = h_2y$

$$\Rightarrow \quad y = \frac{-d_2}{h_2}$$

as $y_2 < 0$, E_2 does not exists.

For

E_3, $\frac{r}{1+ky} - d_1 - h_1x - \frac{my}{a+x^2} = 0, \frac{emx}{a+x^2} - d_2 - h_2y = 0$

(6)

2.1 Jacobian Matrix

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$
$$J_{11} = \frac{\partial f_1}{\partial x} = \frac{r}{1+ky} - d_1 - 2h_1x - \frac{my}{a+x^2} + \frac{2mx^2y}{(a+x^2)^2}$$
$$J_{12} = \frac{\partial f_1}{dy} = \frac{-rkx}{(1+ky)^2} - \frac{mx}{a+x^2}$$
$$J_{21} = \frac{\partial f_2}{\partial x} = \frac{emy}{a+x^2} - \frac{2emx^2y}{(a+x^2)^2}$$
$$J_{22} = \frac{\partial f_2}{\partial y} = \frac{emx}{a+x^2} - d_2 - 2h_2y$$

2.1.1 Stability Analysis of $E_0(0,0)$

$$J_0 = \begin{bmatrix} r-d_1 & 0 \\ 0 & -d_2 \end{bmatrix}$$

Therefore, the eigen values of J_0 are $r - d_1$ and $-d_2$.

Case-I

If $r < d_1$, then $r - d_1 < 0$ & $-d_2 < 0$.

Therefore, both eigen-values are negative and hence the system is stable around the critical point E_0.

Case – 2: if $r > d_1$, then $r - d_1 > 0$ & $-d_2 < 0$.

Therefore, the eigen-values are opposite sign,

Hence, the system becomes saddle around E_0.

2.1.2 Stability Analysis of $E_1(x_1, 0)$:

$$J_1 = \begin{bmatrix} r - d_1 - 2h_1x_1 & rkx_1 - \frac{mx_1}{a+x_1{}^2} \\ 0 & \frac{emx_1}{a+x_1{}^2} - d_2 \end{bmatrix} \qquad (7)$$

Therefore, the eigen-values of J_1 are given by $\det(J_1 - \lambda I) = 0$.

Therefore,

$$\lambda_1 = r - d_1 - 2h_1x_1 = r - d_1 - 2h_1\frac{r-d_1}{h_1} = -r + d_1$$
$$\lambda_2 = \frac{emx_1}{a+x_1{}^2} - d_2 = \frac{h_1em(r-d_1)}{h_1{}^2a + (r-d_1)^2} - d_2$$

i.e. if $-r + d_1 < 0$ and $\frac{h_1em(r-d_1)}{h_1{}^2a+(r-d_1)^2} - d_2 < 0$

i.e. if $r > d_1$ & $h_1em(r-d_1) < d_2\left[h_1{}^2a + (r-d_1)^2\right]$

Then the system becomes stable around the critical point $E_1\left(\frac{r-d_1}{h_1}, 0\right)$

2.1.3 Stability Analysis of $E_2(0, y_2)$:

Since $y_2 < 0$, E_2 does not exists.

2.1.4 Stability Analysis of $E_3(x_3, y_3)$:

$$J_3 = \begin{bmatrix} -h_1x_3 + \frac{mx_3y_3}{a+x_3{}^2} & \frac{-rkx_3}{(1+ky_3)^2} - \frac{mx_3}{a+x_3{}^2} \\ \frac{emy_3}{a+x_3{}^2} - \frac{2emx_3{}^2y_3}{(a+x_3{}^2)^2} & -h_2y_3 \end{bmatrix}$$

Therefore, the eigen-values of J_3 are given by $\det(J_3 - \lambda I) = 0$.

