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An Environmentally Sustainable EOQ Model for Deteriorating Products with Green Quality Dependent Linear Demand Under Learning Effect and Inflation

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Abstract: ***An economic order quantity (EOQ) model with shortages and learning effect under inflationary conditions is developed to address inventory optimization problems and environmental issues. Retailers perform inspection for defective items and are separated at initial stage. Defective items are sold at discount rate at instant after inspection. Models consider instantaneously deteriorating products with linear demand pattern, constant rate of deterioration and holding cost rate to fulfil the consumers demand throughout business cycle length with sole objective of maximizing retailers' profit. Shortages are partially backlogged to fulfil demand of loyal consumers. The learning process is adopted to lump items and prevent incorrect categorization. Green technology is proposed in reducing carbon emissions for a sustainable environment. Models with and without technology adoption are proposed. Numerical examples are presented to validate models. Additionally, sensitivity analysis is performed to study the impact of several parameters on optimal solutions.***

Keywords: Linear Green Quality Dependent (LGQD) Demand, Single Item (SI), Green Technology (GT), Instantaneous Deteriorating Items (IDTs), Carbon Tax Policy (CTP) Partial Backlogging Policy (PBP) and Carbon Emission.

1. Introduction

Items available in the market to fulfil demand of consumers are losing their originality and this happens naturally. Many items damage during transition phase and partially defected. Commodities having deteriorating property need special attention during their storage and transition phase. Some items start deteriorating instantly when it comes to business processes like vegetables, fruits, grains, volatile materials, gasoline, petrol, diesel and many more like commodities. Throughout the business deteriorating commodities lose their whole value over time. The rate of deterioration indicates the degradation in the quality of commodities. Increasing deterioration rate reduces quality of items rapidly. Some of the commodities deteriorated at high rate and resulted in waste material. Some commodities start deteriorating when comes into the business process and therefore termed as instantaneously deteriorating items (IDIs). To maintain the quality and originality of commodities, retailers invest capital in technology known as preservation technology such as fridge and cold storage. Preservation supports maintaining the temperature of storage place at which products life sustains for longer period. At many time commodities are defective during the manufacturing process and supplied to the retailer in a lot size of ordered and, commodities are damaged during transition period or decayed due to lengthy transition period. At retailers' end, received commodities are inspected and defected items are separated at the initial stage. In the present era due to uncertainty of production, demand and continuous varying cost of factors affecting inventory supply chain system, inflation is observed which cannot be ignored. With all these concerns, authors are keen to develop an economic order quantity model (EOQ) for instantaneously deteriorating products with learning in inspection process, holding cost and greener technology to prevent incorrect categorization of the commodities by the retailers which is helpful in minimizing the loss and goodwill in the market and hence maximizing the retailers' profit.

At the early age of developing inventory models, researchers studied inventory control systems under various realistic conditions of deterioration, learning process and inflation. Buzacott (1875) have developed basic EOQ model for deteriorating items with inflation under various policies. Considering defective items in a lot size ordered by retailer, a study was performed by Misra (1975) and analyzed impact of inflation on inventory system under different strategies. Salameh et al. (2000) developed an EOQ model considering some proportion of lot size ordered quantity has imperfect quality and separated by inspection at the time of receiving. Considering allowable shortages under inspection process for defective items, an EOQ model was developed by Jaggi et al. (2013) with credit financing policy. This paper also considers low demand rate as compared to inspection rate. During inspection, the concept of learning concept provides better opportunities to both buyers and sellers during business transactions to choose perfect items. Wright (1936) was first who implemented the concept of learning in the form of quantitative shape which is called as "Learning Curve"

Furthermore, stocking of inventory without technological support yields in business loss as commodities start deteriorating at instant and result in high waste and produces carbon emission affecting environment globally. This also resulted in low supply and increases backlogging over time which also occurred in unrepairable goodwill loss. On certain capital investment, advance technology is used to control deterioration process through the business cycle length and fulfil demand of consumers by reducing backlogging, deterioration support for sustainable environment by reducing emission boosting business profit. Choudhary and Mahata (2022) have developed an inventory model in which deterioration is classified as decay, dryness, evaporation and other forms of ineffectiveness and physical damages. To deal with the situation of deterioration of commodities and to minimize its effect many researchers are incorporating concept of using Preservation Technology and it has become an important tool not only to maintain the quality of product but also in reducing waste material and hence carbon emission while controlling deterioration during storage period. In the present era, the use of advanced preservation technology equipment uses electricity and biomass fuels for generating electricity which produces carbon emissions and increases level of greenhouse gases (GHG) affecting the environment and thus have become centralized attention of researchers. For sustainable environment and reducing GHG, the concept of green technology is introduced which helps in reducing carbon emission and hence global warming.

With the main and primary objective of minimizing carbon emission for a sustainable environment and maximizing retailer's profit for a lot of size ordered quantity with defective items, present study is performed with inflation and learning process in inspection and holding inventory under investment in green technology to reduce carbon emission during business operations and supply chain system. In addition, partial backlogging is considered and imposition of Carbon Tax by Governmental Agencies to control carbon emission is also applied. Further, the aim of the present work is to maximize total retailers' profit with respect to business cycle length, time of vanishing inventory and to decide lot size of ordered quantity. And, to study the impact of learning on the retailer's profit, reduction of holding cost and in reduction of carbon emissions when defective rate follows S-shaped learning curve.

At first instant, present paper discussed the development of models under assumptions and notations and with optimality conditions numerical examples are presented at the next stage and thereafter observations and managerial insights are discussed in the analysis section of this paper. The concavity of the model is also shown through 3D graphs and the impact of some major factors are shown through 2D graphic representations. Sensitivity analysis is also performed to see the impact of parameters considered in developing the present model. Finally, results and future extensions are explained in the conclusion section based on result analysis, observations and sensitivity analysis. Conceptualization and step involved in developing present models are shown in Figure-1.

2. Preliminary

For the development of the EOQ model, authors need to define some prerequisite mathematical expression and are stated as follows:

Learning Curve:
The learning curve introduced by Wright (1936) was considered by researchers as best described curve by a power. The earliest learning curve which represents that the decreasing cost needs to accomplish some repetitive operation. This theory of repetition states that as total produced unit doubles, the cost per unit declines by some constant percentage [Jaber (2006), Yelle (1979)]. Jaber (2006) has presented the debate of various authors on the power versus exponential learning curve. Jordan (1958) Carlson (1973) described the phase involves in the improvement through graphical representation which follows the S-shaped learning curve. Dharmendra et al. (2013) has incorporated the concept of learning curve into holding and ordering cost along with defective items in their inventory model with imprecise market demand under screening error. The learning curve which mostly being considered by many researchers is the S-shaped learning curve. Which involves three phases of process likely to be called inception phase, learning phase and maturity phase. The three phases representing S-shaped learning curve is shown in Figure-2. From figure-2, it may be observed that, in the start of learning phase-1, works get acquainted with the set-up, the tooling, blue-prints, instruction and the condition of process with workplace arrangements.

The mathematical form of S-shaped learning curve is described by

$$\rho(n) = \frac{\eta}{\tau + e^{\zeta n}}$$

Where $\eta, \tau,$ and $\zeta > 0$ are the model parameters, n is the cumulative number of shipments, and $\rho(n)$ is the number of imperfect products present in each lot size received after placing order.

3. Literature Investigation

3.1 Literature review regarding inflation, deterioration and shortages

Inventory management is the most crucial part of supply chain management. Inventories are produced, stocked and supplied to end users. At different level inventory is managed to fulfil regular demand of market and end users and players involved in the inventory management system have the objective of optimizing their respective goals. Research are involved in developing model regard to inventory management to provide an environment to decision makers to decide objectives of players so involved. Harris (1913) was the first one who developed an economic order quantity (EOQ) model incorporating fundamental concept of inventory management system. Further, many researchers developed inventory models incorporating many factors affecting inventory management and explored inventory management framework. Inflation has significant impact on optimizing policy of a firm/company/industry and players involved in supply chain system. Deterioration and inflation together have a push and pull effect on the business cycle length, ordering quantity and thus attract researchers to study its impact on inventory model by incorporating in the model. In this direction, Datta and Pal (1991) developed inventory models incorporating inflationary condition in combination with linear demand and shortages. An inventory model with allowable shortages and inflation was developed by Sarkar and Pan (1994). In this paper they studied the impact of inflation on order quantity. An inventory model for deteriorating items with shortages where order rate is linear function of time is developed by Hariga (1995). Hariga and Ben-Daya (1996) proposed an economic order quantity (EOQ) model for lot-sizing problems under inflationary conditions in a generalized way. Yang et al. (2001) developed deterministic inventory model for deteriorating items considering inflation and shortages under fluctuating demand. A study on inventory models.

