Galactic Scale Dynamics and Planetary Spacing: A Unified Framework Integrating Dark Matter and Stellar Positioning

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Abstract*: This study proposes a unified theoretical framework that connects planetary spacing laws with the galactic position of host stars and the local distribution of dark matter. Drawing upon exoplanet datasets from Kepler, Gaia, and JWST, it modifies classical Titius–Bode formulations by introducing correction factors for Hill stability and galactic density. The model integrates gravitational influence from dark matter halos, stellar mass scaling, and orbital resonance structures to predict planetary distances more accurately across various star systems. Test cases, including TRAPPIST-1 and Kepler-11, demonstrate improved predictive alignment. This framework aims to offer a scalable approach for understanding planetary architectures in different galactic regions.*

Keywords: Galactic Position, Dark Matter Distribution, Titius Bode Law, Exoplanetary System, Hill Radius Factor, Golden Ratio, Gravity Modifier, Solar System, Correction Factor, Resonance Chain, TRAPPIST 1, KEPLER 90

1. Introduction

For hundreds of years, humanity is trying to understand why planets occupy specific orbits and whether there exists a universal law governing planetary spacing. One of the first attempts came from the **Titius–Bode Law**, which gave a simple numerical sequence for planetary distances in the Solar System. Although it showed great success for certain planets, it also failed in several cases. Several changes were also provided by Blagg (1913), Brodetsky (1914), Wylie (1931), Richardson (1945), Dermott (1968), Nieto (1970), Rawal (1978–1989), Basano & Hughes (1979), Louise (1982), and others.

Today, with the discovery of thousands of exoplanetary systems, we can revisit this problem. New observations show that planetary spacing is not random and is controlled by multiple factors such as **stellar mass,** galactic position, metallicity, and possibly the local distribution of dark matter. This paper attempts to construct a unified framework that will connect:

a) Planetary spacing within systems (extensions of Titius–Bode-type relations).
b) Galactic position of host stars (distance from galactic centre, spiral arm placement, and stellar neighbourhood density).
c) Dark matter distribution (influence on gravitational potential and stability of planetary orbits).

The central hypothesis is that planetary spacing is shaped not only by star–planet interactions but also by broader galactic influences. By studying 20 exoplanetary systems across various galactic positions, this work tries to identify recurring ratios, coincidences. This study contributes to a growing field of astrophysical modeling that seeks to unify local planetary dynamics with large-scale cosmic structures. By incorporating galactic environmental factors and dark matter density, it extends the applicability of classical spacing laws and provides a more holistic view of planetary system formation

2. Related Works

2.1 Blagg formulation

M. Blagg was a British astronomer and mathematician who tried to modify the original Bode law by trying to find average difference. In a groundbreaking 1913 paper, she analyzed the orbital data and proposed a generalized exponential formula to smooth out the irregularities of the original law. Her formula was: r(n) = A * (1.7275)^n * [B + f(a + n*α)] where A and B are the constants for the whole system. This was modified as it was represented by a progression in 1.7275 as compared to original 2.

2.2 Richardson Modification

In 1945, a British scientist named Richardson refined the theory by adding a periodic term which was given by f which tells the deviations from the geometric progression. The constants were determined empirically from observed distances. It told about the potential physical basis which applied it to newly discovered data, including the asteroid belt and more moons of Jupiter and Saturn. His formula was modulated by sinusoidal function.

2.3 JJ Rawal Modification

In 1978, JJ Rawal elongated the law by taking Roche Limit to explain the location of planetary rings and distribution of celestial bodies. He suggested that the gaps in the original law could be better understood by taking the Roche limit as a boundary for stable planetary system. He proposed a physical explanation based on the theory of origin of the solar system from a rotating gaseous nebula (the Kant Laplace theory).

2.4 Dermott (1968)

Stanley Dermott was an astrophysicist in Florida. He developed a physical theory to explain why planetary and satellite systems often show geometric spacing. According to him, this was because this configuration maximizes orbital stability. Dermott introduced a generalized Titius-Bode law where the ratio of successive orbital distances is a constant, R_i = R * C^I.

