Comparative Analysis of Time-Domain and Frequency-Domain Methods for Modeling Jamaica's Quarterly GDP

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Abstract: ***This paper presents a comparative analysis of time-domain and frequency-domain approaches for modeling Jamaica's quarterly Gross Domestic Product (GDP) from 2004 to 2015. Using ARIMA models and spectral estimation techniques including periodograms and Daniell kernel smoothing, the study explores the capacity of each method to decompose GDP data into trend, seasonal, and cyclical components. The time-domain approach, while suitable for short-term trend modeling, was limited in isolating periodic behaviours, particularly under high variance. Frequency-domain methods, on the other hand, revealed hidden cycles and enabled clearer interpretation of macroeconomic dynamics. The findings underscore the complementary nature of both approaches in providing comprehensive insights for economic policy formulation, especially in the context of small economies like Jamaica.***

Keywords: time-domain, frequency-domain, GDP modeling, spectral analysis, autoregressive models

1. Introduction

Spectral (spectrum) analysis was first introduced by Isaac Newton in 1664. He looked at the decomposition of light signal into different frequency components as it passes through a glass prism (Brillinger, 2002; Robinson, 1982; Poskitt, 2019). In 1800, Herschel looked at the energy of the sunlight which he averaged to obtain various frequency bands of its associated spectrum. This involved the use of thermometers which were placed alongside Newton's spectrum. During the mid-1800s, the mathematical point of view of the spectrum surfaced by Gouy, who represented white light as a Fourier series. Over time, Michelson and Stratton used the Fourier transform to provide an estimate of the (power) spectrum of the signal as a sum of cosines. This involved the use of the estimate of the spectra where the results facilitated description of the light-emitting sources. Between 1894 and 1898, Schuster offered the use of the periodogram to facilitate statistical analysis of time series data, where individuals could not identify hidden periodicities in the data visually (Brillinger, 2002). However, using the periodogram as is limited deep insights into these periodicities, necessitating improved spectral estimation methods. Therefore, Wiener, Cramer, Kolmogorov, Bartlett, and Tukey worked on more developments to improve the spectral estimator in 1930 (Brillinger, 2002). Tukey, however, proceeded to use spectrum analysis to develop his overall views of data analysis in time series.

Now, due to the nature of macroeconomic time series, which often reflect trends, fluctuations, seasonality, and noise, this type of analysis tends to be considered, particularly for Gross Domestic Product (GDP) data. Traditionally, statisticians have modelled such time series using econometric approaches typically defined by time-domain parametric models, which include Autoregressive Integrated Moving Average (ARIMA) and Vector Autoregression (VAR) (Golyandina & Zhigljavsky, 2020; Hassani & Mahmoudvand, 2018). These models are often found to be reliant on time-domain approaches and normally facilitate forecasting by carefully considering stationary properties. The problem, however, with looking at these methods, is the entanglement involved based on the overlap of the following components of a time series: trend, noise, and cycle. These components are often seen to make the data analysis complicated, especially based on their structure or even characterized to be nonstationary. As such, another framework is considered, which can act as a complement to the time-domain, which is referred to as frequency-domain and subspace methods (Brillinger, 2002; Pollock, 2007; Poskitt, 2019). This framework facilitates the decomposition of the time series into components reflecting fluctuations across differing frequencies. Thus, these models are said to have sinusoidal components, which are time series but can be considered in both the time-domain and frequency domain. Moreover, Spectral analysis indicates a shift from the time-domain approach to the frequency-domain approach by its connection to Fourier transforms. Due to certain properties of Fourier transforms, such as their ability to shift from time to frequency and vice versa, spectral analysis actually encapsulates this transform because it thrives on a frequency point of view, since it analyzes the frequency of signals such as white noise (Brillinger, 2002). Additionally, the Fourier transform exists in the realm of Fourier analysis, which considers a function containing the sum of sine and cosine terms. Based on this function, it can be seen by plots of the graph showing the sine and cosine terms as well as their sum, which projects wave forms of signals that can be identified via frequencies. This involves applying various spectral estimation techniques, such as periodogram and smoothed periodogram, allowing us to identify cycles, periodicities, and oscillations, based on macroeconomic dynamics. Furthermore, macroeconomic data of developing economies are known to be impacted by structural breaks, irregular cycles, or even changes in trends, thereby exhibiting noise. For these types of data, frequency-domain methods are used to facilitate the analysis. In this research, spectral analysis was applied to a time series of data to provide a comparison between the time-domain and frequency-domain using Jamaica's quarterly GDP data for the period of 2004 and 2015.