3.2 Literature review regarding carbon emissions, green technology and screening policy

In the era of global marketing, competition is high and firms/company players in the business have a motive of optimizing their profit in the business focus on production and maximum supply. The system of inventory management is also responsible for generating carbon emission due to various activities involved in managing inventory. Growing level of GHG due to high carbon emissions has become a concern of researchers as well as Government and they are focusing on various strategies applicable in minimizing carbos emissions. Products produced have natural phenomenon of decay in quality. To control decay process firms/companies and business players invest in preservation technology. On the one side technology boosts the life of decaying items and maintains quality for a longer period and on other hand it becomes a source of carbon emissions. Thus, the need to control the release of carbon emissions during inventory management, become a challenge 19th Century and to address these issues, concept of Green Technology (GT) is introduced which supports in reduction of carbon emission as well as deterioration rate. In the beginning issue of carbon footprint in inventory management system was addressed by Hua et al. (2011) in 19th Century. Thereafter, many researchers like Datta (2017), Mishra et al. (2020a), Sepehri (2021a), Sepehri, (2021b), Lou et al. (2015), Taleizadeh et. al. (2020) and Taleizadeh et. al. (2022) has applied the concept GT in the direction of minimizing carbon emissions resulting in a sustainable environment. Requirement. Study of these papers reveals that a capital investment made by a firm/company on green technologies detracts emission with a given rate. Increasing the level of GHG is a global issue and in this direction sustainable development goal (SDG) has been decided in Geneva convention and for a greener production and supply chain system every country has to invest to promote green industrial practices for reducing carbon emissions and, hence, supporting a sustainable environment. Green technologies are a scientific method and are being implied in reduction of carbon emission and hence level of GHG. In green technologies low energy and resources are utilized to increase in the usability of product through manufacturing, recycling and inventory management system. Carbon emissions is produced by industries having major role in enhancing level of GHGs as described by various researchers like Agbede and Aiyelokum (2016); Mulenga and Siziya (2019); Zulu et. al. (2020). Global warming is one of the major outputs of increasing level of GHGs impacting human survival on earth. To limit the release of carbon emissions local Governments introduced various policies to impose on industries as well on the system liable of generating carbon emissions such as carbon cap and Carbon Cap and Trade Policies and imposition of certain Carbon emission tax and other penalty measures. Various studies were performed by researchers like Dietz and Venmanas (2019); Ren et al. (2018) in different countries in diminishing carbon emission. An economic order quantity (EOQ) model for non-instantaneous deteriorating items is proposed by Mishra et. al. (2020). In this paper they have used the concept of preservation technology and green technology. Investment in green technology will lead to a reduced level of carbon emissions from greenhouse operation as suggested by Mashud et. al. (2021).

A joint EOQ and EPQ model f incorporating green technology and circular economy is developed by Su et al. (2021) established. Under carbon emissions regulations a sustainable inventory model for deteriorating items is proposed by Mahato et al. (2024). Under Government policies imposed for reduction of carbon emissions, Mardyana and Mahata (2024) have developed an inventory model. In their paper, for reduction of GHG they have implemented dual carbon emission reduction technology by incorporating Carbon Cap and Tax policy.

3.3 Literature review regarding defective items, screening policy and learning effect

In the production system it is considered that whole produced items are not completely perfect, and they are supplied in the market without any check. Retailers on ordering may receive a mix of perfect and defective items. After receiving lot size, retailers screen defective items from lot-size by investing some amount on as screening cost and screened defective items are sold after complete screening. Screening process helps in sorting defective items which is sold before becoming a waste material generating carbon emissions. Learning is a continuous process and may be applied in improving operations involved in inventory management systems. Screening defective items is not perfect, and error occurs during screening process. From each screening process one can learn and improve further in screening rate by reducing errors of screening. The screening process is a repetitive operation; rate of screening may be increased by learning. Wright (1936) was the first who introduced the learning curve described by power. Most academicians have unanimous agreement on the concept introduced by Wright. In practice "S" shaped learning curve is used more effectively. Jordan (1958) and Carlson (1973) have described "S" shaped learning curve in their paper which involve three phases of improvement in learning process and reducing occurrence of error. Learning may be established as progress in the knowledge with repetitive action on the same platform. It supports the decision-making process when order quantity having defective items is to be decided due to varying quantity in every shipment. Many researchers have reported that the number of shipments is the most significant factor during the transaction of ordered quantity. Jaber and Bonney (2003) have proposed an inventory model and studied the impact of learning curve on lot-size ordered quantity. Incorporating, "S" shaped learning curve, an inventory model for deteriorating and defective items is proposed by Jaber and Goyal (2008) under learning concept of lot-size ordered quantity. Considering learning effect on production cost Khan et al. (2010) established mathematical model with screening rate. Under the concept of learning effect Konstantaras and Jaber (2012) established an inventory model for defective items with shortages vender use of learning effect. Yadav et al. (2013) developed inventory model using learning effect to study its impact on imprecise market demand under screening error. Agrawal et al. (2017) have proposed inventory model for perishable items under consideration of learning effect and allowable shortages. Applying learning effect strategies Nobil et al. (2019) studied a production model with shortages and rework under inspection. An economic order quantity (EOQ) model with learning effect and trade-credit financing policy is developed by Jayaswal et al. (2019). Jayaswal et al. (2019) have developed inventory models for retailer's ordering policy under consideration of deteriorating items and learning effect with trade financing and imperfect quality of items. Jayaswal et al. (2021) have discussed human learning effect on fuzzy based economic order quantity (EOQ) model under trade credit policy and backordering.

3.4 Research Gap and author's contribution

Many researchers have developed inventory models to study the effect learning has on the model incorporating carbon emission, inflation and different demand patterns. The table of comparison developed by authors is presented in Table-1. Various models studied during literature review reveal that a lot of models have been published with inflation, carbon emission effects of learning under various situations. These models are developed considering different demand patterns affecting inventory cost but as per authors' best effort made in reviewing models no model is found developed incorporating carbon emission, inflation, shortages incorporating linear demand pattern for deteriorating imperfect items to study impact of learning. Introduction of learning in the model accelerates the business as seen from various research papers and its impact is seen on reduction of carbon emission while learning involves in operation of advance technologies adopted by a firm/industry or a company. So, authors have considered the waste inventory produced due to deterioration process and invested capital in managing the same for minimization of carbon emission which turns into a sustainable environment.

Authors' contribution is shown in Table-1 and at the bottom of the said Table specified keywords are mentioned. Under present circumstances and pattern positive effect has been observed on the profit function for deteriorating imperfect product when learning process has been adopted. Reduction in the quantity of carbon emission is also seen as investment in green technology is more beneficiary as compared to model only with learning process that support a sustainable environment by reducing carbon emission about 59.60% while there a reduction of 0.125% in case of model with learning and green technology investment.

4. Assumption and Notations

4.1 Assumption

Under following assumptions authors have developed the present EOQ model:

1) Shortages are allowed and partially backlogged depending upon customers' waiting time.
2) Demand is continuous through complete business cycle length.
3) Time horizon is taken to be finite as considered by Osama et al. (2022).
4) Ordered lot size is mixed with perfect and imperfect products as considered by Salameh et al. (2000).
5) Rate of deterioration is constant complete business cycle length.
6) Screening rate is considered more as compared to demand rate.
7) Carbon emission emitted through different source as described in the paper.
8) Carbon regulation is implemented by imposing carbon emission limit and penalty if limit crossed.
9) Replenishment rate is instantaneous, and lead time is negligible.
10) Hundred percent screening is performed for product lot size received in each shipment.
11) Holding cost and screening process follows learning curve.
12) Quantity of carbon emission reduced due to implementation of employees' learning process in Green Technology.
13) Rate of inflation is constant and applied at discounted rate.
14) Single product is considered for development of model.
15) Demand is linear and varies through the business cycle. Demand through business cycle is described as follows:

$D\left(f_p\right) = a + b\, f_p$ During business period $(0\ t_l)$

Since $\lambda_s > D$ therefore $f_q < \frac{\lambda_s - a}{b}$ where f_q is freshness of product, λ_s is screening rate and $a, b > 0$ indicating a is initial demand and b is scaling factor of demand depending upon freshness quality of product.

Green Technology is adopted in preserving products and in controlling carbon emission produced due to use of electricity/electric generator. Mathematically, Green Technology function is defined as follows:

If φ is amount of carbon emission released before adoption of Green Technology, then fraction of reduced carbon emission after investment \$ $\mathcal{G}$ in Green Technology [Bhavani et al. 2022] is

$m(\mathcal{G}) = \varphi\left(\frac{\sigma\,\mathcal{G}}{1+\sigma\,\mathcal{G}}\right)$; where f is factor deciding ability of GT to reduce carbon emission

If $\mathcal{G}$=0 then $TCE_e = \varphi$ and if $\mathcal{G}\to\infty$ then

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4.2 Notations

Q_o	Quantity demanded per business cycle (Units)
O_c	Order placing cost in each cycle ($/year)
U_c	Cost for purchasing per unit for ordered quantity paid by retailer to the supplier ($/unit)
h_c	Cost per unit per unit time beard by retailer for holding items at store ($/unit/year)
c_s	Shortages per unit of quantity backordered per unit time ($/unit/year)
c_l	Opportunity cost per unit of lost sales (($/unit/year)
U_p	Unit selling price for perfect products ($/unit)
ρ	Percentages of imperfect products present in a lot size
U_d	Unit selling price for imperfect products $p_d < p_p$ ($/unit)
λ_s	Screening rate for imperfect products $\lambda_s > D_1$
S_c	Cost of screening products in a lot size ($/unit/year)
x_t	Unit tax imposed on carbon emissions ($/ton)
θ	Constant deterioration rate (per year)
c_d	Cost of deterioration per unit of deteriorated product ($/unit)
c_f	Carbon emission factor for fuels (tons/gallon)
c_e	Carbon emission factor for electricity (tons/Kwh)
v_e	Variable quantity of electricity used to stock one unit of product per unit time (Kwh/year)
W_c	Cost applied to manage waste products derived due to deterioration ($/unit)
r_d	Discount rate at inflation rate i_f
R_d	Inflation due to discount rate that is $R_d - i_f$
t_i	Time of inspecting lots received (year).
t_m	Time epoch at which inventory vanishes (year).
t_l	Total length of business cycle (year)
$S_1(t)$	Inventory level at any time t in the period $[0\ t_i]$
$S_2(t)$	Inventory level at any time t in the period $[t_i\ t_m]$
$S_3(t)$	Inventory level at any time t in the period $[t_m\ t_l]$
Ʈ $(t_m\ t_l)$	Representing business cost of retailers per inventory cycle ($)
ᵽ $(t_m\ t_l)$	Representing profit function of retailers per inventory cycle per unit time where t_m *and* t_l are decision variables ($).

