Analytical Approach Toward the Distribution of Near Square Primes in Relation to Landau's Fourth Problem

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Abstract: ***The 4th Landau's problem also known as the 'near square primes' asked whether there are infinitely many primes which are of the form*** $p = n^2 + 1, n \in \mathbb{N}$***. The solution is yet unresolved and this article attempts to present the solution. This article explores the fourth of Landau's problems, which asks whether there are infinitely many prime numbers of the form*** $n^2 + 1$***. Using tabular data and graphical analysis, the paper investigates the distribution patterns and last-digit trends of such primes, referred to as near square primes. It presents numerical evidence suggesting that although the density of these primes declines with increasing n, they do not cease to occur, thereby implying an infinite continuation. The study employs quadratic modeling and logarithmic regressions to support this hypothesis.***

Keywords: near square primes, Landau's problems, prime number distribution, quadratic equations, number theory,

1. Introduction

At the 1912 International Congress of Mathematicians, Edmund Landau listed four basic problems about prime numbers. These problems were characterised in his speech as "unattackable at the present state of mathematics" and are now known as **Landau's problems.** The 4th Landau's problem also known as the 'near square primes' asked whether there are infinitely many primes which are of the form $p = n^2 + 1, n \in \mathbb{N}$.

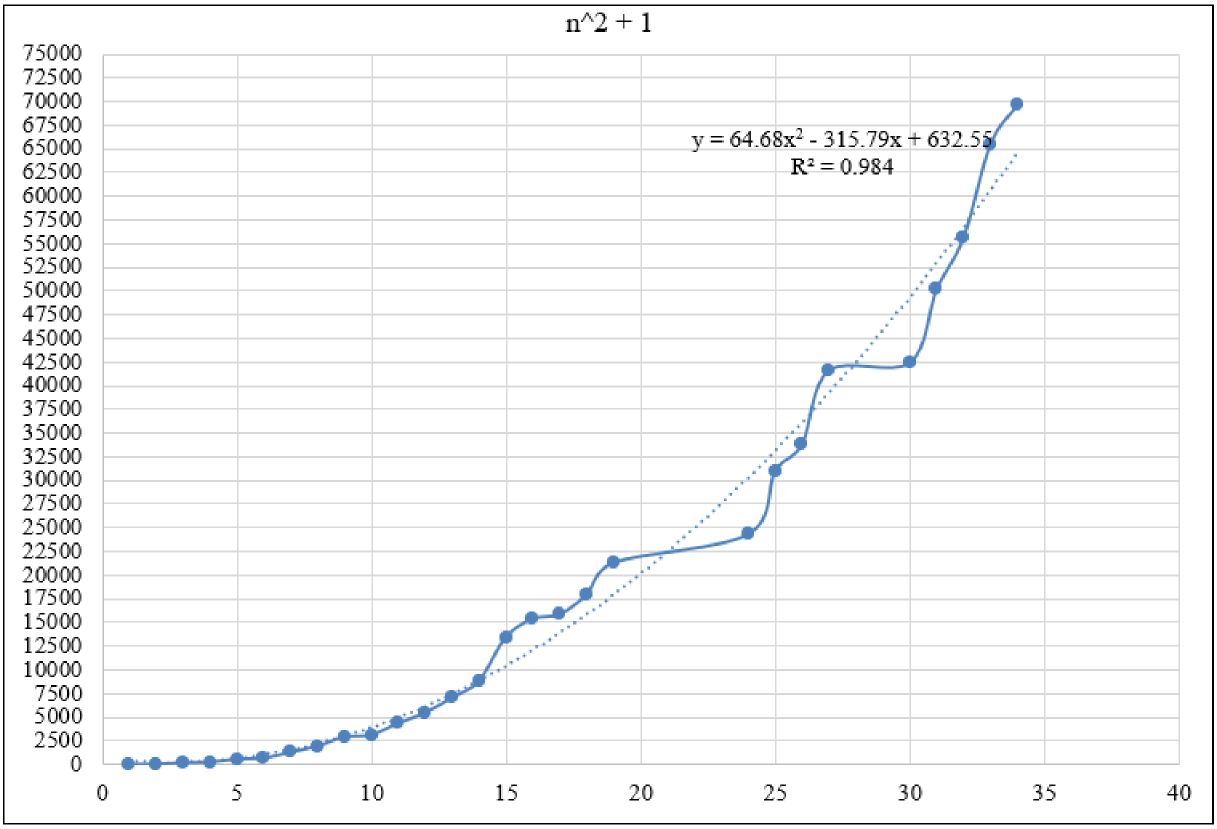
Table 1

Last digit	n^2 + 1 possible	Reason
0	X	Even number
1	√	11, 101 are prime
2	X	Even number
3	X	Perfect squares do not have 2 in unit's place
4	X	Even number
5	X	Not a prime except number 5
6	X	Even number
7	√	16 is prime
8	X	Even number
9	X	perfect squares do not have 8 in unit's place

$n^2 + 1$ cannot have last digit or digit in the unit's place as 0, 2, 3, 4, 5, 6, 8, 9 for reasons mentioned in the table 1 above. Only 7,1 can be the last digit of $n^2 + 1$.

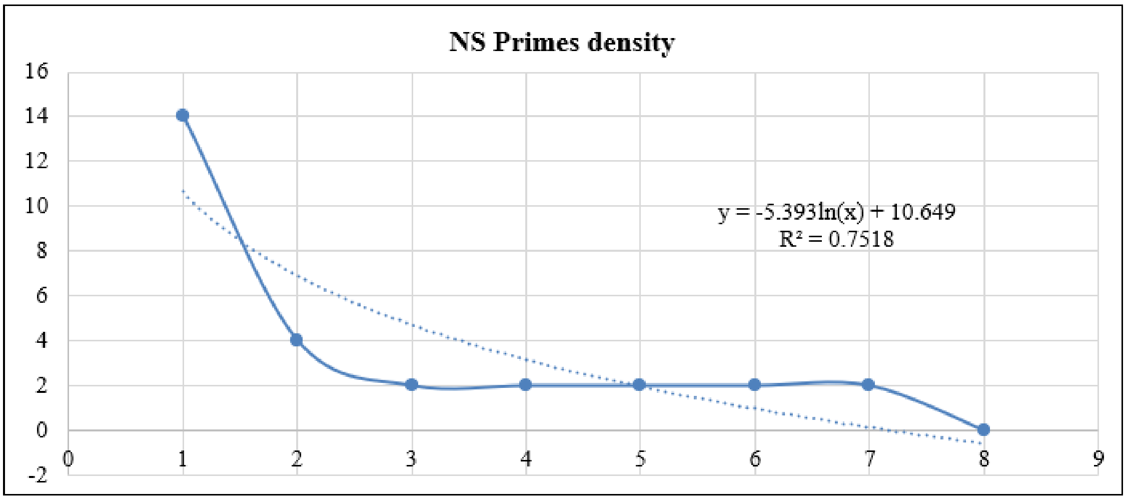
Table 2

No	n^2 + 1
1	17
2	37
3	197
4	257
5	577
6	677
7	1157
8	1297
9	1937
10	2117
11	2917
12	3137
13	4097
14	4357
15	5477
16	5777
17	7057
18	7397
19	8837
20	9217



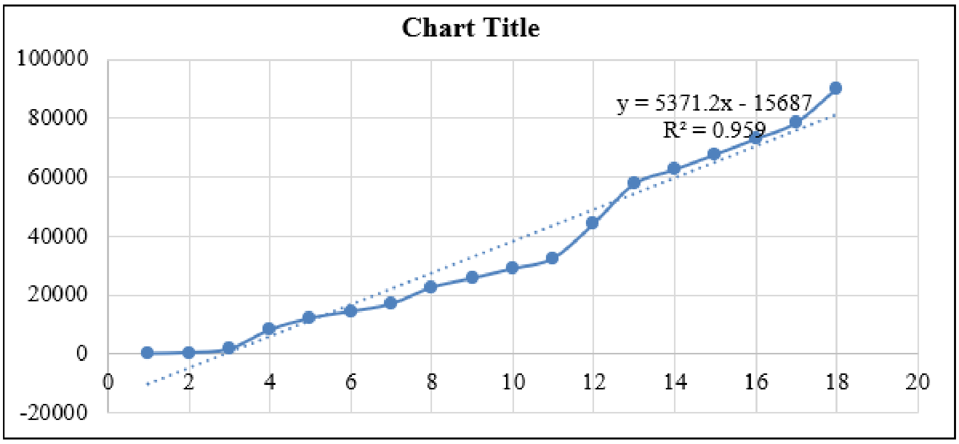
Graph 1

Section 3:



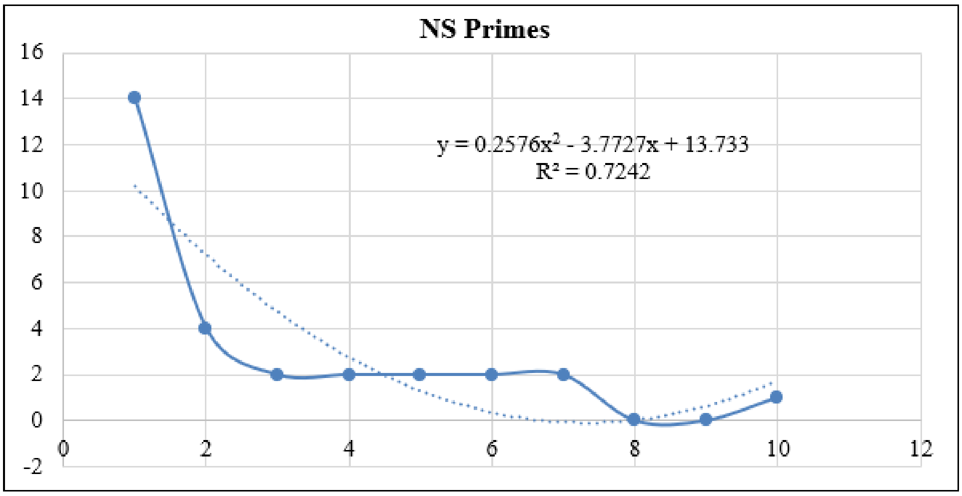
Graph 2

The y axis in graph 2 is number of near square primes with last digit as 7. In x axis first point is 1 x 10,000 and second is 2 x 10,000.



Graph 3

Graph 3 shows near square primes vs. natural numbers



Graph 4

Graph 4 shows reduction in density for near square primes for numbers ending in 1

Table 3

Ten Thousands	NS Primes
1	14
2	4
3	2
4	2
5	2
6	2
7	2
8	0
9	0
10	1

Table 4

Ten Thousands	NS Primes
1	4
2	3
3	3
4	1
5	1
6	1
7	2
8	2
9	0
10	1

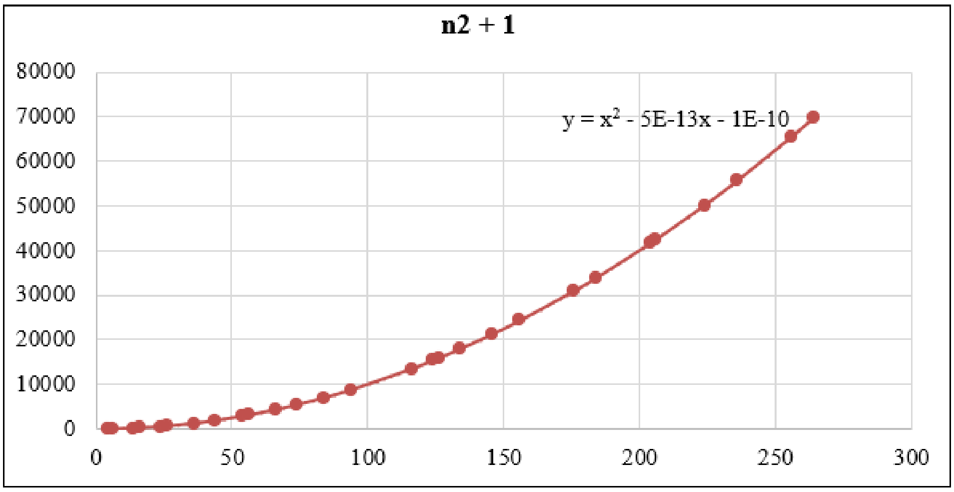
Table 3

n	n^2 + 1
4	17
6	37
14	197
16	257
24	577
26	677
36	1297
44	1937
54	2917
56	3137
66	4357
74	5477
84	7057
94	8837
116	13457
124	15377
126	15877
134	17957
146	21317
156	24337
176	30977
184	33857
204	41617
206	42437
224	50177
236	55697
256	65537
264	69697

Table 4

n	n^2 + 1
10	101
20	401
40	1601
90	8101
110	12101
120	14401
130	16901
150	22501
160	25601
170	28901
180	32401
210	44101
240	57601
250	62501
260	67601
270	72901
280	78401
300	90001

Table 3 and 4 show n (the number to be squared) vs. $n^2 + 1$ for both, primes ending in 1 and 7



Graph 5

Graph of natural numbers vs. $n^2 + 1$ shows the decrease in density of NS primes

Section 3:

Calculations [3]…. The reference link is included in the last section.