Therefore,

$$\begin{vmatrix} -h_1x_3+\dfrac{mx_3y_3}{a+{x_3}^2}-\lambda & \dfrac{-rkx_3}{(1+ky_3)^2}-\dfrac{mx_3}{a+{x_3}^2} \\ \dfrac{emy_3}{a+{x_3}^2}-\dfrac{2em{x_3}^2y_3}{(a+{x_3}^2)^2} & -h_2y_3-\lambda \end{vmatrix}=0$$

$$\Rightarrow \left(-h_1x_3+\frac{mx_3y_3}{a+{x_3}^2}-\lambda\right)(-h_2y_3-\lambda) - \left(\frac{emy_3}{a+{x_3}^2}-\frac{2em{x_3}^2y_3}{(a+{x_3}^2)^2}\right)\left(\frac{-rkx_3}{(1+ky_3)^2} - \frac{mx_3}{a+{x_3}^2}\right)=0$$

$$\Rightarrow \lambda^2-\left(h_1x_3-\frac{mx_3y_3}{a+{x_3}^2}+h_2y_3\right)\lambda + \left(-h_1x_3+\frac{mx_3y_3}{a+{x_3}^2}\right)(-h_2y_3) - \left(\frac{emy_3}{a+{x_3}^2}-\frac{2em{x_3}^2y_3}{(a+{x_3}^2)^2}\right)\left(\frac{-rkx_3}{(1+ky_3)^2} - \frac{mx_3}{a+{x_3}^2}\right)=0$$

Therefore,

$$\lambda_1=\frac{(J_{11}+J_{22})+\sqrt{(J_{11}+J_{22})^2-4(J_{11}.J_{22}-J_{12}.J_{21})}}{2}$$

$$\lambda_2=\frac{(J_{11}+J_{22})-\sqrt{(J_{11}+J_{22})^2-4(J_{11}.J_{22}-J_{12}.J_{21})}}{2}$$

E_3 will be stable if $\lambda_1<0$ and $\lambda_2<0$.

Now if $J_{11}+J_{22}=0$, then λ_1, λ_2 will be purely imaginary. Therefore, $\lambda_1=0+i\beta$ & $\lambda_2=0-i\beta$, where $\beta=\sqrt{4(J_{11}.J_{22}-J_{12}.J_{21})}$.

Therefore, the system will be closed orbit around $E_3(x_3,y_3)$.

3. Numerical Analysis

Now, if $r=1, k=0.1, d_1=0.4, d_2=0.4, h_1=0.1, h_2=0.1, m=1, e=0.9, a=0.2$ and solving the equations $\frac{rx}{1+ky}-d_1x-h_1x^2-\frac{mxy}{a+x^2}=0$ & $\frac{emxy}{a+x^2}-d_2y-h_2y^2=0$,

We get the feasible values of x & y as

$$(x=0,y=0),(x=6,y=0),$$
$$(x=0.09577820312, y=0.1209999144),$$
$$(x=1.673227734, y=1.020200199)$$

3.1 Case-I: For the point $E_0(0,0)$

Case a: $J_{11}=0.6=r-d_1, J_{12}=0, J_{21}=0, J_{22}=-0.4=-d_2$

$$J_0=\begin{bmatrix}0.6 & 0\\ 0 & -0.4\end{bmatrix}$$

Therefore, the eigen-value of J_0 is given by $\det(J_0-\lambda I)=0$. Which gives $\lambda_1=0.6$ & $\lambda_2=-0.4$

Here $\lambda_1>0$ & $\lambda_2<0$. So the Phase Portrait will be Saddle around $E_0(0,0)$.

Case b: again, if $r=1, k=0.1, d_1=1.5, d_2=0.4, h_1=0.1, h_2=0.1, m=1, e=0.9, a=0.2$ and solving equation (3) we get the only feasible solution of x & y as $(x=0,y=0)$.

Now for the point $E_0(0,0)$, $J_{11}=-0.5=r-d_1, J_{12}=0, J_{21}=0, J_{22}=-0.4=-d_2$, we get

$$J_0=\begin{bmatrix}-0.5 & 0\\ 0 & -0.4\end{bmatrix}$$

Therefore, the eigen-value of J_0 is given by $\det(J_0-\lambda I)=0$, i.e. $\lambda_1=-0.5$ & $\lambda_2=-0.4$

Here $\lambda_1<0$ & $\lambda_2<0$. So, the Phase Portrait will be Stable around $E_0(0,0)$.