Decision Variables

t_m	Period at which inventory vanishes (in year).
l_l	Length of business cycle (in year)
Q_o	Quantity ordered

Functions defined in model

$\rho(n)$	Imperfect products following S-shaped learning curve
$D\left(f_q\right)$	Demand depending upon fresh quality of products in first interval of business period.
$m(\mathcal{G})$	Representing green technology function for reduction of carbon emission
$B(x)$	Function representing partially backlogged shortages.
$S_{i=1,2,3}(t)$	Level of inventory during business period at any point of time t
ᵽ $(t_m\ t_l)$	Representing profit function of retailers per inventory cycle per unit time where t_m *and* t_l are decision variables

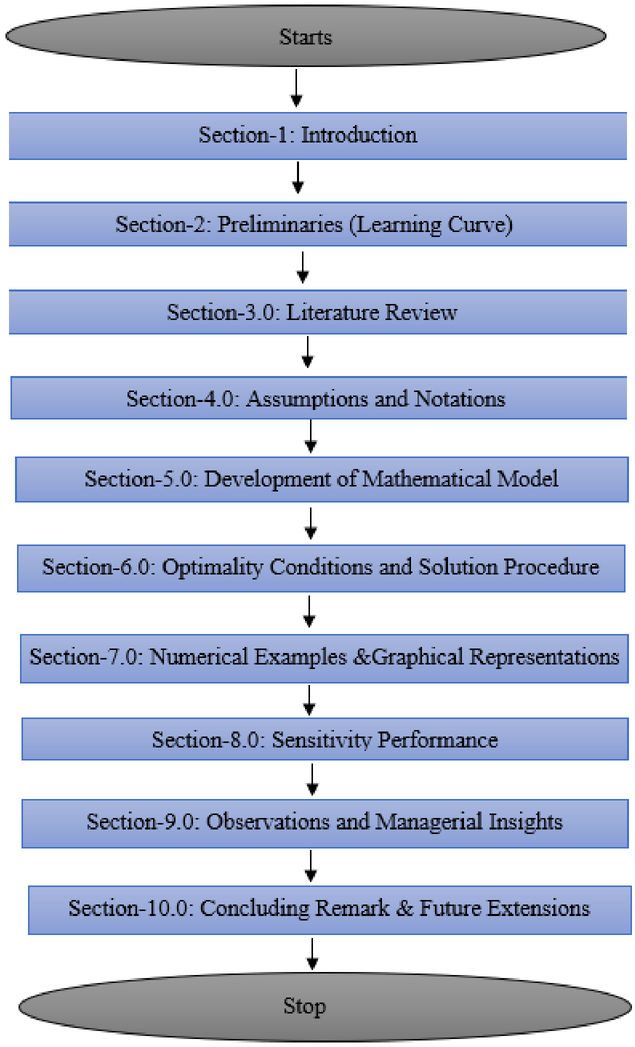

Figure 1: Conceptual representation of inventory model development

Table 1: Depicting summary of the relevant research papers with the present study

Research Paper	Imperfect Products	Learning effect	Demand Pattern	Deterio- -ration	Shortage	Screening Process	Carbon Emission	Sources of Carbon emissions	Green Technology Investment	Inflation
Writ (1936)		Yes								
Salameh and Jaber (2000)	Yes					Yes				
Jaber et al. (2008)	Y	Y				Y				
Khan et al. (2010)	Y	Y				Y				
Jaggi and Khanna (2010)	Y			Y		Y				Y

Jaggi et al. (2011)	Y			Y		Y				Y
Jaggi et al. (2013)	Y					Y				
Jaggi et al. (2017)	Y			Y		Y				Y
Patro et al. (2018)	Y	Y		Y		Y				Y
Daryanto et al. (2019)										
Liao et al. (2000)				Y						Y
Daryanto and Christata (2021)	Y					Y	Y			
Barman et al. (2021)				Y						Y
Jayaswal et al. (2019)	Y	Y								
Jayaswal et al. (2021)	Y	Y		Y		Y				
Mashud et al. (2021)	Y			Y		Y				
Osama et al. (2022)	Y	Y	Constant	Y	N	Y	Y	Holding Inventory	N	Y
Khan et al. (2023)	N	N	PAGD	N	N	N	N	NA	N	N
Mardyana and Mahata (2024)				Constant						
Present Paper	Y	Y	Linear GQD	Y	PB and WTD	Y	Y	Holding, Deterioration	Y	Y

CAPD: Credit and price dependent; CATD: Credit and time dependent; NITD: Non-instantaneous and time dependent; ROD: Rate of deterioration: Non-deterioration period; CAT: Carbon and trade; CT: Carbon Tax; CCAP: Carbon Cap and price; CASD: Credit and stock dependent; SP: Selling Price; PASD: Price and stock dependent; WTD: Waiting time dependent; CPAGD: Credit, price and product greenness dependent; PAGED: Price and greening efforts dependent ,PAGD: Price and greenness dependent; NA: Not applied, GQD: Green Quality Dependent

5. Development of Mathematical Model

As assumed, the inventory lot size, at the beginning of the business cycle at t=0, received is Q_o which may have perfect and imperfect products and thus received lot size screened at a constant rate of λ_s per year to divide Q_o into perfect and imperfect products. The time of screening is considered to be t_i which is calculated as $t_i = \frac{Q_o}{\lambda_s}$. Inventory declines due to demand and deterioration of product during the interval $[0\ t_i]$. In the positive stock period inventory level declines at same patterns in the interval $[t_i\ t_m]$. Due to continuity in the demand of products, shortages occur in the interval $[t_m\ t_l]$. Declination in the level of inventory during entire business cycle is depicted in the Figure-1 and mathematical derivation involving differential equations are represented as follows:

$$\frac{d\,S_1(t)}{dt} + \theta S_1(t) = -D;\ 0 \le t \le\ t_i \quad (1)$$

$$\frac{d\,S_2(t)}{dt} + \theta S_2(t) = -D;\ \ t_i\ \le t \le\ t_m \quad (2)$$

The above first order linear differential equation (1) is solved with the initial condition $S_1\,(0) =\ Q_o$ which gives

$$S_1\,(t) =\ \ Q_o\,e^{-\theta\,t} + \frac{D}{\theta}(e^{-\theta\,t} - 1) \quad (3)$$

Now at $t = t_m$ present level of inventory (PLI) say $S_1\,(\,t_m\,) -\ \rho(n)\,Q_o$. Therefore,

$$\text{PLI} =\ Q_o\,e^{-\theta\,t_m} + \frac{D}{\theta}(e^{-\theta\,t_m} - 1)$$
$$= \{1 -\ \rho(n)\}\,Q_o\ -\ D_1\,t_i \quad (4)$$

Now, solving equation (2) with boundary condition $S_2\,(\,t_m\,)$ of present remaining inventory at $t\ = t_m$, and $t_i = \frac{Q_o}{\lambda_s}$; level of inventory is calculated as

$$S_2\,(t) = \frac{D}{\theta}\left(e^{\theta\,(t_m - t)} - 1\right) + \left\{(1 -\ \rho(n))\,Q_o\ - D\,t_i\right\}e^{\theta\,(t_m - t)} \quad (5)$$

Also at $t\ = t_m\,,\ S_2\,(\,t_m\,) = 0$ gives

$$S_2\,(t) = \frac{D}{\theta}\left(e^{\theta\,(t_m - t)} - 1\right);\ t_i\ \le t \le\ t_m \quad (6)$$

Equating equation (4) and (6) with $t_i = \frac{Q_o}{\lambda_s}$, initially ordered quantity is obtained as

$$Q_o\ = \frac{\lambda_s\,D\left(e^{\theta\,t_m} - 1\right)}{\theta\{\lambda_s\,(1 - \rho(n)) + D e^{\theta\,t_m}\}} \quad (7)$$

It has been assumed that shortages occurs due to continuity of demand in the market which was partially backlogged at next replenishment cycle and supplied to the royal waiting customers in the beginning of the business cycle. The level of inventory in the interval $t_i\ \le t \le\ t_m$ is described by the following differential equation

$$\frac{d\,S_3(t)}{dt} = -D\,\left(e^{-\delta\,(t_l - t)}\right);\ \ t_m\ \le t \le\ t_l \quad (8)$$

Solving equation (8) with boundary condition at $t = t_l$; $S_3\,(\,t_m\,) = 0$, the obtained solution is

$$S_3\,(t)\ = \frac{D}{\delta}\left(e^{-\delta\,(t_l - t_m)} - e^{-\delta\,(t_l - t)}\right); \quad (9)$$

Associated inventory cost and sales revenue collected are given as follows:

Initially ordered lot size received at the retailer's end and stocked at suitable place where inventory are screened and preserved to fulfil demand of customers. Stocking of inventory cost some charge per unit per unit of time for the facilities used for longer life and to reduce deterioration. Since inflation and discount offered on inflation is one of the key factor affecting inventory cost at the time of decision taken place and hence included in each cost factor. The charge for holding inventory is termed as holding cost and calculated for the entire business cycle is as under:

$$CH = h_c\left(\int_0^{t_i} e^{-R_d t}\,S_1(t) + \int_{t_i}^{t_m} e^{-R_d t} S_2(t)\right)$$

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Substituting level of inventory at different interval in above equation and on simplification, holding cost is obtained as

$$CH = h_c\left[\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right) + \frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\right]$$

During stock period deterioration takes place at certain rate as assumed and thus it described as deterioration cost and calculated as:

$$CD = c_d\left(Q_o - \int_0^{t_m} e^{-R_d t}\,D\,dt - \rho(n))Q_o\right)$$
$$= c_d\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \rho(n))Q_o\right)$$