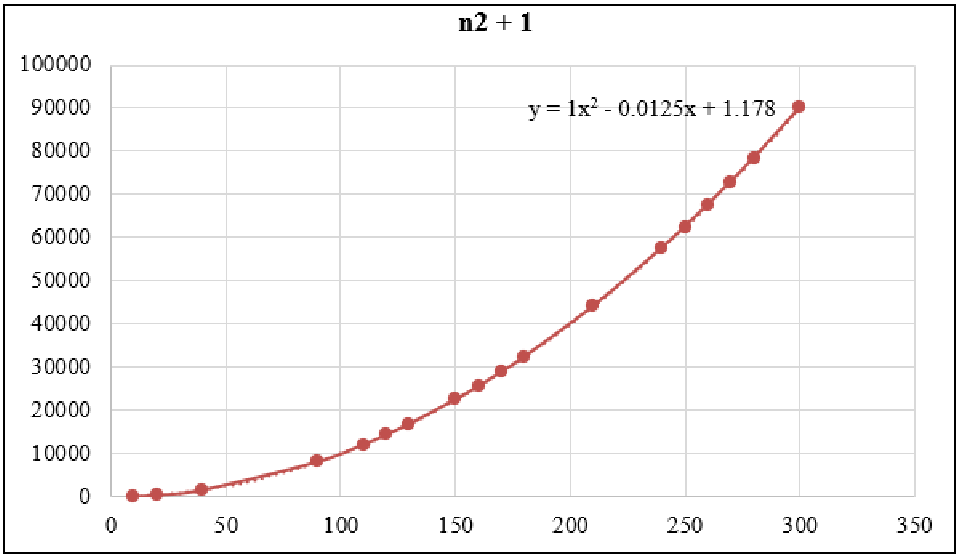
For near square primes ending in 1 refer to graph 2.

$Y = 0.053.x^2 – 0.8864.x + 4.6333$
$= 53/1000.\ x^2 – 0.8864 + 4.6333$
$= 53/1000.\ x^2 – 8464/10000.\ x + 4.6333$
$= 53/1000.\ x^2 – 8464/10000.\ x +46333/10000$
$=1/10000(530x^2 – 8864.x + 46333)$
$1/10000 \{ +8864 \pm\sqrt{8864^2 – 4.\ 530.\ 4.6333)\}2.\ 8864}$

78570496 – 9822.59
78560673
X1 = 78560673 / 8863.4 * 17728*10000
X2 = 78580318.6/ 8863.4*17728*10000

Calculations for graph 2
$-5.393\ln(x) + 10.649 = 0$
$\mathrm{Ln}(x) = 10694/5393$
$X = e^{10694/5393}$
$x \approx 7.20$

The logarithmic approximation gives Y = 0 at 7.2, which is 72000 but more near-square primes are found after 80,000. Hence, Near Squre primes appears to approach infinity.



Graph 6

2. Conclusion

The equation y = 0 is never possible other than n = 0 itself. Hence, the calculations give a near – zero answer. The near-square primes will continue to exist to infinity. The near-square primes do not end in the 60,000 to 70,0000 range. Hence, Near Square primes approach infinity. The near-square primes density only approaches the 0 number. Hence, we do not have natural number roots to the quadratic equation. Hence, near-square primes approach infinity. The density falls to 0 but never stops. It continues to infinity.

This study examined the existence of near square primes of the form $n^2 + 1$, providing empirical evidence through numerical sequences and graphical trends. Although the density of such primes diminishes over large n, no absolute cessation is observed. The results suggest an asymptotic continuation of these primes, supporting the hypothesis that near square primes persist infinitely. However, the findings remain exploratory and should be further tested using rigorous mathematical proofs.

References

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