Spectral Analysis is known to be developed in time series to ascertain the cycles and trends that may exist in the dataset. Typically, this involves the use of the Fourier transform and its inverse by leveraging different parameters, such as the smoothing parameter and methods such as periodograms. This is done by observing the frequency components defined by sinusoidal waves associated with periods, amplitudes, and phase (Brillinger, 2002; Pollock, 2007). Consequently, the descriptive properties of Spectral analysis allow individuals to recognize time series structure and cyclical behaviour at different time scales through filtration. This is typically done by preserving the data points that may be lost in the filtering process and avoiding bias. As such, spectral tools facilitate analysis of the co-movements among series and the frequency of the business cycle. Pomenkova and Marsalek (2012) stated that these types of analyses are done using the time-domain and frequency-domain approach, but that when cycles are nested, time-domain approaches are not enough because the data is not broken down into fragments for careful examination. They argued that to see these cycles structurally and based on their behaviour, a frequency-domain approach should be considered to get richer insights into the analysis.

Purpose of the Study

The purpose of this study is to compare time-domain and frequency-domain approaches for modeling Jamaica's quarterly GDP, intending to evaluate their individual strengths and complementary roles in economic analysis.

Significance of the Study

This study is significant as it applies advanced spectral analysis methods to a developing economy, offering rare insights into cyclical and structural behaviours in Jamaica's GDP that traditional models may overlook. It bridges a notable research gap in Caribbean macroeconomic modeling.

Preliminaries

Time Series (Chatfield, 2016):

1) A time series $\{x_t, t \geq 0\}$ is a set of observations representing the value of a variable at consecutive points in time t.
2) A time series $\{x_t, t \geq 0\}$ is
   a) Continuous if observations are made continuously;
   b) Discrete if observations are made at discrete time points;
   c) Deterministic if past values allow the prediction of the future values with certainty;
   d) Stochastic if future values cannot be predicted with certainty due to random influences.

Autoregressive Process (AR(p)): The stochastic process $X = \{X_t\}_{t\in\mathbb{N}}$ is an autoregressive process of order p if it can be expressed in the form

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + Z_t$$

where $\{Z_t\}_{t\in\mathbb{N}}$ is a purely random process with mean 0 and variance σ_Z^2 (Chatfield, 2016).

Kernel: This is a special type of probability density function with the added property that it must be even (Diebold, 2007; Moses & Stoica, 2005).

Daniell kernel: This is an approach involving the use of a symmetric moving average, where all the weights assigned to the frequencies are of equal value (Moses & Stoica, 2005).

Modified Daniell kernel: This is an approach, unlike the Darnel kernel involves the use of a symmetric moving average, where greater weights are assigned to the central frequencies with the intent of avoiding bias related to the peaks of dominant cycles (Moses & Stoica, 2005).

Spectrum: The Spectrum $f(\omega)$ can be estimated using either a parametric approach or a non-parametric approach. Let $\hat{f}(\omega)$ be an estimation of the spectrum $f(\omega)$. Then the non-parametric estimation is based on the following formula

$$\hat{f}(\omega) = \sum_{m=-h}^{h} \varkappa\left(\omega_{j+1}, \omega_j\right) I_n\left(\omega_{j+m}\right); \quad \sum_{m=-h}^{h} \varkappa\left(\omega_{j+1}, \omega_j\right) = 1;$$

$$I_n^*\left(\omega_j\right) = \frac{1}{\pi}\left[\hat{\gamma}(0) + \sum_{k=-1}^{T-1} \varkappa_k^* \hat{\gamma}(k)\cos\omega_j k\right]$$

where $\omega_j = \frac{2\pi j}{T}$, with h defined as the bandwidth used to smooth the periodogram $I_n\left(\omega_j\right)$, generally representing the number of frequencies needed to facilitate the best estimation (Bátorová, 2012; Diebold, 2007; Iacobucci, 2003; Priestley, 1981; Velasco, 1998; Wang, 1999). Here, $\varkappa_k^*$ is also called the kernel, the weight used to facilitate the smoothing process: $\varkappa_k^* = \begin{cases} 1 - \frac{|k|}{h}, \ for \ |k| \leq h \\ 0, \ otherwise \end{cases}$.

2. Literature Review

This section explores studies that are focused time-domain and frequency domain approaches. It provides literature that makes comparison of the two approaches in terms of their advantages and limitations.

2.1 Time-Domain Approach vs Frequency-Domain Approach

In the time-domain, one can examine a variable (time series) to see how it changes over time, normally done using a graph. While in the frequency-domain, the same variable is assessed using frequencies to observe patterns of cycles using graphs to identify repetition patterns and interpret the outcomes (Pollock, 2007). To understand frequency-domain methods, one must familiarize oneself with the foundation building blocks of sinusoids, or sines and cosines, since the frequencies or wavelengths are directly linked to them. In other words, every frequency component of a time series is related to sinusoids whose innate nature is characterized by cyclical behaviours, due to their waveforms. The frequency-domain representations reveal the strength of each cycle, the number of times it is repeated, and how each phase shift is aligned to the waves generated from sine and cosine functions. When the frequency is at its highest point for a spectral analysis, it is called the Nyquist frequency (Chatfield, 2016; Pollock, 2007). It allows the breakdown of time series data into annual trends, business cycles or even short-term shocks to assess the patterns observed, as well as the variance levels evidenced in the dataset.

