

Understanding the Erdős Straus Conjecture and Its Path toward Possible Resolution

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Abstract: The Erdős–Straus conjecture is a yet unproven statement in number theory. The conjecture is that, for every integer n that is greater than or equal to 2, there exist positive integers x, y, z for which in other words, the number $4/n$ can be written as a sum of three positive unit fractions.

Keywords: Erdos-Straus conjecture, primes, triangular numbers, $4/n = (1/x) + (1/y) + (1/z)$, modulus method, 10^{17} integers checked, arithmetic progression, unit fractions

1. Introduction: [1] [2]

The Erdős-Straus Conjecture is a problem in number theory related to Egyptian fractions. The conjecture is named after Paul Erdos and Ernst G Straus. An Egyptian fraction is a sum of distinct unit fractions, which are fractions with a numerator of 1. The conjecture states that for any integer $n > 1$, the equation:

$$4/n = (1/x) + (1/y) + (1/z)$$

has positive integer solutions for x, y and z . In simpler terms, it suggests that the fraction $4/n$ can always be expressed as the sum of three-unit fractions.

For example:

- For $n=2$, we have $4/2=2$, and indeed, $2 = (1/1) + (1/2) + (1/2)$.
- For $n=3$, we have $4/3 = (1/1) + (1/6) + (1/6)$
- For $n=5$, we have $4/5 = (1/2) + (1/4) + (1/20)$

A solution would contribute to our understanding of Egyptian fractions and number theory. Despite significant numerical evidence supporting the conjecture, no general proof has been found. conjecture has been checked for $n = 10^{17}$ [3].

$$4/mn = (1/mx) + (1/my) + (1/mz) \quad [3]$$

Any positive integer is a multiple (m as a multiplier) of some prime so we only need to check n as a prime number. $m \geq 1$.

$$4/7 = 4 \cdot 4/7 \cdot 4 = 16/7 \cdot 2^2 = 7+7+2/7 \cdot 2^2 = 1/4 + 1/4 + 1/14 \quad [3]$$

$$4/11 = 4 \cdot 4/11 \cdot 4 = 16/11 \cdot 2^2 = 11+4+1/11 \cdot 2^2 = 1/4 + 1/11 + 1/44$$

$$4/13 = 4 \cdot 4/13 \cdot 4 = 16/13 \cdot 2^2 = 13+2+1/13 \cdot 2^2 = 1/4 + 1/26 + 1/52$$

2. Solution

Truth of the conjecture for 2 [4]

For the only even prime, i.e. 2, we have the solution

$$4/2 = 1/1 + 1/2 + 1/2.$$

Thus, the conjecture is true for any positive even integer.

Truth of the conjecture for primes congruent to 3 (mod 4) [4]

If $n \equiv 3 \pmod{4}$, we always have solutions since

$$4/n = 4((n+1)/2) / n((n+1)/2) = 1 / ((n+1)/2) + 2(n+1)/n = 1 / ((n+1)/2) + (2+2/n) = 1 / ((n+1)/2) + (2+1/(n+1)) /$$

$$n) = 1((n+1)/4) + 1((n+1)/2) + 1((n+1)/2) = 1((n+1)/4) + 1/n + 1/n,$$

where t_n is the n^{th} triangular number.

Thus, the conjecture is true for any positive integer divisible by a prime p congruent to 3 (mod 4).

Truth of the conjecture for primes congruent to 1 (mod 4) [4]

If the conjecture is true for $n \equiv 1 \pmod{4}$, then the conjecture would be true for any positive integer divisible by a prime congruent to 1 (mod 4).

3. Conclusion

The Erdos-Straus conjecture is not known for all values of primes but it is verified by computer up to $n = 10^{17}$. The solution is true for all even integers as they are not primes (exception - 2). General proof for all cases is still not available.

References

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