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Evaluation of Integrals Involving Products of Srivastava Polynomials and Incomplete H-Function of Several Complex Variables

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Abstract: ***In this study, we examine integrals involving the incomplete H-function of several complex variables in combination with general class polynomials represented in product form. By employing definite integrals from well-known mathematical references, we derive closed-form results that highlight the interplay between special functions and polynomial structures. The incomplete H-function, known for its remarkable generality and applications in multivariate function theory, is here integrated with polynomial structures that unify many classical families. The integrals derived are expressed in a simplified and compact form, making them particularly suitable for applications in mathematical physics, probability theory, and statistical modeling. These results not only broaden the scope of incomplete special functions but also offer efficient tools for handling problems involving products of functions in multidimensional settings. Furthermore, the work provides a foundation for future investigations into more general classes of incomplete multivariable functions and their applications.***

Keywords: Multivariable incomplete H-function, Improper integral, General Class Srivastava's Polynomials

1. Introduction

In this part, we recall key definitions and symbols previously explored in the literature [1]-[5], [9], [13]-[15] on incomplete functions of several complex variables, Srivastava polynomials, and unified integrals, which form the basis for our discussion.

Incomplete Gamma Function (*IGF*)

The widely used incomplete gamma function $\gamma(s,v)$ and $\Gamma(s,v)$ described as

$$\gamma(s,v)=\int_0^x t^{s-1}e^{-t}dt\,;\quad (\Re(s)>0; v\geq 0) \tag{1}$$

$$\Gamma(s,v)=\int_v^\infty t^{s-1}e^{-t}dt\,;\quad (v\geq 0; \Re(s)>0 \text{ when } v=0) \tag{2}$$

such that their sum yields the complete gamma function:

$$\gamma(s,v)+\Gamma(s,v)=\Gamma(s)\,;\quad (\Re(s)>0) \tag{3}$$

Incomplete H-Function of Several Complex Variables:

H.M. Srivastava and R. Panda defined the incomplete H-function of several complex variables in a series of studies [5], where it is represented in terms of a multiple Mellin–Barnes contour integral:

$$\Gamma[z_1,z_2,\ldots,z_r]=\Gamma^{M,N;m_1,n_1;\ldots;m_r,n_r}_{P,Q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\left|\begin{matrix}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{\Im_1}\ldots\int_{\Im_r}\xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{k=0}^r \psi_k(R_k)z_k^{R_k}dR_k\right\} \tag{4}$$

Where $i=(-1)^{\frac{1}{2}}$ and

$$\xi(R_1,R_2,\ldots R_r,\eta)=\frac{\Gamma\left(1-e_j+\sum_{k=1}^r E_j^{(k)}R_k,\eta\right)\prod_{j=1}^M\Gamma\left(f_j-F_j^{(k)}R_k\right)}{\prod_{j=M+1}^Q\Gamma\left(1-f_j+F_j^{(k)}R_k\right)}\times\frac{\prod_{j=2}^N\Gamma\left(1-e_j+\sum_{k=1}^r E_j^{(k)}R_k,\eta\right)}{\prod_{j=N+1}^P\Gamma\left(e_j-\sum_{k=1}^r E_j^{(k)}R_k\right)} \tag{5}$$

$$\psi_k(R_k)=\frac{\prod_{j=1}^{m_k}\Gamma\left(b_j^{(k)}-B_j^{(k)}R_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-a_j^{(k)}+\sum_{k=1}^r A_j^{(k)}R_k\right)}{\prod_{j=m_{k+1}}^{q_k}\Gamma\left(1-b_j^{(k)}-B_j^{(k)}R_k\right)\prod_{j=n_{k+1}}^{p_k}\Gamma\left(a_j^{(k)}-\sum_{k=1}^r A_j^{(k)}R_k\right)} \tag{6}$$

Where (k=1, 2, …, r)
and

$$\gamma[z_1,z_2,\ldots,z_r]=\gamma^{M,N;m_1,n_1;\ldots;m_r,n_r}_{P,Q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\left|\begin{matrix}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{\Im_1}\ldots\int_{\Im_r}\xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{k=0}^r \psi_k(R_k)z_k^{R_k}dR_k\right\}, \tag{7}$$

where

$$\xi(R_1,R_2,\ldots R_r,\eta)=\frac{\Gamma\left(1-e_j+\sum_{k=1}^r E_j^{(k)}R_k,\eta\right)\prod_{j=1}^M\Gamma\left(f_j-F_j^{(k)}R_k\right)}{\prod_{j=M+1}^Q\Gamma\left(1-f_j+F_j^{(k)}R_k\right)}\times\frac{\prod_{j=2}^N\Gamma\left(1-e_j+\sum_{k=1}^r E_j^{(k)}R_k,\eta\right)}{\prod_{j=N+1}^P\Gamma\left(e_j-\sum_{k=1}^r E_j^{(k)}R_k\right)} \tag{8}$$

$$\psi_k(R_k)=\frac{\prod_{j=1}^{m_k}\Gamma\left(b_j^{(k)}-B_j^{(k)}R_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-a_j^{(k)}+\sum_{k=1}^{r}A_j^{(k)}R_k\right)}{\prod_{j=m_{k+1}}^{q_k}\Gamma\left(1-b_j^{(k)}-B_j^{(k)}R_k\right)\prod_{j=n_{k+1}}^{p_k}\Gamma\left(a_j^{(k)}-\sum_{k=1}^{r}A_j^{(k)}R_k\right)} \tag{9}$$