The Fourier transform facilitates linkages between time-domain and frequency-domain using functions that allows conversion of the time function into a sum of sine waves of different frequencies. The 'spectrum' is estimated based on the frequency component indicated by the waves, based on the signal (Hassani & Zhigljavsky, 2008; Pollock, 2007). Note that this conversion process is not only one-way, from time-domain to frequency-domain but also vice versa using the inverse Fourier transform.

2.2 Limitations Associated with Using Time-Domain Models for Macroeconomic Time Series

Classical Time-Domain Models have been traditionally used to facilitate analysis of macroeconomic time series data. These models, as previously mentioned, include ARIMA and VAR as well as structural time-series. The models have been utilized to provide forecasting or even impulse-response analyses (Chatfield, 2016). However, these types of models over time become too complex to handle based on the nature of the time series, where it may not be stationary or heavily affected by structural changes, restricting the applications of assumptions such as stationarity, constant variance, and linearity. In other words, these models do not work because the following remain still visible in the time series: cycles, long-term trends, noise, and seasonal variations (Chatfield, 2016). Furthermore, it has been argued that if the time-domain models used are mis-specified, then there may be misleading inference based on the time series data, particularly concerning trend behaviour or business cycles. Based on the limitations of time-domain models, alternative frameworks are considered to not only deal with the sequences based on time but also their frequency structure.

2.3 Frequency-Domain Advantages

Thereby, calling for the use of frequency-domain methods, notably the singular spectrum analysis (SSA). According to Golyandina & Zhigljavsky (2020), SSA is particularly useful for economic time series that do not follow the stationarity assumptions or may be non-linear. SSA is typically employed due to its non-parametric decomposition characteristics that are flexible. It involves the integration of classical time-series analysis, signal processing, and multivariate statistics. This method is effective for extracting the long-term trends and cycles from the noisy data. The model provides reconstruction of the “signal sub-space” that facilitates forecasting. One recent application involving the use of SSA is shared in a research by Hassani & Zhigljavsky (2008) titled “Singular spectrum analysis based on the minimum variance estimator”. This application involved time series data focusing on monthly accidental deaths in the USA, to facilitate reconstruction and forecasting of the series. Based on their investigation, they reported that SSA is a very effective nonparametric method to deal with noise and non-stationary found in economic datasets. This is particularly evident based on its ability to decompose economic time series to show the trend, noise, and oscillatory components.

3. Data Description and Reprocessing

Figure 1 below shows a plot of Quarterly Gross Domestic Product (GDP) at market prices time series data. This figure is a graphical method, which is one aspect of time-domain modeling. It revealed that there exists an upward linear trend, indicating that the variance is increasing with time, showing signs of heteroscedasticity. The study uncovered that throughout the period of 2004 to 2015, Jamaica experienced economic growth. This is evident from the consistent growth pattern over time, with noticeable changes during the periods 2010 and between 2013 and 2014. External shocks typically cause these changes. The GDP in 2010 was negatively affected by the global economic crisis, which occurred in 2008, as well as Hurricane Nicole, which immensely affected tourism, mining, and remittances (International Monetary Fund, 2010). By 2013, Jamaica was severely burdened with debts (Malcolm & Schmid, 2016). Furthermore, it indicates that multiplicative seasonality with the upward trend as well as non-stationarity attributes, though in the short run. The characteristics of multiplicative seasonal effects or business cycles are typically reflected based on the variability scales, where variance keeps increasing with time. As such, even with the aid of time-domain methods, the data would not be fully decomposed to clearly show the seasonality distinctively as opposed to the trend or the noise, particularly due to its high variance.

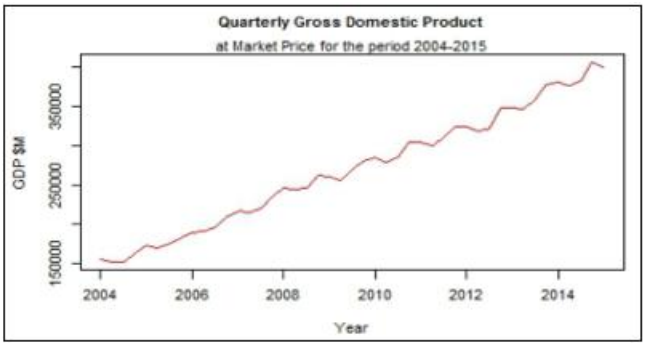

Figure 1: Quarterly Gross Domestic Product at market price for the period 2004-2015

Therefore, a logarithmic transformation was applied to the GDP time series to stabilize the variance and linearise the exponential growth seen. This was done to make it easier to deal with the time series for both time-domain and frequency-domain analysis, as well prepare the data so the spectral analysis can be done. This decision equally aligns with both Brillinger’s (2002) and Hassan & Zhigljavsky’s (2008) work, which carefully considers best practices in conducting time series and spectral analysis.