Due to continuous demand, shortages occur in the interval $t_m \le t \le t_l$, shortages quantity backlogged. Shortages partially backlogged which bears some cost called backlogging cost and quantity lost due to non-waiting customers also bears some cost which is called as opportunity cost. Both cost are calculated as under:

$$CS = \int_{t_m}^{t_l} -e^{-R_d t}S_3(t)\,dt$$
$$= c_s\left[\frac{D}{\delta}\left(\frac{e^{-\delta t_l\,(R_d-\delta)\,t_m} - e^{-R_d\,t_l}}{R_d-\delta} + \frac{e^{-(R_d+\delta)\,t_l+\delta\,t_m} - e^{-\,\delta\,t_l+(R_d+\delta)\,t_m}}{R_d}\right)\right];$$

And

$$CL = c_l\;D\left[\int_{t_m}^{t_l} e^{-R_d t}(1-e^{-\delta\,(t_l-t)})\,dt\right];$$
$$= c_l\;D\left[\left(\frac{e^{-R_d\,t_m}-e^{-R_d\,t_l}}{R_d} + \frac{e^{-R_d\;t_l}-e^{-\,\delta\,t_l-(R_d-\delta)\,t_m}}{(R_d-\delta)}\right)\right];$$

Since, received lot size may contain imperfect products with perfect one, thus for screening lot size a screening cost is beard by retailer and is calculated as

$$COS = S_c\,Q_o$$

Deteriorated products are collected in the form of wastage and are disposed off so for disposing the same a waste management cost is included in total inventory cost which is calculated as

$$WMC = w_c\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \rho(n))Q_o\right);$$

Retailer's Purchased cost is calculated as

$$CP = U_c\,Q_o;$$

Retailer's ordering cost

$$O_c$$

As high technology is used for preservation of products to control rate of deterioration. Preservation Technology runs on electric energy/electric generator and due to use of electricity/electric generator, carbon emission releases. Thus, carbon emission produced due different sources which are calculated as

Carbon Emission produced due to use of electric energy during stock hold is

$$CE_{hE} = c_e\,v_e\left[\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right) + \frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\right] \quad (10)$$

Carbon Emission produced due to use of electric generator during stock hold is

$$CE_{hF} = c_f\,v_e\left[\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right) + \frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\right] \quad (11)$$

Carbon Emission produced due to deteriorated product before it disposed of and is calculated as

$$CE_D = c_{fd}\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \rho(n))Q_o\right) \quad (12)$$

Total amount of carbon emission produced from all sources during business cycle is recorded as

$$TCE_e = CE_{hE} + CE_{hF} + CE_D$$
$$= c_e\,v_e\left[\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right) + \frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\right] + c_f\,v_e\left[\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right) + \frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_{i-(\theta+R_d)\,t_m}}\right)}{R_d+\theta} + (1-$$

$$\rho(n))Q_o\left\{\frac{\left(e^{-R_d\,t_i}\right)-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta}+\right.$$
$$\left.\frac{\left(e^{-R_d\,t_m}-e^{-R_d\,t_i}\right))}{R_d}\right\}\Bigg]+c_{fd}\left(Q_o-\frac{D}{R_d}\left(1-e^{-R_d\,t_m}\right)-\right.$$
$$\left.\rho(n))Q_o\right) \quad (13)$$

Cost of carbon emission in terms of tax imposed by the local government on carbon emission produced due to various activities of inventory management is

$$CEC =$$
$$x_t\left\{c_e\,v_e\left(\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right)+\frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta}+\right.\right.\right.$$
$$\left.\frac{\left(e^{-R_d\,t_i}-1\right))}{R_d}\right)+\frac{D}{\theta}\left(\frac{\left(e^{-R_d\,t_i}\right)}{R_d+\theta}\right)+\frac{\left(e^{-R_d\,t_m}-e^{-R_d\,t_i}\right))}{R_d}-$$
$$\frac{\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta}+(1-\rho(n))Q_o\left\{\frac{\left(e^{-R_d\,t_i}\right)-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta}+\right.$$
$$\left.\left.\frac{\left(e^{-R_d\,t_m}-e^{-R_d\,t_i}\right))}{R_d}\right\}\right)+c_f\,v_e\left(\frac{Q_o}{R_d+\theta}\left(1-e^{-(R_d+\theta)\,t_i}\right)+\right.$$
$$\frac{D}{\theta}\left(\frac{\left(1-e^{-(R_d+\theta)\,t_i}\right)}{R_d+\theta}+\frac{\left(e^{-R_d\,t_i}-1\right))}{R_d}\right)+\frac{D}{\theta}\left(\frac{\left(e^{-R_d\,t_i}\right)}{R_d+\theta}\right)+$$
$$\frac{\left(e^{-R_d\,t_m}-e^{-R_d\,t_i}\right))}{R_d}-\frac{\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta}+(1-$$
$$\left.\rho(n))Q_o\left\{\frac{\left(e^{-R_d\,t_i}\right)-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta}+\frac{\left(e^{-R_d\,t_m}-e^{-R_d\,t_i}\right))}{R_d}\right\}\right)+$$
$$\left.c_{fd}\left(Q_o-\frac{D}{R_d}\left(1-e^{-R_d\,t_m}\right)-\rho(n))Q_o\right)\right\} \quad (14)$$

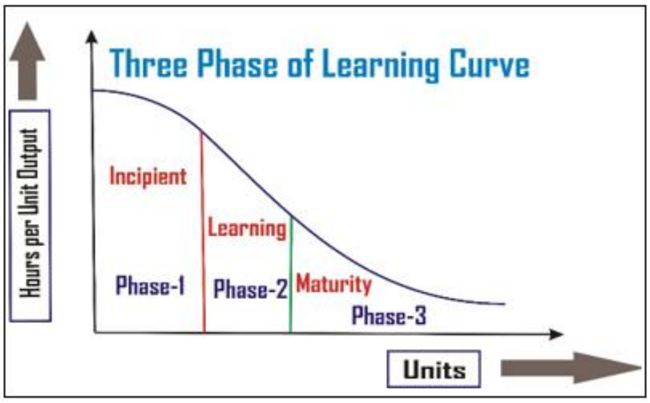

Figure 2: Representing three phases of learning curve

$$Ʈ\,(t_m\,t_l)=\begin{pmatrix}\text{Ordering Cost+Purchasing Cost+Holding Cost+Carbon Emission Cost}\\ \text{+Backlogging Cost+Opportunity Cost+Green Technology Cost}\\ \text{+Deterioration Cost+Waste management Cost}\end{pmatrix}$$

Revenue received by retailer during business cycle from different sources is calculated as

Revenue collected from sales of imperfect products is

$$RI_p = U_d\,\rho(n))Q_o e^{-R_d\,t_m}$$

Revenue collected from sales of perfect products is

$$RP_p = U_p\,\frac{D}{R_d}\left(1-e^{-R_d\,t_m}\right)$$

Total sales revenue received is

$$TSR = U_p\left(\frac{D}{R_d}\left(1-e^{-R_d\,t_m}\right)\right)+U_d\,\rho(n))Q_o e^{-R_d\,t_m}$$

Model without Green Technology Investment

Average profit of retailer's business is calculated as

$$ꝑ\,(t_m\,t_l)=\frac{1}{t_l}\left[\text{Total Sales revenue}-\begin{pmatrix}\text{Ordering Cost+Purchasing Cost+Holding Cost+Carbon Emission Cost}\\ \text{+Backlogging Cost+Opportunity Cost+Green Technology Cost}\\ \text{+Deterioration Cost+Waste management Cost}\end{pmatrix}\right]$$

$$\text{₱}\,(t_m\, t_l) = \frac{1}{t_l}\Bigg[U_p\left(\frac{D}{R_d}(1-e^{-R_d t_m})\right) + U_d\, \rho(n))Q_o e^{-R_d t_m} - \Bigg(O_c + h_c \Bigg[\frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\, t_i}) + \frac{D}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\, t_i})}{R_d+\theta} + \frac{(e^{-R_d\, t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\, t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\, t_m} - e^{-R_d\, t_i}))}{R_d} - \frac{(e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\, t_i}) - (e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + \frac{(e^{-R_d\, t_m}-e^{-R_d\, t_i}))}{R_d}\right\}\Bigg] + U_c\, Q_o + c_d\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\, t_m}) - \rho(n))Q_o\right) + c_s\left[\frac{D}{\delta}\left(\frac{e^{-\delta t_l-(R_d-\delta)\, t_m} - e^{-R_d\, t_l}}{R_d-\delta} + \frac{e^{-(R_d+\delta)\, t_l+\delta\, t_m} - e^{-\delta\, t_l+(R_d+\delta)\, t_m}}{R_d}\right)\right] + c_l\, D\left[\left(\frac{e^{-R_d\, t_m}-e^{-R_d\, t_l}}{R_d} + \frac{e^{-R_d\, t_l} - e^{-\delta\, t_l-(R_d-\delta)\, t_m}}{(R_d-\delta)}\right)\right] + S_c\, Q_o + w_c\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\, t_m}) - \rho(n))Q_o\right) + \{CEC\}\Bigg)\Bigg] \tag{15}$$

Where

$$CEC = x_t\Bigg\{c_e\, v_e\Bigg(\frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\, t_i}) + \frac{D}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\, t_i})}{R_d+\theta} + \frac{(e^{-R_d\, t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\, t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\, t_m}-e^{-R_d\, t_i}))}{R_d} - \frac{(e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\, t_i})-(e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + \frac{(e^{-R_d\, t_m}-e^{-R_d\, t_i}))}{R_d}\right\}\Bigg) + c_f\, v_e\Bigg(\frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\, t_i}) + \frac{D}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\, t_i})}{R_d+\theta} + \frac{(e^{-R_d\, t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\, t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\, t_m}-e^{-R_d\, t_i}))}{R_d} - \frac{(e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\, t_i})-(e^{\theta\, t_i-(\theta+R_d)\, t_m})}{R_d+\theta} + \frac{(e^{-R_d\, t_m}-e^{-R_d\, t_i}))}{R_d}\right\}\Bigg) + c_{fd}\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\, t_m}) - \rho(n))Q_o\right)\Bigg\}$$