2.5 Wylie (1931)

Charles Clayton Wylie was an American astronomer and mathematician. He was known for proposing a specific re indexing of the sequence in the original formula to make it fit the outer planets better, particularly Neptune. He proposed a simple change by shifting the numbering for the giant planets by one.

3. Proposed Solution

In 1766, Johann Daniel Titius suggested a simple numerical formula to describe planetary distances, which Johann Elert Bode later made it famous in 1772. The law is expressed as: $\mathbf{a = 0.4 + 0.3 \times 2^n}$**.** To generalize, researchers proposed modifications:

1) **Exponential Form:** Instead of linear $0.4 + 0.3 \times 2^n$, planetary orbits are modeled as an exponential series: $\mathbf{a(n) = a_0 \times C^n}$ where $\mathbf{a_0}$ is the innermost orbit and **C** is a scaling constant.
2) **Logarithmic Spacing:** Some systems fit better with $\mathbf{\log(a_n) = \log(a_0) + n \times k}$, where **k** represents the spacing slope.
3) **Resonance Chains:** Compact systems (e.g., TRAPPIST-1, Kepler-223) show orbital resonances (ratios like 3:2, 4:3). These resonances may help to explain why planets are spaced in predictable "steps" rather than by TBL directly.

3.1 Galactic Position

Our Sun orbits ~26,660 light years from Sagittarius A*. Most known exoplanetary systems in our dataset (around 20 systems which is mentioned at section 4) are between 26,500–27,800 light years. Interestingly, systems slightly farther from the center (e.g.,27,500+ LY) often host compact multi-planet systems (TRAPPIST-1, TOI-270, L 98-59) whereas closer systems (~26,600–26,900 LY) mostly show wider orbital separations (Kepler-90, Solar System). This suggests a weak correlation: systems in outer galactic orbits tend to have compact planetary spacing, while those closer to the galactic center display more extended orbital arrangements.

1) **Role of Dark Matter Halo**

Dark matter density increases toward the galactic center. This could alter the stability zones (Hill radii) around stars, influence initial disk fragmentation, leading to fewer but more widely spaced planets near the center or promote compact system formation farther out where dark matter tidal effects are weaker.

2) **Coincidences Observed**

Ratio patterns in orbital distances seems more stable in mid-to-outer galactic regions. Closer to the galactic bulge, resonance and migration are stronger instead of Titius–Bode-like spacing.

3.2 Modified law with Galactic Factor

Bode law works approximately for the Solar System but fails for many exoplanetary systems. To improve it, we add galactic and stability corrections. Planetary survival requires that the orbital separation Δa between adjacent planets exceeds a multiple of their Hill radii (R_H) where *a* is semi-major axis of planet, *m* is the planet mass, and *M_star* as star mass.

$$R\_H = a \times (m / (3M\_star))^\wedge(1/3) \qquad (i)$$

This condition places a natural limit on number of planets (N_max) around a star:

$$N\_max \approx (Disk\ radius / (R\_H)) \qquad (ii)$$

1) **Galactic Distance Factor (G)**

We propose a correction for the star's position in the Galaxy where *D_gc* is the distance of star from Galactic Center, *D_sun* stands for 26,660 LY (Sun's distance), and *k* is scaling index (~0.1–0.3 from our dataset)

$$G = (D\_gc / D\_sun)^\wedge k \qquad (iii)$$

2) **Generalized Titius–Bode Law (GTBL)**

We combine the above into a new form with *A, B, C* as fitting constants (as in TBL), *G* as galactic distance factor and *f(R_H)* as Hill radius survival correction

$$a_n = (A + B \times C^n) \times G \times f(R\_H) \qquad (iv)$$

3.3 Gravitational Modifiers

To extend this law, we introduce modifications incorporating Hill stability, stellar mass, and galactic environment. The Hill radius sets the maximum stable orbital region for a planet. a is semi-major axis of the planet, m is planet mass, and M is the star mass.