Figure 2, however, provides the raw periodogram of the original data for the quarterly GDP data over 2004 and 2015. This display provides the raw spectral estimate of the time series, with the spectral density represented on the y-axis and the frequency on the x-axis. This representation allows us to recognize the periodic components and understand the distribution of power across frequencies. A careful look at the figure reveals jaggedness and high-magnitude fluctuations, indicating an unsmoothed periodogram. The jaggedness of the graph implies that there is high variation within the spectral estimate, causing inconsistencies due to the existing noise. Further insights reflect that the calculation of each frequency is done without averaging, facilitating these inconsistencies, throwing off the estimation, making the periodogram not a true estimator of the spectrum. Additionally, there exist dominant peaks at specific frequencies, implying that cyclical components exist based on the periodic behaviour in the GDP data. For instance, a frequency of approximately 0.25 corresponds to a strong peak, indicating that in a year, there is a cycle of about 4 quarters [the regular annual economic cycle]. This is closely tied to the expected economic seasonality for a quarterly data set. Furthermore, based on the comparative analogy of the original data, even if the GDP data were transformed using logarithms, there still would not be any difference with the spectral estimate. This is evidently seen in the shape of the spectrum and the preservation of the underlying periodicities. This characterization is consistent with theory, where the logarithmic transformation is usually monotonic, not changing the frequency or periodicity.

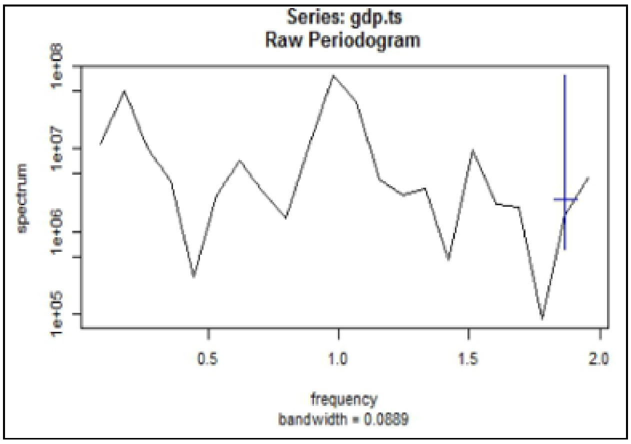

Figure 2: Logged Quarterly GDP

Moreover, the frequency across the x-axis ranges from 0 to 2 cycles per quarter (time period), indicating that the lower the frequency, power is most concentrated. This, based on theory, coincides with longer-term cyclical and trend components. The cycles are revealed to be 1-year for a frequency of 0.25; however, at a frequency of 0.5, there exists a 6-month cycle. As we move across the frequencies and reach 1.0, a 3-month cycle is represented, while 2.0 corresponds to a 1.5-month cycle. This is in line with the general behaviour of GDP data, which tends not to have high-frequency behaviour, mainly having frequencies between 0-1. Now, the next step most suited to address the high variance and obtain a better spectral estimate is through the application of smoothing techniques such as the Daniel Kernel and modified Daniel Kernel, which will be discussed later on. This approach, according to Brillinger (2022), is necessary to investigate the noise in the dataset. Following the computation of the smoothed periodogram, the spectral estimators with be calculated, specifically AR(1), which is needed to compare the parametric and non-parametric versions of the spectrum.

4. Time-Domain Modelling

Another approach for estimating the spectrum of the gross domestic product (GDP) is the parametric approach. This approach entails finding the best-fitting autoregressive (AR) model for the series and then plotting the spectrum of the model. For this to be done, the order p of the AR process is first needed. This was achieved by evaluating Akaike's information criterion (AIC) and minimizing its value across the various model orders 1 through 30. Generally, the metric formula used is $AIC = \frac{2l}{n} + \frac{2p}{n}$, where l is defined as the likelihood while p is the number of parameters [also known as the model order]. So, a plot for the model selection criterion AIC (as seen in Figure 3) as a function of order p for the autoregressive models fitted to the GDP series was done. The order was found to be p = 1, indicating lag-1 dependency. This AR(1) model indicates that there is dependence across quarter's of the GDP data, a previous quarterly data is used to provide context to the present data set being observed. We see, compared to Figures 1 and 2, that there is no jaggedness in the data set, but it still constitutes the upward trend. In other words, we see that as AIC increases, p also increases, having a smoothed parametric spectrum. Also, the low-frequency components previously seen in Figure 2 are now isolated. This parametric approach facilitates a smoother spectrum that has fewer complexities affecting interpretation as opposed to the jagged periodogram in Figure 2. This approach is advantageous since it allows the reduction of the high variance that affected the spectral estimate, as well as being able to identify the dominant frequency aspects, and it bridges the gap between the two domains through the integration of time-domain autoregressive modeling.

