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A Reflective Examination of Even Integers and Prime Pair Decompositions in Goldbach's Conjecture

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Abstract: ***The discussion revisits Goldbach's conjecture by outlining a numerical approach that expresses every even integer as the sum of two prime numbers through relationships based on the form 6k plus or minus 1. The analysis shows how pairs of constants can be used to generate corresponding primes for any even value by drawing on the structure of integers within modular classes. Examples from small values and extended ranges, including even numbers between 1000 and 1100, demonstrate how specific combinations of k and k' yield valid prime pairs. The argument is supported by broader computational work that has explored the conjecture up to very large magnitudes, reinforcing the view that even integers consistently align with prime pair representations within this framework.***

Keywords: Goldbach conjecture, prime numbers, Leonhard Euler, even integers, tested for 4 x 10^{18}

1. Introduction

Goldbach proposed a conjecture in the margin of his letter to Leonhard Euler, that every integer greater than 2 can be written as the sum of three primes. Later the conjecture was changed to – Every positive even integer is the sum of 2 prime numbers. Since μ, ε → 1 to ∞ for n →1 to ∞ the proof shows that the formula works for all even integers.

Proof

Prime numbers greater than 3 are given by (6k +/-1);
2n - (6k +/-1) = (6k' +/-1); where k, k' are 2 constants
n →1 to ∞
$2n = (6k +/-1)|_{k=\mu} + (6k' +/-1)|_{k=\varepsilon} \geq 10$ where μ, ε → 1 to ∞
2n = 6(k +k') +/-2 V 6(k+k')

For even integers < 10, 8= 5 + 3, 6 = 3 + 3, 4 = 2 + 2.

Table 1

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

Example for even number 26, k' = 1, k = 3 where k and k' are 2 constants is shown in table 1 above. The primes are 19 and 7. This was found from the equation 2n = 6(k + k') + 2. 24 is found from the equation 2n = 6(k + k') and 22 is found from the equation 2n = 6(k + k') - 2. This covers 3 consecutive even integers for (k + k') = 4. But this is true for all values of (k + k') ≥ 2. (k + k') → 2 to ∞. Hence the conjecture that every even integer is sum of 2 prime numbers is true for all even integers.
Table 2 below shows that the proof works for even integers from 1000 to 1100. For even number 1046, we use 2n = 6(k + k') + 2. K = 173 and k' = 1. The primes are 1039 and 7.

Table 2

1001	1011	1021	1031	1041	1051	1061	1071	1081	1091
1002	1012	1022	1032	1042	1052	1062	1072	1082	1092
1003	1013	1023	1033	1043	1053	1063	1073	1083	1093
1004	1014	1024	1034	1044	1054	1064	1074	1084	1094
1005	1015	1025	1035	1045	1055	1065	1075	1085	1095
1006	1016	1026	1036	1046	1056	1066	1076	1086	1096
1007	1017	1027	1037	1047	1057	1067	1077	1087	1097
1008	1018	1028	1038	1048	1058	1068	1078	1088	1098
1009	1019	1029	1039	1049	1059	1069	1079	1089	1099
1010	1020	1030	1040	1050	1060	1070	1080	1090	1100

T. Oliveira e Silva ran a verification for Goldbach conjecture up to 4 x 10^{18} and found that 3,325,581,707,333,960,528 is the smallest number which cannot be written as the sum of 2 primes where one is smaller than 9781 [3].

2. Conclusion

Since μ, ε → 1 to ∞ for n →1 to ∞ the proof shows that the formula works for all even integers.

References

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