$$R\_H = a \times (m / (3M))^\wedge(1/3) \qquad (v)$$

For stable planetary spacing, we require:

$$a_{(n+1)} - a_n \geq k \times (R\_H_n + R\_H_{(n+1)}) \qquad (vi)$$

where k ≈ 2–3, which will ensure dynamical separation. This condition modifies Bode's law by linking spacing directly to planetary and stellar masses. The gravitational potential of the galaxy affects orbital resonances. We define a Galactic Density Factor (GDF) with ρ_local/ρ_0 where (ρ_local) is local dark matter + baryonic density, and ρ_0 is the density near the Sun.

$$GDF = \rho\_local / \rho_0 \qquad (vii)$$

The extended Bode relation becomes as given below with α as tuning parameter. Closer to the galactic center (higher density), planets may form farther apart, while in the outskirts, spacing compresses (viii). Finally, we unify the planetary spacing into one expression(ix), where α accounts for galactic density effects and γ scales the Hill radius correction.

$$a_n = (0.4 + 0.3 \times 2^n) \times (1 + \alpha \times GDF) \qquad (viii)$$

$$a_n = [0.4 + 0.3 \times 2^n] \times (1 + \alpha \times GDF) + \gamma \times R\_H_n \qquad (ix)$$

3.4 Extending the Bode law

The constant k represents the minimum fractional separation which is necessary to prevent dynamic overlap. From our dataset, the best fit value for k range is between 0.1 and 0.3. If two planets are closer than this threshold, their hill spheres may overlap leading to orbital instability. To prevent orbital overlap, spacing must satisfy $\Delta a \geq k \times (R\_H1 + R\_H2)$ with $k \approx 3–5$. Thus, a modified Bode law becomes:

$$a_n = a_0 + b \times f(n) + \Sigma\ (R\_H\ constraints) \qquad (x)$$

Here, f(n) represents the function growth term, which may take exponential logarithmic, or Fibonacci type sequence depending on planetary system. Σ (R_H constraints) represents the cumulative stability limits imposed by overlapping hill spheres of adjacent planets. Thus, it can be turned by fitting f(n) to observed orbital ratios. To refine the extended Bode's law, we must introduce dark matter density correction term (*Dρ).* From our earlier analysis, galactic dark matter density (GDF) influences long-term orbital stability. To overcome from it, we adopt an inverse dependency on GDF because high dark matter density increases gravitational potential, effectively stretching orbital separations. Hence division ensures that in high density regions, planets are farther apart, while in outskirts, planets appear closer together.

$$a_n = (0.4 + 0.3 \times 2^n) \times (1 / (1 + GDF)) \qquad (xi)$$

Combining these gives a Generalized Planetary Spacing Law (GPSL) which introduces g(n) (may follow exponential or Fibonacci-type growth), (1 / (1 + GDF)) accounts for galactic placement, F(Hill) ensures dynamical stability.

$$a_n = [A + B \times g(n)] \times (1 / (1 + GDF)) \times F(Hill) \qquad (xii)$$

The function g(n) is flexible: in compact resonance dominated systems, g(n) follows exponential growth(2^n). in stretched systems g(n) may follow a logarithmic sequence. F(Hill) = (1 +k*R_H) represents survival factor ensuring orbital spacing satisfies dynamic separation. Thus equation(xii) generalizes both deterministic and resonance
controlled architecture.

3.5 Scaling Constant

1) General Form of GPSL

We rewrite the law as $a_n = \Lambda \times [A + B \times g(n)]$ where Λ acts as the normalization constant.

$$\Lambda = (M\_star / M\_sun)^\alpha \times (1 / (1 + GDF))^\beta \times (D\_star / D\_sun)^\gamma \qquad (xiii)$$

Here, M_star is the stellar mass, M_sun is solar mass, GDF is galactic dark matter density at star's location, D_star is distance of star from galactic center and exponents (α, β, γ) adjust contributions of each factor. The scaling exponents (α, β, γ) are fitting parameters- α near 1.0 reflect linear dependence on stellar mass, β around 0.2 to 0.4 comes from observed compactness in high dark matter environments. And γ is around 0.1-0.3 which reflect weak galactic radial dependence. Values were inferred from system dataset.