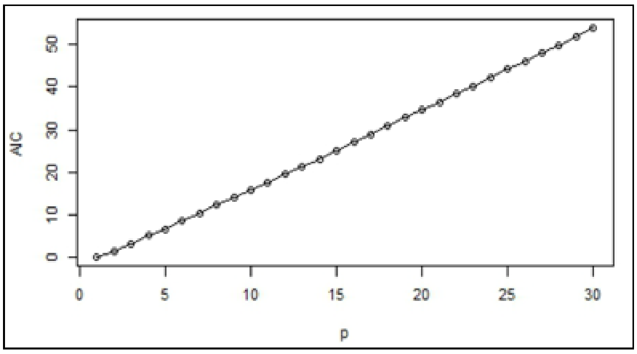

Figure 3: Model Selection Criterion AIC

5. Frequency-Domain Domain Analysis using Spectral Estimation

Now, the Quarterly Gross Domestic Product (GDP) time series was examined exactly as it is. The periodogram was generated for it without any form of transformation to assess it in its natural sense. This frequency-domain approach reveals the hidden cycles that were not visible in the time-domain plot provided in Figure 1. The periodogram in Figure 4 shows that there are three profound peaks with strong cycles, making the spectral structure more visible. The dominant peak occurs at a medium-frequency $\omega_1 = 0.97777778$. The period for this value is $\frac{1}{\omega_1} = \frac{1}{0.97777778} = 1.023$, indicating that it takes three months (1 quarter) for a complete cycle. This dominant peak indicates a strong quarterly pattern having regular seasonal effects. These effects come in the form of profound intra-year variation, where GDP is influenced by tourism, taxes, or even consumer demand. At low frequency, $\omega_2 = 0.08888889$ gives the second peak, and this corresponds to period $\frac{1}{\omega_2} = \frac{1}{0.08888889} = 11.25$, indicating that it takes thirty months (11 quarters) for a complete cycle. Here, we see that the economic cycle is longer, which may be attributed to business cycles, policy-related adjustments, or even investment cycles. While at high-frequency, $\omega_1 = 1.60000000$, gives the third peak with corresponding period $\frac{1}{\omega_3} = \frac{1}{1.60000000} = 0.625$, indicating that it takes approximately two months (0.625 quarters) for a complete cycle. This is the shortest cycle observed, which is unusual, but may be a result of the noise or even the high-frequency aspect. This periodogram is jagged, having high-frequency fluctuations. As such, to get an estimate of the spectrum, based on the non-parametric method, it is necessary to smooth the periodogram to deal with the roughness as displayed by peaks. This is necessary to minimize the noise and help to make the non-parametric spectral estimate more stable and interpretable.

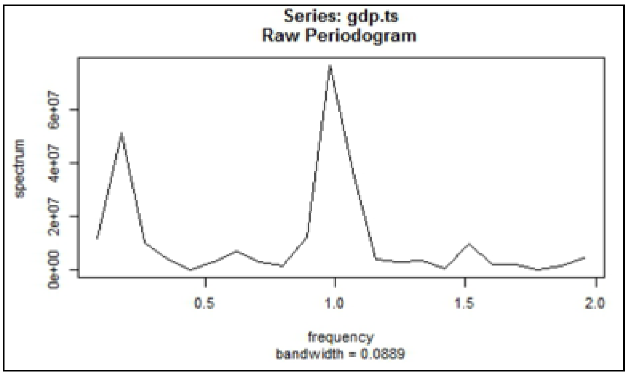

Figure 4: Periodogram of Quarterly GDP

To improve the spectral estimate, smoothing the data is necessary, as previously mentioned and discussed based on Figures 2 and 4. Now, the process of smoothing the data involves using the Daniell kernel or the modified Daniell kernel with a smoothing parameter m. As can see, the bandwidth is 0.18 which was used for both smoothing method resulting in moderate smoothing. From the diagrams in Figure 5, the modified Daniell kernel produces a more effectively smoothed periodogram than the Daniell kernel. The dominant peaks seen in Figure 4 have now been refined, reducing the variance. This is true since the unmodified Daniell kernel tends to make the center values weigh slightly less heavily than in the modified Daniell kernel. As such, the preferred kernel used is the modified one. This kernel makes the spectrum more accurate and interpretative, which is carefully in line with best practices established in Tukey's foundational work (Brillinger, 2002). These results indicate that the use of frequency-domain approaches is better to use to reveal the periodic or cyclical aspects of the dataset that were not so visible using time-domain models, as seen in Figures 1 and 2. Furthermore, there is now one visible peak at a frequency of approximately 1, facilitating a more distinguishable feature while indicating a strong quarterly cycle in the data when compared to Figures 2 and 4. Now, the dominant cycles have a better sense of the accuracy of the amplitude estimates.