Model with Green Technology Investment and Learning Effect

/Total inventory cost of business when green technology investment is made to control release of carbon emission

$$\text{Ʈ}\,(t_m\, t_l) = \left(\begin{array}{c}\text{Ordering Cost+Purchasing Cost+Holding Cost with learning effect+Carbon Emission Cost}\\ \text{+Backlogging Cost+Opportunity Cost+Green Technology Cost}\\ \text{+Deterioration Cost+Waste management Cost+Green Investment Cost}\end{array}\right)$$

Here labours are trained about advance technology equipment and hence learning impact holding cost and carbon emission cost reduced as well due to investment in green technology and hence average profit in this case is

$$\text{₱}\,(t_m\, t_l) = \frac{1}{t_l}\left[\text{Total Sales revenue} - \left(\begin{array}{c}\text{Ordering Cost+Purchasing Cost+Holding Cost with learning effec+Carbon Emission Cost due to GT}\\ \text{+Backlogging Cost+Opportunity Cost+Green Technology Cost}\\ \text{+Deterioration Cost+Waste management Cost+Green Technology Cost}\end{array}\right)\right]$$

$$
\text{ꝑ}(t_m\, t_l) = \frac{1}{t_l}\left[\left(\begin{aligned}
& U_p\left(\frac{D}{R_d}(1-e^{-R_d\, t_m})\right) + U_d\, \rho(n))Q_o e^{-R_d\, t_m} - \\
& O_c + h_c\, j^{-l}\left[\begin{aligned}
& \frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\,t_i}) + \frac{D}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\,t_i})}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) \\
& + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\, t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + \\
& (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}
\end{aligned}\right] + \\
& U_c\, Q_o + c_d\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \rho(n))Q_o\right) + \\
& c_s\left[\frac{D}{\delta}\left(\frac{e^{-\delta t_l\,(R_d-\delta)\,t_m} - e^{-R_d\,t_l}}{R_d-\delta} + \frac{e^{-(R_d+\delta)\,t_l+\delta\,t_m} - e^{-\delta\,t_l+(R_d+\delta)\,t_m}}{R_d}\right)\right] + \\
& c_l\; D\left[\left(\frac{e^{-R_d\,t_m}-e^{-R_d\,t_l}}{R_d} + \frac{e^{-R_d\,t_l} - e^{-\delta\,t_l-(R_d-\delta)\,t_m}}{(R_d-\delta)}\right)\right] + S_c\,Q_o + \\
& w_c\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \rho(n))Q_o\right) + \mathcal{G}\;t_l \\
& \{CEC\}
\end{aligned}\right)\right]
$$

Where

$$
\begin{aligned}
CEC = {} & m(\mathcal{G})x_t\left\{c_e\, v_e\left(\frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\,t_i}) + \frac{D_1}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\,t_i})}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D_2}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \right.\right. \\
& \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \frac{\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\Bigg) + \\
& c_f\, v_e\left(\frac{Q_o}{R_d+\theta}(1-e^{-(R_d+\theta)\,t_i}) + \frac{D}{\theta}\left(\frac{(1-e^{-(R_d+\theta)\,t_i})}{R_d+\theta} + \frac{(e^{-R_d\,t_i}-1))}{R_d}\right) + \frac{D}{\theta}\left(\frac{(e^{-R_d\,t_i})}{R_d+\theta}\right) + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d} - \right. \\
& \frac{\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + (1-\rho(n))Q_o\left\{\frac{(e^{-R_d\,t_i})-\left(e^{\theta\,t_i-(\theta+R_d)\,t_m}\right)}{R_d+\theta} + \frac{(e^{-R_d\,t_m}-e^{-R_d\,t_i}))}{R_d}\right\}\Bigg) + c_{fd}\left(Q_o - \frac{D}{R_d}(1-e^{-R_d\,t_m}) - \right. \\
& \left.\left.\rho(n))Q_o\right)\right\}
\end{aligned}
$$

6. Optimality Condition and Solution Algorithm

6.1 Optimality Conditions

To determine the optimal value of decision variables for optimization of goal of inventory modelling. The following conditions are applied and checked for optimum profit. That is

To maximize $\text{ꝑ}(t_m\, t_l)$

Subject to: $t_m > 0, t_l > 0$

$$
\frac{\partial\, \text{ꝑ}(t_m\, t_l)}{\partial t_m} = 0;\ \frac{\partial\, \text{ꝑ}(t_m\, t_l)}{\partial t_l} = 0 \tag{17}
$$

For maximum profit the necessary and sufficient conditions must be satisfied which are given below

$$
\frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial\, {t_l}^2} < 0; \text{ and } \frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial\, {t_m}^2} < 0; \tag{18}
$$

The given below Hessian matrix be positive definite at the value of decision variables where profit is considered to be maximum

$$
\left(\frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial\, {t_m}^2}\right)\left(\frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial\, {t_l}^2}\right) - \left(\frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial t_l\, \partial t_m}\right)\left(\frac{\partial^2\, \text{ꝑ}(t_m\, t_l)}{\partial t_m\, \partial t_l}\right) > 0; \tag{19}
$$

6.2 Solution Procedure

Step-1: Input value of parameters in the model developed at equations (15) & (16) at initial stage.
Step-2: Differentiate profit function partially with respect to t_l *and* t_m using equation (15) & (16) for two different models.
Step-3: Equate $\frac{\partial\, \text{ꝑ}(t_m\, t_l)}{\partial t_l}$ *and* $\frac{\partial\, \text{ꝑ}(t_m\, t_l)}{\partial t_m}$ to zero (as equation 17) and solve to find value of t_l *and* t_m for both Models separately.
Step-4: Put value of t_l *and* t_m in the profit function and calculate profit in case of both models.
Step-5: Check optimality conditions given at equations (18) and (19) at value of t_l *and* t_m calculated for both models separately.
Step-6: If step-5 satisfied, go to Step-7 else go to Step-1.
Step-7: Compare value of profit calculated for both models.
Step-8: Declare maximum profit for the selected model amongst two and go to Step-9
Step-9: Stop.

7. Numerical Examples and Graphical Representations

To validate model and objective of present work, optimum value of decision variables in two different models are calculated using Mathematica-11.3 Software. Further retailer's profit of obtained by applying decision variables into profit function for two models described by equations (10) and (11). Since models developed are highly non-linear and proof of attainment of maximum profit is analytically not possible, therefore using graphical representations for developed model (with learning and green technology investment) are depicted in this section. Graph depicted in Figure-4 shows the concavity of the model that has global maxima showing maximum profit at certain point which are declared as optimal decision variables and the value. Following numerical examples are presented to validate optimality of profit function for the developed models.

Example 1: The following set of data in their respective unit have been considered for model validation in case model developed with learning curve and without GTI. Data are chosen randomly to validate the models are:

Demand is taken as $D(f_p) = a + b\, f_q$ unit per year; $a = 100$; $b = 0.05$.; $f_q = 0.5$; $\lambda_s = 1000$; $S_c = \$0.15\ per\ unit$ $\zeta = 0.25$; $O_c = \$\,200$ per order; $U_c = \$15$; $h_c = \$\,30$ per unit/time/year; $c_s = \$\,3$ per unit per year; $c_l = \$\,2$ per unit per year ; $x_t = \$35$ per Kg CO2; $U_p = \$100$ per unit; $U_d = \$20$; $c_d = \$15$; $\theta = 0.015$; $c_f = 0.0016$ kg CO_2 per unit per unit time; $c_e = 0.003$ kg CO_2 per kWh;$v_e = 1.04$ kWh; $W_c = \$5$ per unit; $R = R_d - r_d = \$\,0.008$ per unit ; $f = 3$; ; $\alpha = 2\ lit\ per\ day$; $C_{fd} = 0.08$;; $n = 5$; $\eta = 0.25$; $i = 0.675$; $\eta = 0.003$; $\delta = 6.5$; $c_{fd} = 0.03$; $\gamma = 0.6$;

After calculation using Mathematica -11.3 software following optimal obtained results are:
$t_m^* = 132.395$; $t_l^* = 494.989$; $Q_0^* = 118018.0$; $CE = 3{,}38{,}332\ kg$; $\text{ᵽ}^*\,(t_m\, t_l\,) = \$2.26017X10^8$ $\rho(5) = 0.000571429$

Example 2: The following set of data in their respective unit have been considered for model validation in case model developed with Learning Curve and with Green Technology Investment for reducing carbon emission. Data are chosen randomly to validate the models are:

Demand is taken as $D(f_q) = a + b\, f_q$ unit per year; $a = 100$; $b = 0.05$.; $f_q = 0.5$; $\lambda_s = 1000$; $S_c = \$0.15\ per\ unit$; $O_c = \$\,200$ per order; $U_c = \$15$; $h_c = \$\,30$ per unit/time/year; $c_s = \$\,3$ per unit per year; $c_l = \$\,2$ per unit per year ; $x_t = \$35$ per Kg CO2; $U_p = \$100$ per unit; $U_d = \$20$; $c_d = \$15$; $\theta = 0.015$; $c_f = 0.0016$ kg CO_2 per unit per unit time; $c_e = 0.003$ kg CO_2 per kWh;$v_e = 1.04$ kWh; $W_c = \$5$ per unit; $R = R_d - r_d = \$\,0.008$ per unit ; $f = 3$; ; $\alpha = 2\ lit\ per\ day$; $C_{fd} = 0.08$; $f = 0.003$; $n = 5$; $\zeta = 0.25$; $t_i = 0.675$; $\eta = 0.003$; $\delta = 6.5$; $c_{fd} = 0.03$; $\sigma = 0.8$; $G = \$180$per cycle;$\gamma = 0.6$; $\varphi = 0.005$.