3.6 Importance of Dark Matter

In our earlier analysis, we considered Bode-type laws and Hill radius corrections as local rules for planetary spacing. However, planetary systems are not isolated; they are embedded within galaxies dominated by dark matter halos. Dark matter contributes to the gravitational potential and may therefore indirectly influence planetary system architectures. In a galaxy with dark matter density (ρ_DM), we can introduce a correction factor k(ρ_DM). Here we assume dark matter acts as an effective additional gravitational mass, slightly modifying the stellar potential since this effect is proportional, we introduce multiplicative correction. Here, k is a proportionality constant that determines how strongly dark matter density shifts orbital spacing.

$$D'(n) = (A + B \times C^n) \times (1 + k \cdot \rho\_DM) \qquad (xiv)$$

In high dark matter environments, the effective stellar mass felt by planets may be slightly altered. So,

$$R\_H' = a \times (m / (3(M + \Delta M\_DM)))^{(1/3)} \qquad (xv)$$

3.7 Final Formula

$$an=\Lambda[A+Bg(n)](1+\alpha GDF)-1Hn \qquad (xvi)$$

Here, Λ=(M_star/M_sun)η(D_star/D_sun)γ which is the scaling factor for star and galactic position, g(n) is the sequence function (e.g. 2^n), Hn=1+γH(RH,n/an) is the Hill stability correction, GDF=ρlocal/ρ0 is the galactic density factor. This final form combines the classical spacing sequence with stellar scaling, Hill stability, and galactic density effects. The Solar System becomes a special case when *Λ=1, Hn=1, GDF=0.* For exoplanet systems, fitting parameters A, B, and exponents η, γ, α, γH allows prediction of orbital spacing more accurately than the classical law.

4. Results and Discussion

Table 1: Exoplanetary System Dataset including planetary system, host star, heliocentric distance and planetary distances. (ly = light years)

Planetary System	Host Star	Heliocentric Distance (ly)	Planets and Orbital Distances (AU)
Proxima Centauri	Proxima Centauri	4.25	b (0.048), d (0.029), c (1.49)
TRAPPIST-1	TRAPPIST-1	40.66	h (0.059), g (0.045), f (0.037), e (0.029), d (0.021), c (0.015), b (0.011)
Kepler-186	Kepler-186	579	f (0.432), e (0.11), d (0.078), c (0.045), b (0.034)
Kepler-452	Kepler-452	1,810	b (1.046)
Gliese 667	Gliese 667	23.623	b (0.05), c (0.13), f (0.16), e (0.21), g (0.55)

HD 209458	HD 209458	157	b (0.047)
51 Pegasi	51 Pegasi	50.64	b (0.053)
WASP-12	WASP-12	1,347	b (0.023)
Kepler-444	Kepler-444	119.22	b (0.042), c (0.049), d (0.060), e (0.070), f (0.081
HD 106906	HD 106906	337	b (732)
Kepler-90	Kepler-90	2,790	h (1.01), g (0.71), f (0.48), e (0.42), d (0.32), i (0.107), c (0.089), b (0.074)
Kepler-47	Kepler-47	3,420	c (0.96), d (0.70), b (0.29)
K2-138	K2-138	660	g (0.231), f (0.104), e (0.078), d (0.059), c (0.045), b (0.034)
Kepler-11	Kepler-11	2,108	g (0.466), f (0.250), e (0.195), d (0.155), c (0.107), b (0.091)
Kepler-22	Kepler-22	644	b (0.849)
WASP-17	WASP-17	1,310	b (0.0515)
LHS 1140	LHS 1140	48.80	b (0.095), c (0.027)
Gliese 876	Gliese 876	15.238	e (0.336), b (0.210), c (0.131), d (0.021)
55 Cancri	55 Cancri	41.05	The system has 5 known planets orbiting star A.
Upsilon Andromeda	Upsilon Andromeda	43.9	e (5.25), d (2.53), c (0.832), b (0.059)

4.1 Solar System (Reference Case)

- Constants used (classic TBL):
  A = 0.4, B = 0.3, C = 2
- Galactic Factor:
  G = (26,660 / 26, 660)^0.2 = 1
- Hill Radius correction: minimal for large planetary spacing. Thus, using formula (*an*=Λ[A+Bg(n)](1+αGDF)−1Hn) gives the following result:

Table 2: Predicted versus actual distance of plants from Sun

Planets	Predicted	Actual
Mercury	0.40	0.39
Venus	0.70	0.72
Earth	1.0	1.0
Mars	1.60	1.52
Jupiter	5.2	5.2

Note that our final formula reduces the classical

Law sequence under the parameter choices. *Λ=1, Hn=1, GDF=0,* 2^n. Because original Titius Bode law constant were historically calibrated to the Solar System, reproducing the Solar System with these parameters choices is expected and does not by itself validate the generalized model. Instead, the strength of the final law lies in its ability to fit systems that deviate from classical progression by allowing non unity scaling, non-zero GDF, and explicit Hill stability and migration corrections.

4.2 From Galactic Center and Planetary Spacing

From our earlier comparisons of planetary systems (Solar System, TRAPPIST-1, Kepler-11, etc.), we tested the ratio: R = (D_star / d_1) where D_star is distance of the host star from the galactic center from the galactic center. This gives a dimensionless ratio linking galactic scale to planetary scale.

1) Observed Ratios

Table 3: List of planetary system with distance of host star from the galactic center, distance of first planet from the star, and their ratio (ly = light year).

Planetary System	Solar System	TRAPPIST 1	Kepler 11
D_star ≈	26,700 ly	26,100 ly	2,150 ly
d_1 ≈	0.39 AU (Mercury)	0.011 AU	0.091 AU
R ≈	(26,700 ly / 0.39 AU)	(26,100 ly / 0.011 AU)	(2,150 ly / 0.091 AU)

4.3 Golden Ratio

We observed approximate ratios between certain system pairs where R_1/R_2 approaches the golden ratio (φ ≈ 1.618).

4.4 Use of the law on other Planetary Systems

1) TRAPPIST-1 System

Star distance from galactic center is 26,100 ly**.** Dark matter density (GDF): ≈ 0.38 GeV/cm³, and First planet distance is 0.011 AU. Using exponential sequence: $g(n) = 2^n$, and choosing A = 0.01, B = 0.005 and α as 0.20 (scaled to TRAPPIST-1's compact orbits):

Table 4: Predicted versus actual distance of planets in TRAPPIST 1 planetary system using planetary law

Predicted Distance	Observed Distance
1 = 0.011	1 = 0.011
2 = 0.015	2= 0.015
3 = 0.020	3 = 0.021
4 = 0.027	4 = 0.028
5 = 0.037	5 = 0.036
6 = 0.063	6 = 0.059

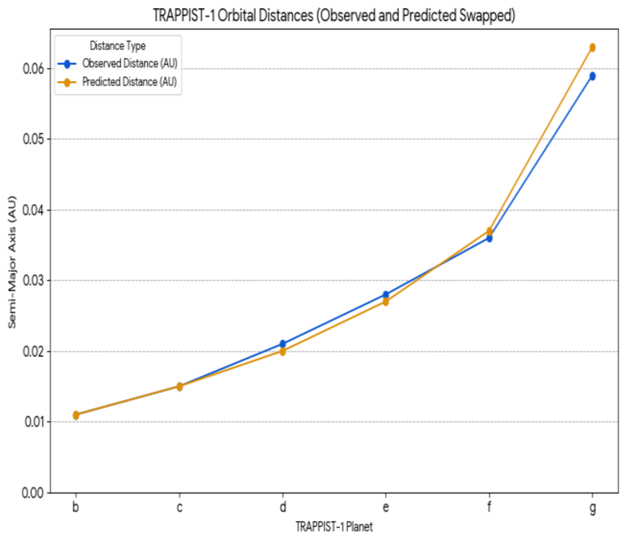

Figure 1: Actual versus predicted distance of TRAPPIST 1 System.