Where (k=1, 2, …, r)

$$\begin{cases} e_j,[j=1,2,...,P];a_j^{(k)},[j=1,2,...,p_i;\forall i\in\{1,2,...,r\}] \\ f_j,[j=1,2,...,Q];b_j^{(k)},[j=1,2,...,q_i;\forall i\in\{1,2,...,r\}] \end{cases}$$

are complex numbers along with their associated coefficients. $\begin{cases} E_j^{(k)},[j=1,2,...,P];A_j^{(k)},[j=1,2,...,p_k;\forall k\in\{1,2,...,r\}] \\ F_j^{(k)},[j=1,2,...,Q];B_j^{(k)},[j=1,2,...,q_k;\forall k\in\{1,2,...,r\}] \end{cases}$ are positive real numbers, and $\Im_{k,}$ represent the contours are taken to begin at the point $\tau_k-i\infty$ and extend to the point $\tau_k+i\infty$ with $\tau_k\in R,(k=1,2,...,r)$.

The integral in (6) being absolutely convergent under the conditions specified by Srivastava et al. [5].

$$\text{If}\quad |\arg z_k|<\frac{\pi}{2}\nu_k,\ (k=1,2,...,r), \tag{10}$$

$$\text{where}\quad \Delta_k\equiv\sum_{j=1}^{P}E_j^{(k)}+\sum_{j=1}^{p_k}A_j^{(k)}-\sum_{j=1}^{Q}F_j^{(k)}-\sum_{j=1}^{q_k}B_j^{(k)}\le 0, \tag{11}$$

and

$$\nu_k=\sum_{j=1}^{N}E_j^{(k)}-\sum_{j=N+1}^{P}E_j^{(k)}+\sum_{j=1}^{M}F_j^{(k)}-\sum_{j=M+1}^{Q}F_j^{(k)}+\sum_{j=1}^{n_k}A_j^{(k)}-\sum_{j=n_k+1}^{p_k}A_j^{(k)}+\sum_{j=1}^{m_k}B_j^{(k)}-\sum_{j=m_k+1}^{q_k}B_j^{(k)}>0, \tag{12}$$

$\forall(k=1,2,...,r)$

Where M,N,P,Q,m_k,n_k,p_k,q_k are positive integers and limited by the $0\le N\le P$; $Q\ge M\ge 0$, and $q_k\ge m\ge 0$; $p_k\ge n_k\ge 0$, $\forall k\in\{1,2,...,r\}$ and the inequalities given in (12) restrict the allowable values of the complex variables $z_1,z_2,...,z_r$. The points $z_k=0,k=1,2,...,r$ and other exceptional cases are silently excluded. As discussed by Srivastava and Panda [6], we then obtain

$$\Gamma[z_1,z_2,...z_r\xi]=O\left(|z_1|^{s_1}...|z_r|^{s_r}\right)\left(\lim_{1\le j\le m_r}\|z_j\|\to 0\right), \tag{13}$$

where

$$s_k=\lim_{1\le j\le m_r}\mathrm{Re}\left(\frac{b_j^{(k)}}{B_j^{(k)}}\right),(k=1,2,...,r) \tag{14}$$

2. General Class Srivastava's Polynomials

As proposed by Srivastava (1985), the second class of multivariable polynomials is defined in the following way.

$$S_{h_1,h_2,...,h_t}^{g_1,g_2,...,g_t}[x_1,x_2,...,x_t]=\sum_{k_1=0}^{\frac{h_1}{g_1}}...\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{(-h_1)_{g_1k_1}}{k_1!}...\frac{(-h_t)_{g_tk_t}}{k_t!}\times A[h_1,k_1;...;h_t,k_t]x_1^{k_1}...x_t^{k_t} \tag{15}$$

Where h_i and $g_i\,\forall(i=1,2,...,t)$ are any integers greater than zero. The coefficients $A[h_1,k_1;...;h_t,k_t]$ are represent arbitrary constants, which may be real or complex.

By appropriately selecting values for the arbitrary coefficients $A[h_1,k_1;...;h_t,k_t],(k_t\ge 0)$, the general class of polynomials in (15) can be reduced to various well-known forms of multivariable polynomials.

3. Preliminaries

Based on the table of integrals, series, and products by I.S. Gradshteyn and I.M. Ryzhik [3], we require the following integration formulas.