After calculation using Mathematica -11.3 software following optimal obtained results are:

$t_m^* = 132.388$; $t_l^* = 438.741$; $Q_0^* = 118014.0$; $CE = 1{,}36{,}698\ kg$; $\text{ᵽ}^*\,(t_m\, t_l\,) = \$2.25734X10^8$ $\rho(5) = 0.000571429$

Example 3: The following set of data in their respective unit have been considered for model validation in case model developed without Learning Curve and with Green Technology Investment for reducing carbon emission. Data are chosen randomly to validate the models are:

Demand is taken as $D(f_p) = a + b\, f_q$ unit per year; $a = 100$; $b = 0.05$.; $f_q = 0.5$; $\lambda_s = 1000$; $S_c = \$0.15\ per\ unit$; $O_c = \$\,200$ per order; $U_c = \$15$; $h_c = \$\,30$ per unit/time/year; $c_s = \$\,3$ per unit per year; $c_l = \$\,2$ per unit per year ; $x_t = \$35$ per Kg CO2; $U_p = \$100$ per unit; $U_d = \$20$; $c_d = \$15$; $\theta = 0.015$; $c_f = 0.0016$ kg CO_2 per unit per unit time; $c_e = 0.003$ kg CO_2 per kWh;$v_e = 1.04$ kWh; $W_c = \$5$ per unit; $R = R_d - r_d = \$\,0.008$ per unit ; $\alpha = 2\ lit\ per\ day$; $C_{fd} = 0.08$; $f = 0.003$; $t_i = 0.675$; $c_{fd} = 0.03$; $\sigma = 0.8$; $G = \$180$per cycle; $\gamma = 0.6$; $\varphi = 0.005$.

After calculation using Mathematica -11.3 software following optimal obtained results are:
$t_m^* = 132.587$; $t_l^* = 590.520$; $Q_0^* = 35463.30$; $CE = 1{,}38{,}129\ kg$; $\text{ᵽ}^*\,(t_m\, t_l\,) = \$2.28289\ X10^8$ $\rho(5) = 0.000571429$

Graphical representations
(For model with Learning and Green Technology Investment)

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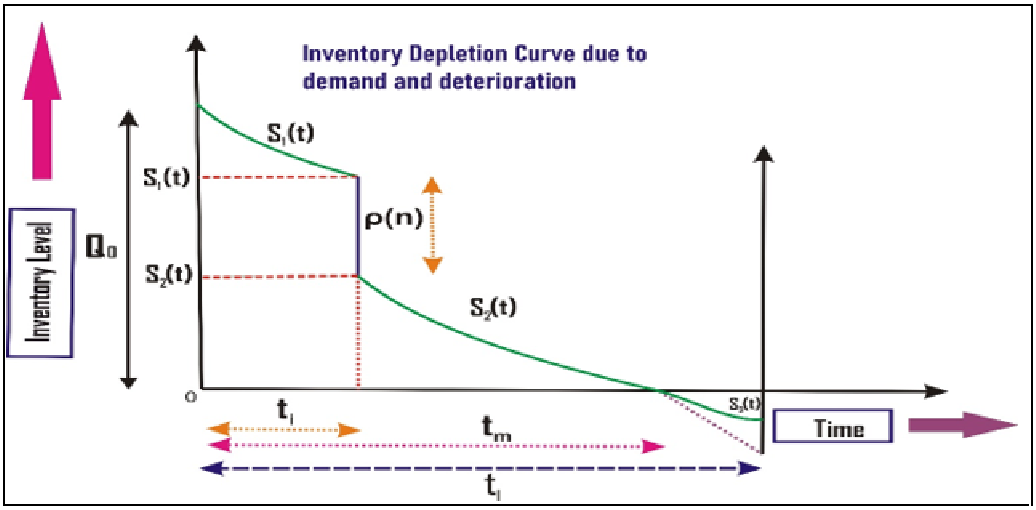

Figure 3: Showing depletion of inventory during complete cycle length under inspection process