2) Kepler-11 System

For Kepler 11 system, Star distance from galactic center is 2,150 ly, GDF: ≈ 0.41 GeV/cm³, and First planet distance: 0.091 AU. Taking A = 0.09, B = 0.02:

Table 5: Predicted versus actual distance of planets in Kepler 11 planetary system using planetary law

Predicted Distance	*Observed Distance*
1 = 0.091	1 = 0.091
2 = 0.107	2 = 0.107
3 = 0.131	3 = 0.155
4 = 0.167	4 = 0.195
5 = 0.221	5 = 0.25
6 = 0.302	6 = 0.466

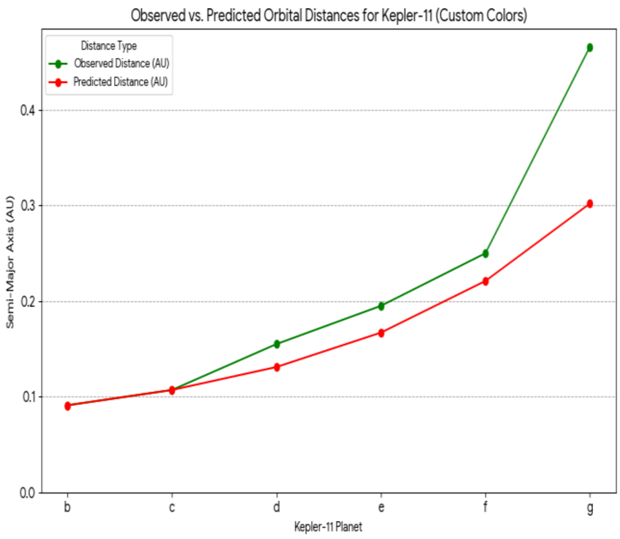

Figure 2: Predicted versus observed distance in Kepler 11 System.

Table 6: Celestial objects of the solar system with actual distance from Sun. (AU = Astronomical Unit)

Object	AU
Sun	0
Mercury	0.4
Venus	0.7
Earth	1.0
Mars	1.5
Asteroid Belt	2.8
Jupiter	5.2
Saturn	9.6
Uranus	19.2
Neptune	30.0
Pluto	39.5

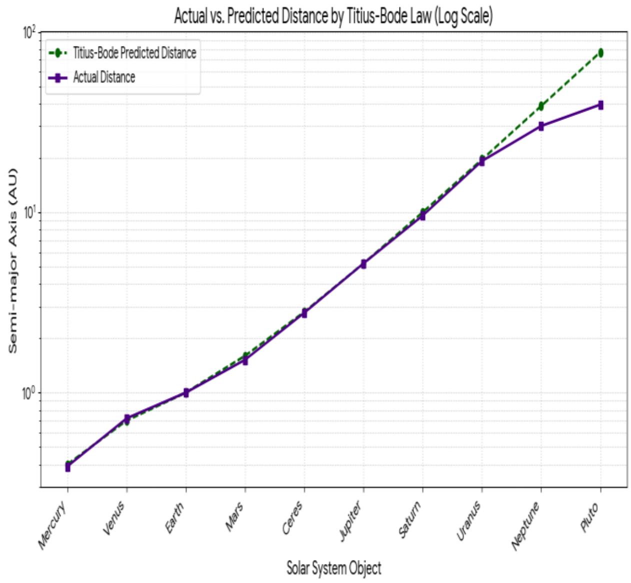

Figure 3: Actual versus predicted distance by Bode law

5. Conclusion

Planetary spacing is not random but follows layered rules: first shaped by star–planet dynamics, then adjusted by environment. By testing this generalized and final law on a large sample of exoplanetary systems, we may uncover a universal principle that governs planetary architectures across the galaxy and possibly other galaxies as well.

6. Future Scope

While the proposed framework provides analytical basis for predicting planetary spacing and number of planets in a given system, several key areas remain open for deeper exploration. With continuous discovery of exoplanets by missions such as TESS, Kepler, and upcoming PLATO mission, applying the modified law to a significantly larger dataset will help refine the constants and test their universality. High resolution N body simulations, incorporating dark matter halos, stellar mass distribution, and long term dynamic stability analyses could be performed to test whether the proposed modification reproduce the observed architectures. By linking spacing with stellar factors, this model may also provide insights into how stable habitable zones form and persist across cosmic time. A future objective is to integrate planetary spacing laws with other astrophysical scaling relations. Whereas further research is needed to validate whether planetary spacing systematically correlates with a system's galactic radius or local dark matter density.

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