3.2 Case-II: For the point, $E_1(x_1,0)$, *i.e.* $E_1\left(\frac{r-d_1}{h_1},0\right)$, *i.e.* $E_1(6,0)$

$$J_{11}=-0.6, J_{12}=0.4342541436,$$
$$J_{21}=0, J_{22}=-0.25087293$$

$$J_1=\begin{bmatrix}-0.6 & 0.4342541436\\ 0 & -0.25087293\end{bmatrix}$$

Therefore, the eigen-value of J_1 are $\lambda_1=-0.6$ & $\lambda_2=-0.25087293$.

Here $\lambda_1<0$ & $\lambda_2<0$. So, the Phase Portrait will be Stable around $E_1(6,0)$.

3.3 Case -III: The point $E_2(0,-4)$ does not exists, since $y_2<0$.

3.4 Case -IV: For the point $\boldsymbol{E_3(x_3,y_3)}$, i.e. $E_3(0.09577820312, 0.1209999144)$

$$J_{11}=0.04582669491, J_{12}=0.4485387022, J_{21}=-0.4749557624,$$
$$J_{22}=-0.01209999144$$

$$J_3=\begin{bmatrix}0.04582669491 & 0.4485387022\\ -0.4749557624 & -0.01209999144\end{bmatrix}$$

Therefore, the eigen-value of J_3 is given by $\det(J_3-\lambda I)=0$,

$$\lambda_1=\frac{3372670347-5i\sqrt{3395154656342955811 19}}{200000000000}\ \&$$

$$\lambda_2=\frac{3372670347+5i\sqrt{339515465634295581119}}{200000000000}$$

Since, λ_1 & λ_2 are complex conjugate, the Phase portrait will be Unstable Focus.

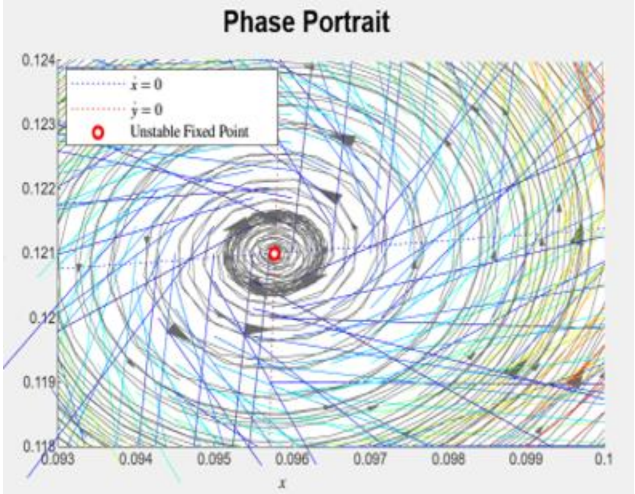

Figure 3: Unstable Focus around the critical point $\boldsymbol{E_3(x_3,y_3)}$.

Again, if $r = 0.9, k = 0.2, d_1 = 0.5, d_2 = 0.3, h_1 = 0.7, h_2 = 0.7, m = 0.9, e = 0.8, a = 0.3$ and solving equation (3), we get only the feasible values of x & y as $(x = 0, y = 0)$,

$$(x = 0.5714285714, y = 0.09681936868), (x = 0.1675844793, y = 1.020200199)$$

Now for the point $\boldsymbol{E_3}(\boldsymbol{x_3}, \boldsymbol{y_3})$ **i.e.** $\boldsymbol{E_3}(0.5714285714, 0.09681936868)$**,** the Jacobian is given by

$$J_1 = \begin{bmatrix} -0.07279963099 & -0.4887470001 \\ 0.1760990588 & -0.06777355787 \end{bmatrix}$$

Therefore, the eigen values are given by

$$\lambda_1 = \frac{-7028659443 - 4i\sqrt{53788482097613505779}}{100000000000} \;\&$$

$$\lambda_2 = \frac{-7028659443 + 4i\sqrt{53788482097613505779}}{100000000000}$$

Since, λ_1 & λ_2 are complex conjugate, with $\alpha < 0$, the Phase portrait will be Stable Focus.