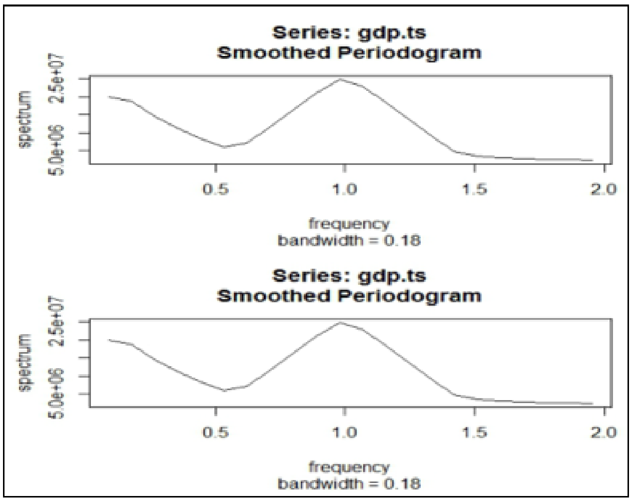

Figure 5: Smoothed Periodogram of Quarterly GDP with the use Daniell kernel and modified Daniell kernel respectively for $m = 2$

Further smoothing was considered to improve the accuracy of the spectral estimate for the data. Now, the choice of the smoothing parameter m, is done manually in order to find the one that smooths the periodogram nicely. This parameter identifies the number of adjacent frequency points that are necessary to be averaged in the modified Daniell kernel. The first value of the parameter chosen was $m = 2$ with bandwidths of 021 and 0.296, which did not smooth the periodogram nicely, even when two passes of the modified Daniell kernel were done using the same value as seen in Figure 6. It indicates inadequate smoothing since it still

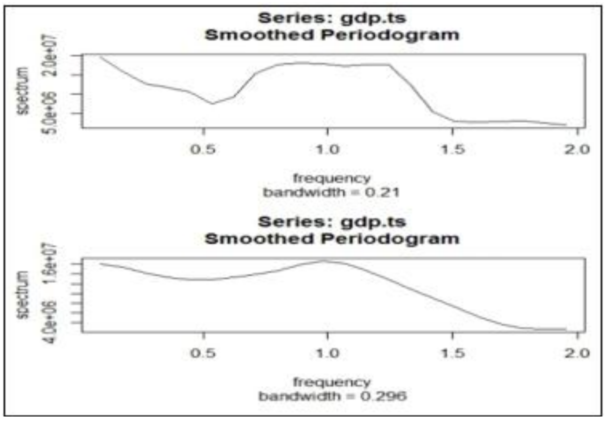

Figure 6: Smoothed Periodogram for m = 2 with modified Daniell kernel

exhibited noise, and the peaks were difficult to identify. This was mostly visible between the 0.5 and 1.5 frequency bands. This level of smoothing is not much different from that shown in Figure 5.

As such, another value for the parameter was considered, which is $m = 4$, where two passes [bandwidths 0.21 and 0.296] were made in smoothing the periodogram, which gives a much better outcome for the periodogram as seen in Figure 7. The dominant peak, just like Figure 5, was still found to be at a frequency of about 1.0, where the variance has now been suppressed. The more the periodogram is smoothed is the more it looks like its spectrum, even though it should be noted that in creating the value of the smoothing parameter, it can cause an inability to identify peaks. After the periodogram is smoothed enough, one can identify where the dominant peaks are located. Therefore, it is crucial to carefully select an appropriate smoothing parameter so that the peaks do not become masked while ensuring under-smoothing does not occur, where the noise would still be there. The study revealed that more noise is retained when less smoothing occurs, that is, when smaller m are used. However, larger m are best suited but should be properly gauged to avoid masking spectral peaks. As such, finding the optimal m is paramount.

