International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Impact Factor 2024: 7.101

Understanding the Erdős Straus Conjecture and Its Path toward Possible Resolution

Abhijit Manohar

Kolhapur, Maharashtra, India
Corresponding Author Email: *armanohar0977[at]gmail.com*

Abstract: ***The Erdős–Straus conjecture is a yet unproven statement in number theory. The conjecture is that, for every integer that is greater than or equal to 2, there exist positive integers and for which in other words, the number can be written as a sum of three positive unit fractions.***

Keywords: Erdos-Straus conjecture, primes, triangular numbers, 4 / n = (1 / x) + (1 / y) + (1 / z), modulus method, 10^{17} integers checked, arithmetic progression, unit fractions

1. Introduction: [1] [2]

The Erdős-Straus Conjecture is a problem in number theory related to Egyptian fractions. The conjecture is named after Paul Erdos and Ernst G Straus. An Egyptian fraction is a sum of distinct unit fractions, which are fractions with a numerator of 1. The conjecture states that for any integer n>1, the equation:
4 / n = (1 / x) + (1 / y) + (1 / z)

has positive integer solutions for x, y and z. In simpler terms, it suggests that the fraction 4n\frac{4}{n}n4 can always be expressed as the sum of three-unit fractions.

For example:
- For n=2, we have 4/2=2, and indeed, 2= (1/1) +(1/2) +(1/2).
- For n=3, we have 4/3= (1/1) +(1/6) +(1/6)
- For n=5, we have 4/5= (1/2) +(1/4) +(1/20)

A solution would contribute to our understanding of Egyptian fractions and number theory. Despite significant numerical evidence supporting the conjecture, no general proof has been found. conjecture has been checked for n = 10^{17} [3].
4/mn = (1/mx) + (1/my) + (1/mz) [3]

Any positive integer is a multiple (m as a multiplier) of some prime so we only need to check n as a prime number. m ≥1.
$4/7=4\cdot4/7\cdot4=16/7\cdot2^2=7+7+2/7\cdot2^2=1/4+1/4+1/14$ [3]
$4/11=4\cdot4/11\cdot4=16/11\cdot2^2=11+4+1/11\cdot22=14+111+144$
$4/13=4\cdot4/13\cdot4=16/13\cdot2^2=13+2+1/13\cdot2^2=1/4+1/26+1/52$

Solution:
Truth of the conjecture for 2 [4]
For the only even prime, i.e. 2, we have the solution
4 / 2 = 1 / 1+1 / 2+1 / 2.

Thus, the conjecture is true for any positive even integer.

Truth of the conjecture for primes congruent to 3 (mod 4) [4]
If n≡3(mod4), we always have solutions since
4 / n=4((n+1) / 2) / n((n+1) / 2)=1 / ((n+1) / 2)2(n+1)n=1 / ((n+1) / 2) x (2+2 / n)=1 / ((n+1) / 2)(2+1 / (n+1) / n)=1((n+1) / 4)+1((n+1) / 2)n+1((n+1) / 2)n=1((n+1) / 4)+1tn+1tn,

Where tn is the n^{th} triangular number.

Thus, the conjecture is true for any positive integer divisible by a prime p congruent to 3 (mod 4).

Truth of the conjecture for primes congruent to 1 (mod 4) [4]

If the conjecture is true for n≡1(mod4), then the conjecture would be true for any positive integer divisible by a prime congruent to 1 (mod 4).

2. Conclusion

The Erdos-Straus conjecture is not known for all values of primes but it is verified by computer up to n = 10^{17}. The solution is true for all even integers as they are not primes (exception - 2). General proof for all cases is still not available.

References

[1] Top 10 Unsolved Math Problems of All Time. Last updated 23rd of Jul 2015. https://www.geeksforgeeks.org/blogs/top-unsolved-math-problems-of-all-time/#8-the-twin-prime-conjecture
[2] "Erdos-Straus Conjecture." Wikipedia, Wikimedia Foundation, 12th of May 2025, https://en.wikipedia.org/wiki/Erd%C5%91s%E2%80%93Straus_conjecture
[3] "Erdos-Straus Conjecture". Online encyclopaedia of Integer Sequences (OEIS). Founded in 1964 by N. J. A. Sloane. OEIS Foundation Inc. (2025), The On-Line Encyclopaedia of Integer Sequences, Published electronically at https://oeis.org. **URLs** https://oeis.org/wiki/Erd%C5%91s%E2%80%93Straus_conjecture
[4] "Truth of the Conjecture for primes". Online encyclopaedia of Integer Sequences (OEIS). OEIS Foundation Inc. (2025), The On-Line Encyclopedia of Integer Sequences, Published electronically at https://oeis.org. **URLs** https://oeis.org/wiki/Erd%C5%91s%E2%80%93Straus_conjecture