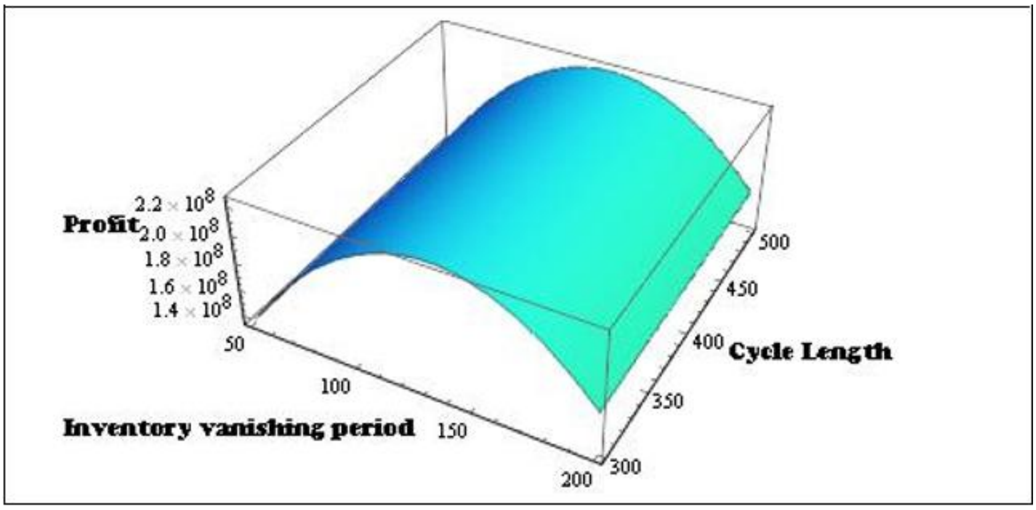

Figure 4: Cycle Length & Inventory vanishing period Vs Profit Function

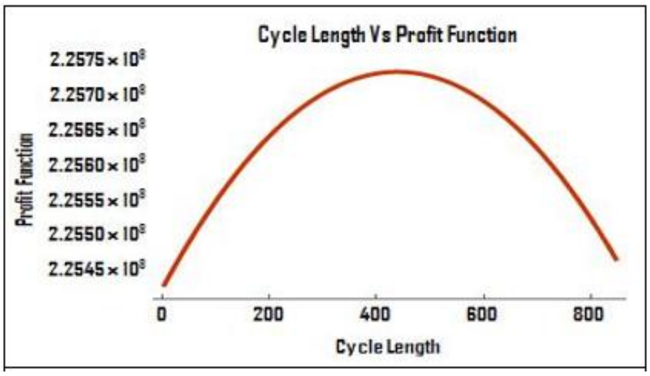

Figure-5: Cycle Length Vs Profit Function

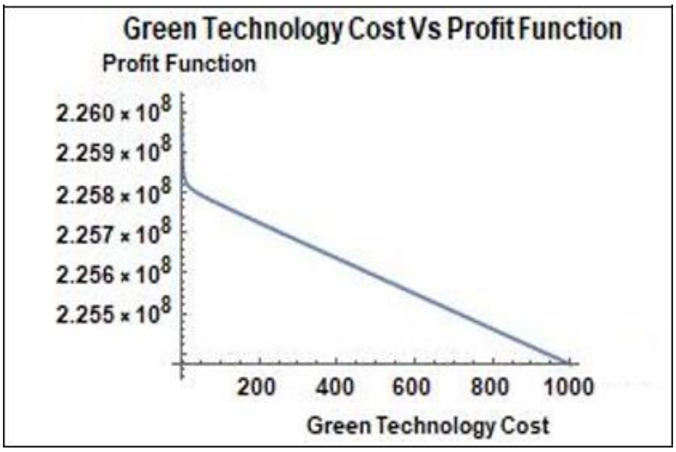

Figure 6: Green Technology Cost Vs Profit Function

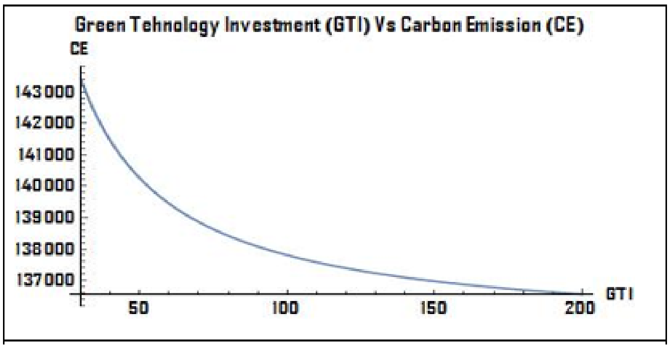

Figure 7: Green Technology Investment Vs Carbon Emission

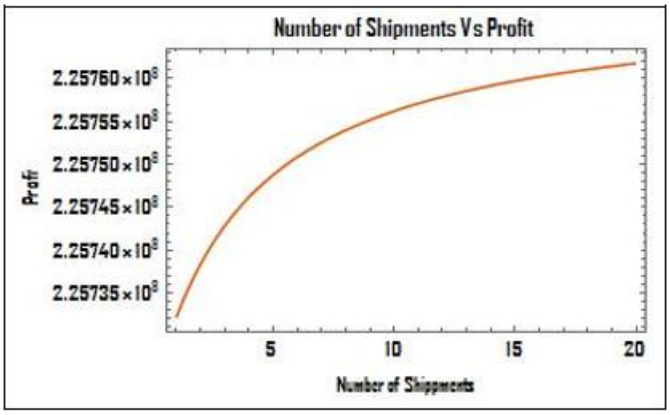

Figure 8: Number of shipment Vs Profit Function

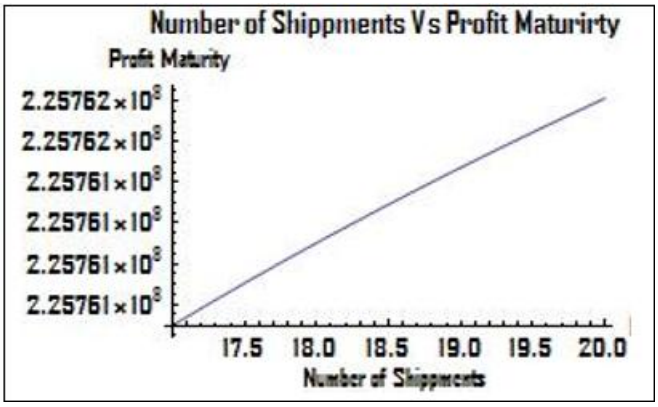

Figure 9: Number of shipment Vs Profit Maturity

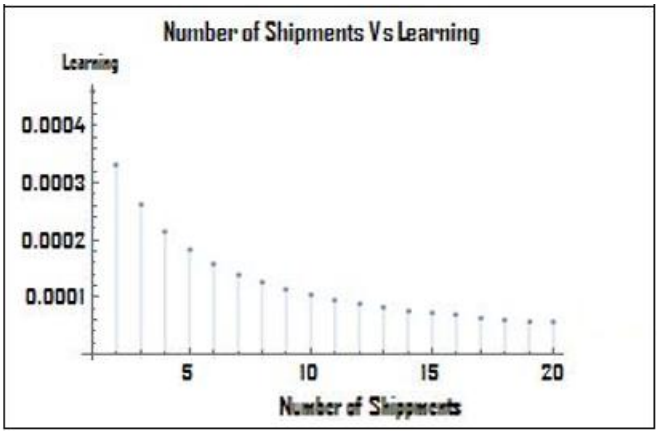

Figure 10: Number of shipment Vs learning output

8. Sensitivity Performance

In this section, sensitivity analysis is performed for parameters' value varying between -50% to 50%, and impact of parameters on profit of retailers, cycle length of business and selling quantity of green product are recorded in the Table- 2 at base value of model developed with learning and green technology investment and further authors recorded their findings in the observation section separately.

Table 2: Sensitivity analysis based on parameters' value ranging from -50% to 50%

Value of decision variables taken at base value $n = 5$ & $f_q = 1$						Percentages Change	
Parameter	Value	t_m^*	t_l^*	Q_0^*	$\text{p}^*\,(t_m^*\, t_l^*)$	Q_0^*	$\text{p}^*\,(t_m^*\, t_l^*)$
O_c	300	132.388	438.741	118059	2.25734x10^8	0.038	0
	100	132.407	438.727	118058	2.25743x10^8	0.037	0.00398
a	150	136.976	457.562	102932	2.39190x10^8	-12.780	5.96099
	50	127.550	382.224	137512	2.12524x10^8	16.526	-5.85202
b	0.075	132.407	438.727	118058	2.25743x10^8	0.037	0.00399
	0.025	132.406	438.726	118058	2.25743x10^8	0.037	0.00399
U_c	22.5	132.256	438.727	117977	2.24858x10^8	-0.031	-0.38807
	7.5	132.557	438.727	118139	2.26629x10^8	0.106	0.39649
h_c	45	135.651	438.727	119777	2.26319x10^8	1.492	0.25915
	15	129.331	438.727	116380	2.25433x10^8	-1.385	-0.13334
σ	1.2	140.438	410.617	61216.4	1.25630x10^8	-48.128	-44.34600
	0.4	140.438	410.617	61216.4	1.25630x10^8	-48.128	-44.34600
c_s	3	132.406	445.935	118058	2.25752x10^8	0.037	0.00797
	1.5	132.407	431.519	118058	2.25733x10^8	0.037	-0.00044
c_l	4	132.407	292.485	118058	2.25628x10^8	0.037	-0.04695
	1	132.406	877.454	118058	2.26063x10^8	0.0372	0.14574
α	3	140.838	438.741	61216.4	1.25632x10^8	-48.128	-44.34511
	1	140.838	438.741	61216.4	1.25632x10^8	-48.128	-44.34511

c_{fd}	0.045	140.832	438.741	61214.9	1.25632x10^8	-48.129	-44.34511
	0.015	140.844	438.741	61217.8	1.25632x10^8	-48.127	-44.34511
λ_s	1500	132.034	438.741	132478	2.24921x10^8	12.256173	-0.36016
	500	132.960	438.741	88817.8	2.27372x10^8	-24.739	0.72563
U_p	150	133.034	678.987	118683	2.28902x10^8	0.567	1.40342
	50	131.252	198.468	117433	2.22850x10^8	-0.492	-1.27760
c_f	0.0024	132.388	438.741	118059	2.25734x10^8	0.038	0
	0.0008	132.388	438.741	118059	2.25734x10^8	0.038	0
φ	0.9	140.838	438.741	61214.6	1.25576x10^8	-48.129	-44.36992
	0.3	140.845	410.617	61218.1	1.25576x10^8	-48.126	-44.36992
U_d	30	132.388	438.768	118059	2.25735x10^8	0.0381	0.00044
	10	132.388	438.714	118059	2.25733x10^8	0.0381	-0.00045
θ	0.0225	141.243	438.741	65316.2	1.69028x10^8	-44.654	-25.12071
	0.0075	123.791	438.741	267221	3.95580x10^8	126.432	75.24166
S_c	0.225	132.386	438.741	118058	2.25725x10^8	0.0373	-0.00398
	0.075	132.389	438.741	118060	2.25743x10^8	0.039	0.00398
c_d	22.5	130.174	438.727	116844	3.08641x10^8	-0.991	36.72774
	7.5	137.622	438.727	120796	1.43156x10^8	2.357	-36.58199
W_c	7.5	131.471	438.741	117564	2.53349x10^8	-0.381	12.23342
	2.5	133.588	438.727	119756	1.98102x10^8	1.476	-12.24095
R_d	0.012	85.6359	292.408	87138.0	9.29982x10^7	-26.163	311.98135
	0.004	272.433	877.739	155030	9.61719x10^8	31.366	326.04082
x_t	52.5	132.393	438.741	118062	2.25735x10^8	0.040	0.00044
	10	132.388	438.741	118057	2.25733x10^8	0.036	-0.00044
t_i	1.0125	132.111	438.741	117912	2.24313x10^8	-0.086	-0.62950
	0.3375	132.383	438.741	118207	2.27166x10^8	0.164	0.63437
G	270	132.407	410.603	118058	2.25704x10^8	0.037	-0.01328
	90	132.407	466.851	118058	2.25785x10^8	0.037	0.02259
c_e	0.0045	132.393	438.741	118062	2.25735x10^8	0.041	0.00044
	0.0015	132.383	438.727	118057	2.25733x10^8	0.036	-0.00044
v_e	1.56	132.393	438.741	118062	2.25735x10^8	0.041	0.00044
	0.004	132.383	438.727	118057	2.25733x10^8	0.036	-0.00044
f	4.5	132.404	438.729	118058	2.25742x10^8	0.038	0.00354
	1.5	132.358	438.763	118061	2.25720x10^8	0.0398	-0.00620
η	0.0045	140.797	438.768	61217.6	1.25618x10^8	-48.127	-44.35131
	0.0015	140.879	438.771	61215.1	1.25645x10^8	-48.129	-44.33935
δ	9.75	140.926	273.659	61238.2	1.25612x10^8	-48.109	-44.35397
	3.25	140.672	933.988	61175.1	1.26402x10^8	-48.163	-44.00400
ζ	0.375	140.847	438.735	61216.2	1.25637x10^8	-48.128	-44.34289
	0.125	140.827	438.748	61216.2	1.25628x10^8	-48.128	-44.34688

Table-3: Representing variation in the decision variables and profit based on Learning parameter (at $\eta = 0.25$) with increasing number of shipments

Variation in the value of decision variables, Ordered Quantity and Profit Function with effect to learning factor & at $\rho(5)$						
$\eta = 0.25$			Order quantity and profit		Percentage change	
n	t_m^*	t_l^*	Q_0^*	ꝑ* $(t_m^*\ t_l^*)$ in \$	Q_0^*	ꝑ* $(t_m^*\ t_l^*)$ in \$
1	132.37	438.754	118060	2.25726x10^8	0.039	-0.00354
2	132.376	438.750	118060	2.25728x10^8	0.039	-0.00266
3	132.38	438.747	118060	2.25730x10^8	0.039	-0.00177
4	132.384	438.744	118059	2.25732x10^8	0.038	-0.00088
5	132.388	438.741	118059	2.25734x10^8	0.038	0
6	132.391	438.738	118059	2.25736x10^8	0.038	0.00088
7	132.394	438.736	118059	2.25737x10^8	0.038	0.00134
8	132.397	438.734	118058	2.25738x10^8	0.037	0.00177
9	132.400	438.732	118058	2.25740x10^8	0.037	0.00266
10	132.402	438.730	118058	2.25741x10^8	0.037	0.00310
11	132.404	438.729	118058	2.25742x10^8	0.037	0.00354
12	132.407	438.727	118058	2.25743x10^8	0.037	0.00399
13	132.408	438.726	118058	2.25744x10^8	0.037	0.00444
14	132.410	438.724	118058	2.25745x10^8	0.037	0.00488
15	132.412	438.723	118058	2.25745x10^8	0.037	0.00488
16	132.414	438.722	118058	2.25746x10^8	0.037	0.00533
17	132.415	438.721	118057	2.25747x10^8	0.036	0.00577
18	132.416	438.720	118057	2.25748x10^8	0.036	0.00621

19	132.418	438.720	118057	2.25748x10^8	0.036	0.00621
20	132.419	438.18	118057	2.25749x10^8	0.036	0.00665

Table 4: Representing variation in the decision variables and profit based on Learning parameter (at $\eta = 1.5$) with increasing number of shipments