Case – V: Again for the point $\boldsymbol{E_3}(\boldsymbol{x_3}, \boldsymbol{y_3})$ **i.e.** $\boldsymbol{E_3}(0.1675844793, 1.020200199)$ we will get, $J_{11} + J_{22} = 0.2997249002 > 0$ & $J_{11}.J_{22} - J_{21}.J_{12} < 0$. So the phase portrait will be unstable.

3.5 Case – VI: If $r = 0.54, k = 0.1, d_1 = d_2 = 0.4, h_1 = h_2 = 0.1, m = 1, e = 0.9, a = 0.2$ and solving the equation (3), we get the feasible values of x & y as $(x = 0, y = 0)$, $(x = 0.09339069218, y = 0.02696864202)$.

Now for the point $\boldsymbol{E_3}(\boldsymbol{x_3}, \boldsymbol{y_3})$ **i.e.** $\boldsymbol{E_3}(x = 0.09339069218, y = 0.02696864202)$

$J_{11} + J_{22} \approx 0$ & $J_{11}.J_{22} - J_{21}.J_{12} > 0$, so the phase portrait will be closed orbit around $\boldsymbol{E_3}(\boldsymbol{x_3}, \boldsymbol{y_3})$**.**

4. Result & Conclusion

The fear of predators has been incorporated into a predator-prey model in this study using either the linear functional response or the Holling type IV functional response, depending on the model. The cost of dread does not alter the model's dynamical behaviours in the case of the linear functional response, according to mathematical findings, and when it does exist, a unique positive equilibrium is asymptotically stable.

However, the cost of fear has a variety of effects on predator-prey interactions in the model with the Holling type IV functional response. If the birth rate of the prey is not high enough to support fluctuations, analytical methods reveal that there exists a globally stable positive equilibrium. No of how susceptible the prey is to possible threats from predators, in this situation, the populations of prey and predators tend to finally yield positive constants. If the level of fear is high, the positive equilibrium of the predator-prey system is locally asymptotically stable when the birth rate of prey is great enough to support oscillations. By excluding periodic fixes in these situations, the cost of fear can be stabilise the predator-prey relationship. In order to prevent the "paradox of enrichment" in ecosystems, this provides a new mechanism. When the dread is somewhat modest, periodic solutions are still possible. Indicating that the cost of dread will not affect the existence of Hopf bifurcation but also modify the direction of Hopf bifurcation, conditions for the existence of Hopf bifurcation and circumstances influencing the direction of Hopf bifurcation are obtained. In fact, we have shown that Hopf bifurcation can be both supercritical and subcritical in the model adding the cost of fear, in contrast to the traditional predator-prey models that do not take into account the consequences of predation risk, where Hopf bifurcation can only be supercritical.

By releasing one or two more factors than just k, numerical simulations are run to demonstrate the potential impact that fear effects can have on predator-prey interactions. When the Hopf bifurcation is present, rising anxiety levels have the potential to shift the direction of the bifurcation from supercritical to subcritical when prey birth rates rise in tandem. Depending on the initial population size, fear can produce a variety of complex dynamical behaviours, such as bi-stability, where the solutions trend to a steady state or fluctuate periodically. According to numerical simulations, the prey is less susceptible to perceived predation risk when the prey birth rate is large, regardless of how other factors vary. Additionally, when the assault (i.e. predation) rate is high, the prey would be more inclined to exhibit anti-predator defences and would perceive fewer possible risks as the predator death rate increases. Simulations of the cost of dread using various functions show that our findings in this work hold true even when using other broad monotone-decreasing functions.

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