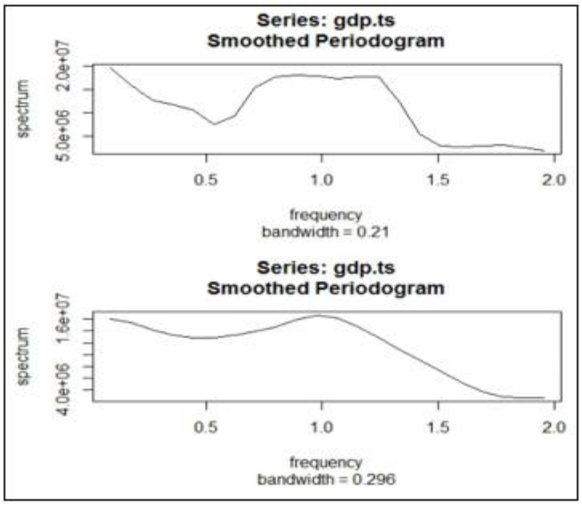

Figure 7: Smoothed Periodogram for $m = 2$ with modified Daniell kernel

6. Comparison and Interpretation

This study facilitates the comparison between the time-domain and frequency-domain through the examination of GDP time series data for the period of 2004 and 2015. As previously mentioned, Figure 1 was generated using time-domain methods to analyse the trend of the dataset. Figure 2, however, shows the jagged appearance of the data through the unsmoothed periodogram with a bandwidth of 0.0889. It displayed various cyclical behaviours at different frequencies, revealing GDP fluctuations. This is the advantage of using frequency-domain approaches, where hidden components such as variation and periodicities are usually revealed that otherwise would not be visible in time-domain models. Figure 3, however, involves selecting the AR(p) model using time-domain features via AIC. Figure 3 captures the visual result of obtaining the optimal AR(p) for the GDP data, where the order p is known. This model is later transformed to that of the frequency-domain, as seen in Figure 8, to obtain the autoregressive spectral estimator for the GDP time series. Figure 8 illustrates a steep decline of the spectrum as frequencies increase, where power is more concentrated at lower frequencies. The spectrum for this model suggests that the parameter for the AR(1) process would be $\alpha > 0$ since the graph decreases exponentially, indicating that power is concentrated at low frequencies; as such, there exists a low-frequency spectrum. This aligns with the slower cycles and long-term trends. Evidently, AR models create a connection between both the time-domain and frequency-domain, indicating moments where these domains complement each other.

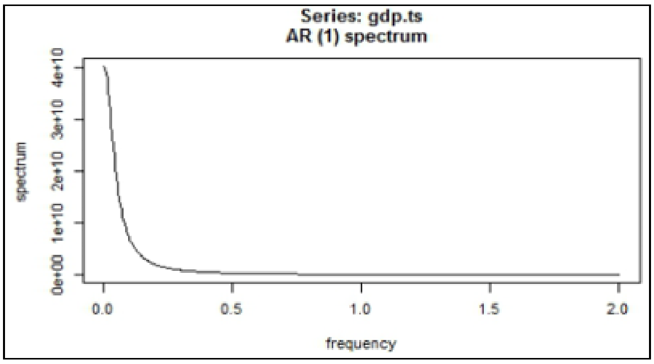

Figure 8: AR (1) Spectral Estimator for GDP

Figures 4 and 5, as previously mentioned, resulted from smoothed periodograms using the modified Daniell kernel, a frequency-domain approach providing more reliable smoothing. Thus, addressing spurious peaks that stemmed from the noise. Figures 6 and 7, however, were also generated using a frequency-domain approach for fine-tuning smoothing but with manual selection of the parameter m. Figure 6 was obtained through less smoothing, implying low bias where the peaks are retained, while Figure 7 had a higher bias as more smoothing occurs, implying a bias-variance trade-off. This reflects that while the high variance is addressed through more smoothing, high bias occurs as the curve gets flatter for better spectral estimation, but can cause divergence regarding the true underlying spectrum. Smoothing introduces some bias, the severity of which is dependent on the size of the bandwidth h. The bandwidth and the variance have an inverse relationship. Additionally, the peaks may become narrower where distinct cyclical behaviour may not be visible. Consequently, there needs to be a balance between bias and variance, two competing phenomena as documented by Chatfield (2016) and Diebold (2007) by who caution against over-smoothing. When non-parametric frequency-domain methods like smoothing, a modified Daniell kernel are used, the model benefits from flexibility through the adjustment of the smoothing parameter values, which Hannan (1990) suggests is a practical approach, especially for an economic time series study. However, the flexibility of time-domain methods does not exist because the pre-existing nature of these methods is already predefined in terms of assumed fixed structures.

Another frequency-domain analysis is evident based on Figure 9, where the techniques facilitate the decomposition of the data into short-run, medium-run, and long-run dynamics [business cycles]. Figure 9 shows a dominant spike at a medium frequency $\omega_1 = 0.97777778$, corresponding to a medium-run business or seasonal cycle. This appears to occurs for over 1-year periods. The second peak, on the other hand, which occurs at a low-frequency $\omega_2 = 0.08888889$, corresponds to a short-run business cycle. These cycles are often irregular and affected by external shocks, but not for an extensive time period. While the third peak occurs at a high frequency $\omega_3 = 1.6000000$, which indicates a long-run business cycle. This indicates that persistent macroeconomic dynamics are at play, affecting GDP. Throughout the period of 2004 and 2015, the GDP was affected by the debt burden faced where the country went through deficit (International

Monetary Fund, 2010). However, this type of isolation in to various frequencies is not possible for time-domain models like ARIMA even though they are profound, unless additional conditions are considered such as differencing or even seasonal lags. Even so, it may have to be done multiple times, and still might not be very effective. Policymakers can benefit from frequency-domain approaches where they can identify temporary disruptions and long-term structural changes as well as differentiate them.