Variation in the value of decision variables, Ordered Quantity and Profit Function with effect to learning factor & at $\rho(5)$						
$\eta = 1.5$			Order quantity and profit		Percentage change	
n	t_m^*	t_l^*	Q_0^*	$\text{ᵽ}^*(t_m^*\, t_l^*)$	Q_0^*	$\text{ᵽ}^*(t_m^*\, t_l^*)$
1	132.384	438.744	118059	2.25732x10^8	0.038	-0.00089
2	132.397	438.734	118058	2.25738x10^8	0.037	0.00177
3	132.407	438.727	118058	2.25743x10^8	0.037	0.00399
4	132.414	438.722	118058	2.25746x10^8	0.037	0.00532
5	132.419	438.718	118057	2.25749x10^8	0.036	0.00665
6	132.423	438.715	118057	2.25751x10^8	0.036	0.00754
7	132.427	438.712	118057	2.25753x10^8	0.036	0.00843
8	132.430	438.710	118056	2.25754x10^8	0.035	0.00887
9	132.429	438.708	118056	2.25755x10^8	0.035	0.00932
10	132.434	438.707	118056	2.25756x10^8	0.035	0.00976
11	132.436	438.705	118056	2.25757x10^8	0.035	0.01020
12	132.438	438.704	118056	2.25758x10^8	0.035	0.01065
13	132.439	438.703	118056	2.25759x10^8	0.035	0.01109
14	132.441	438.702	118056	2.25759x10^8	0.035	0.01109
15	132.442	438.701	118056	2.25760x10^8	0.035	0.01153
16	132.443	438.701	118056	2.25760x10^8	0.035	0.01153
17	132.444	438.700	118056	2.25761x10^8	0.035	0.01198
18	132.445	438.699	118056	2.25761x10^8	0.035	0.01198
19	132.445	438.699	118056	2.25761x10^8	0.035	0.01198
20	132.4446	438.698	118055	2.25762x10^8	0.034	0.01242

Table-5: Representing variation in the decision variables and profit based on Learning parameter (at $\eta = 1.5$) with increasing number of shipments

Variation in the value of decision variables, Ordered Quantity and Profit Function with effect to learning factor & at $\rho(5)$						
$\eta = 2.5$			Order quantity and profit		Percentage change	
n	t_m^*	t_l^*	Q_0^*	$\text{ᵽ}^*(t_m^*\, t_l^*)$ in \$	Q_0^*	$\text{ᵽ}^*(t_m^*\, t_l^*)$ in \$
1	132.402	438.730	118058	2.25741x10^8	0.037	0.00310
2	132.419	438.718	118057	2.25749x10^8	0.036	0.00665
3	132.428	438.711	118056	2.25753x10^8	0.035	0.00842
4	132.434	438.707	118056	2.25756x10^8	0.035	0.00976
5	132.439	438.704	118056	2.25758x10^8	0.035	0.01065
6	132.442	438.701	118056	2.25760x10^8	0.035	0.01154
7	132.444	438.700	118055	2.25761x10^8	0.034	0.01198
8	132.446	438.698	118055	2.25761x10^8	0.034	0.01198
9	132.448	438.697	118055	2.25763x10^8	0.034	0.01287
10	132.449	438.696	118055	2.25763x10^8	0.034	0.01287
11	132.450	438.695	118055	2.25764x10^8	0.034	0.01331
12	132.451	438.695	118055	2.25764x10^8	0.034	0.01331
13	132.452	438.694	118055	2.25764x10^8	0.034	0.01331
14	132.452	438.694	118055	2.25764x10^8	0.034	0.01331
15	132.453	438.693	118055	2.25765x10^8	0.034	0.01376
16	132.454	438.693	118055	2.25765x10^8	0.034	0.01376
17	132.454	438.692	118055	2.25766x10^8	0.034	0.01419
18	132.454	438.692	118054	2.25766x10^8	0.033	0.01419
19	132.455	438.692	118055	2.25766x10^8	0.034	0.01419
20	132.455	438.692	118055	2.25766x10^8	0.034	0.01419

9. Observations and Managerial Insights

9.1 Observations

From numerical section it is observed that the profit of the retailer earned in case of model developed under learning concept is significantly higher than profit earned in the case of model developed under learning concept and green technology investment together. In addition, inventory vanishing period, business cycle length and product quantity ordered is considerably higher in case of model with learning concept than the model with learning and green technology investment. Further, authors observed that amount of carbon emission produced in case of model with learning concept is much more as compared to model with learning and green technology investment. Investment in the green technology is more beneficiary as compared to model only with learning process that support a sustainable environment by reducing

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carbon emission about 59.60% while there a reduction of 0.125% in case of model with learning and green technology investment. Model only with green technology investment and yields 0.99% more profit as compared to model with learning and 1.251% more to the model equipped with both policies while 1.04% more carbon emissions & 59.60% less carbon emission as compared to model with learning and green technology investment and model only with learning concept respectively. Also, cycle length and quantity ordered in case of model only with green technology investment policy are much more than rest of two models. This reveals that model developed with both policies is more beneficial for retailers as well as environmental perspective and supports human healthy survival.

Table-2 reveals that profit function is considerably sensitive to the demand parameter a and is directly proportional to this while quantity ordered is indirectly proportional to this parameter and is moderately sensitive. Profit function is considerably sensitive to screening rate while quantity ordered is highly sensitive to this and both are directly proportional to this parameter. Waste management cost moderately impact profit function and considerably to ordered quantity and have direct impact on both. Profit function and ordered quantity are moderately sensitive to other remaining parameters while highly σ, α, c_{fd}, φ, m, δ, & η and are decreasing in both cases of decrease or increasing value of these parameters. Parameters R_d and d_θ have much more impact on profit function as well as on quantity purchased as compared other parameters and have direct impact on both.

Effect of learning parameter on inventory vanishing period, business cycle length, ordered quantity and profit function can be noted from Table-3, Table-4 and Table-5. At different value of learning parameter values of vanishing period, business cycle length, ordered quantity and profit function have been calculated. In each case considerable increment on all the output decision variables, quantity ordered, and profit function have been recorded. With increase in the number of shipments maturity towards learning process have been observed as profit become consistent after certain number of shipments. Saturation of profit is observed at low number of shipment when learning parameter $\eta = 2.5$.

Form Figure-9 & Figure-10 shows maturity in the learning process and learning output resulting consistent retailer's profit after certain number of shipments. Figure-8 reveals that profit increases as number of shipments increases at other fixed parameters' value. From Figure-6 it is observed that investment in the green technology decreases profit value with very low percentages while amount of carbon emission decreases at moderate pace. Figure-5 shows that retailer's Profit increases as business cycle length increases up to a fixed length and thereafter starts decreasing proving global maxima having optimum profit. Concavity of the model function is shown in Figure-4. Profit function is optimal with respect to cycle length and inventory vanishing period.

9.2 Managerial insight

Present research work provides wider scope to managerial team to balance between profit and environmental issues. Environmental concerns are more significant as compared to individual profit and thus managers can choose model developed with learning process and green technology investment. Models indicate that if the manager's nature is of high learning from past activities of inventory management system, then company/retailers' profit will grow at faster pace. Manager can set parameter's value for maximising retailer's profit and minimizing amount of carbon emissions released during inventory operations. Ordered quantity may be lowered for more profit with low carbon emission as present model is helpful in this direction. Use of green technology by investing some capital for managing storage facilities for deteriorating products resulted in low deterioration and less waste disposal cost as well as very low release of carbon emissions having global impact and providing sustainable environment. Performing Sensitivity will provide a base to choose range of parameters to fix for balancing profit output and release of carbon emission as well as business cycle length and quantity ordered.

Greenness quality dependent demand is considered so managers have lay out to decide the quality of a product of be fixed for optimum profit lower business cycle length and carbon emission. Manager have been provided wider opportunity to choose model either profitable or for betterment sustainable environment. Selection of cost-effective components will lead to profit as well as sustainable environment with the present developed models. Controlling on rate of deterioration and discount rate managers can increase profit with selected model and reduce quantity of carbon emissions.

10. Concluding Remarks and future elongations

The present research work has examined the effect of learning and green technology investment on the optimal size of ordered quantity, business cycle length and retailer's profit. Model is developed under inflationary condition with discount rate and learning process along with greening product and investment in green technology to control rate of deterioration in resulting low carbon emissions. Retailers profit is maximized with respect to quantity ordered and business cycle length. The present study reveals that carbon emission is affected by green product as well as investment in the green technology. In addition, carbon emission and profit are also affected due to deterioration rate and cost occurred there at so higher investment in green technology will reduce carbon emission resulting sustainable environment and balanced profit. High learning capacity of managers will lead to saturation in the profit with less shipments. Learning concepts suggest retailers to manage shipments with high learning rate till maturity phase. Model yielding low amount of carbon emissions (59.60%) as compared to model releasing high (59.60%) is more beneficial. Table-3, Table-4 and Table-5 shows that retailer's profit $\text{ᵽ}^*\,(t_m^*\, t_l^*\,)$ follows the "S"- shaped learning curve and achieve the maturity with variable shipment and learning rate. Furthermore, retailer's profit and ordered quantity affected by inflation rate and deterioration rate which are discussed in sensitivity analysis section. The present study is important for those who want sustain environment with optimal business cycle length, ordered quantity and significant profit under carbon tax imposed by governmental agencies. The developed model is highly non-linear function of decision variables and analytical solutions are obtained, and concavity of the model is depicted through 3D graphs. Observation revealed that if there is more imperfect/defective products then retailer should be more vigilant while placing order.

11. Future Extensions

This paper can be extended by applying learning concepts to the holding cost, setup cost and transportation system for reducing carbon emissions for goal of achieving sustainable environment as per today's need. Additionally, this work may be extended adding non-linear, stochastic, probabilistic and fuzzy demand pattern and different carbon regulations imposed by regularity authorities or government. Furthermore, it can be enriched by incorporating different trade-credit policy such as discount policy on advance payment or discount on purchase cost when full payment is made in advance or quantity discount on bulk purchase.

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