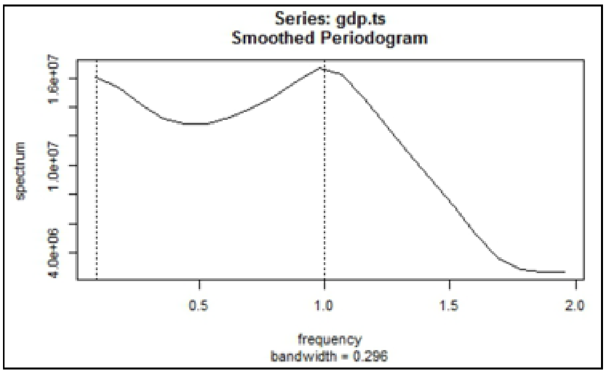

Figure 9: Peak Densities of Smoothed Periodogram for $m = 4$

Further insights into the AR(1) spectral density revealed in Figure 8 illustrate that time-domain methods could capture the long-term trend. The drawback, however, is that these methods cannot provide the frequencies visible in Figure 8, medium and high frequencies. While the AR(1) model was vital based on the nature of the GDP dataset having past values, the cycle patterns were hidden, but because of the estimate of the spectrum, they could be uncovered.

Overall, the study highlights that each method serves specific analytical needs, which is reasonably true of the time-domain and frequency-domain methods. In other words, each has its place and importance. For instance, in looking at the cyclical behaviour and period patterns in GDP data, the frequency-domain approach is much better considered. This is because data can be isolated graphically into business cycles, irregular fluctuations, or seasonal variations, using spectral density functions and smoothing tools like periodogram. On the other hand, to assess shock response, facilitating short-term forecasting and serial correlation can be addressed using time-domain methods, including AR(1) or ARIMA. These methods allow us to make analyses looking at trends and lag structures, but overlook frequency-specific variation. Nonetheless, complementing both, instead of replacing, becomes essential to researchers and policymakers for much richer insight into macroeconomic dynamics, especially for small economies that are often impacted by external shocks and seasonality, which tend to overlap.

7. Implications for Economic Policy

The study revealed some implications from the comparative analysis of time-domain and frequency-domain. In order to establish proactive policies, it is imperative that economic cycles are understood. This was implied based on the frequency-domain analysis, which revealed the existence of three different cycles in Jamaica's GDP time series data. These cycles are short-run, medium-run, and long-run, which were not explicitly visible when time-domain approaches such as AR(1) were used. This suggests the need for policymakers to capitalize on spectral analysis to complete the time-domain models for forecasting to address cyclical downturns and upturns in the economy more effectively. Furthermore, based on the results for medium-run cycles, fiscal or monetary policies may need to be used to address these cycles using countermeasures by carefully considering the timing and what stimulates these cycles. Also, based on the high-frequency signals found in the data, there is a need for policies to guide interventions into sectors that may be volatile based on short-term fluctuations, such as tourism or agriculture. As such, there is a need for these sectors to become resilient against external shocks, particularly since Jamaica has a small economy. This can be addressed through improved structural planning using long-run spectral analyses to help facilitate interventions such as diversifying exports more and more, improving productivity across sectors, especially crucial ones, and modernizing the infrastructure. Though this analysis might be in slow motion to cause reformation, it can help to facilitate sustainable growth in the long run if used strategically. Finally, there is real-time monitoring of GDP volatility, which can be done by smoothed spectral methods to create early-warning systems. In this way, the economy can indicate when macroeconomic factors become unstable or even predict recessionary activities. This, however, will need calibration of smoothing parameters appropriately to avoid making false positive predictions. Overall, adopting mixed-methods forecasting approaches is relevant for spectral decomposition using frequency-domain approaches followed by time-domain forecasting to make the data analyses richer and more accurate. Consequently, individuals will need to receive training on how to use these approaches.

8. Conclusion

The study concludes that both domain approaches are crucial in analyzing GDP data if an economy desires to have economic policies that are more responsive, forward-looking, and evidence-based. While the domains have their own unique advantages, where one falls short in providing analyses, the other can make up for this to provide a much richer insight into the data set. In other words, when combined, a full dissection of the complex dynamics of the GDP can be done to identify crisis management solutions based on early warning signs facilitated by structural planning and reform agendas for smarter long-term decision-making processes.

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