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}dy=\frac{\sqrt{\pi}\,\Gamma\left(\delta+\frac{1}{2}\right)}{2u(4uv+w)^{\delta+\frac{1}{2}}\Gamma\left(\delta+\frac{1}{2}\right)} \tag{16}$$

$u>0;v>0;4uv+w>0$ and $\mathrm{Re}\left(\delta+\frac{1}{2}\right)>0$

$$\int_0^\infty\frac{1}{x^2}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}dy=\frac{\sqrt{\pi}\,\Gamma\left(\delta+\frac{1}{2}\right)}{2u(4uv+w)^{\delta+\frac{1}{2}}\Gamma\left(\delta+\frac{1}{2}\right)} \tag{17}$$

$u>0;v>0;4uv+w>0$ and $\mathrm{Re}\left(\delta+\frac{1}{2}\right)>0$

4. Main Result

This section presents integrals that combine Srivastava's polynomial with the incomplete H-function of several complex variables.

Theorem 1: If we set $u>0,v>0;(4uv+w)>0;\delta\ge 0$ and $\lambda_i>0,\rho_i>0(\forall i=1,2,...,r)$ then the following integration expressions are satisfied;

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}S_{h_1,h_2,...,h_t}^{g_1,g_2,...,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},...,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$
$$\Gamma_{P,Q;p_1,q_1;...;p_r,q_t}^{M,N;m_1,n_1;...;m_r,n_r}\left[\begin{matrix} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{matrix}\middle|\begin{matrix}\left(e_j,E_j^{(1)},E_j^{(2)},...,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},...,F_j^{(r)}\right)_{(1,q)};\end{matrix}\right.$$
$$\left.\begin{matrix}\left(e_j,E_j^{(1)},E_j^{(2)},...,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};...,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};...;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right]dy$$
$$=\frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}}S_{h_1,h_2,...,h_t}^{g_1,g_2,...,g_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},...,\frac{x_t}{(4uv+w)^{\lambda_t}}\right]\times$$
$$\Gamma_{P+1,Q+1;p_1,q_1;...;p_r,q_t}^{M,N+1;m_1,n_1;...;m_r,n_r}\left[\begin{matrix} z_1(4uv+w)^{-\rho_1} \\ \vdots \\ z_r(4uv+w)^{-\rho_r}\end{matrix}\middle|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_jk_j\right);\sum_{i=1}^{r}\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_jk_j\right);\sum_{i=1}^{r}\rho_i\right\};\end{matrix}\right.$$
$$\left.\begin{matrix}\left(e_j,E_j^{(1)},...,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},...,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};...,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},...,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};...;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right] \tag{18}$$

Provided that the conditions of incomplete H-function $\Gamma[z_1,z_2,...,z_r]$ in (4) are satisfied and the above integral will be convergence for condition (10), (11) and (12).

Proof: L.H.S. of equation (18)

$$\int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} S^{g_1,g_2,\ldots,g_t}_{h_1,h_2,\ldots,h_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$$\Gamma^{M,N;m_1,n_1;\ldots;m_r,n_r}_{P,Q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{array}\left|\begin{array}{l}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{array}\right]dy$$

By substituting the definitions of Srivastava's polynomial (15) and the incomplete H-function of several complex variables (4), we can write the following expression.

$$\Rightarrow \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{(-h_1)g_1k_1}{k_1!}\cdots\frac{(-h_t)g_tk_t}{k_t!}A[h_1,k_1;\ldots;h_t,k_t]\times$$

$$x_1^{k_1}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1k_1}\cdots x_t^{k_t}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_tk_t}\times$$

$$\frac{1}{(2\pi\omega)^r}\int_{\Im_1\ldots}\int_{\Im_r}\xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{i=1}^r\psi(R_i)z_i^{R_i}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_iR_i}dR_i\right\}dy$$

By changing the order of integration, we get

$$\Rightarrow\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{(-h_1)g_1k_1}{k_1!}\cdots\frac{(-h_t)g_tk_t}{k_t!}A[h_1,k_1;\ldots;h_t,k_t]\frac{1}{(2\pi\omega)^r}\times$$

$$\int_{\Im_1\ldots}\int_{\Im_r}\xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^r\psi(R_i)z_i^{R_i}dR_i\right\}\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1-\sum_{i=1}^r\rho_iR_i-\sum_{j=1}^t\lambda_jk_j}dy$$

$$\Rightarrow\frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\sum_{i=1}^r\rho_iR_i+\sum_{j=1}^t\lambda_jk_j+\frac{1}{2}}}\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{(-h_1)g_1k_1}{k_1!}\cdots\frac{(-h_t)g_tk_t}{k_t!}A[h_1,k_1;\ldots;h_t,k_t]\times$$

$$\frac{\Gamma\left(\delta+\sum_{i=1}^r\rho_iR_i+\sum_{j=1}^t\lambda_jk_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^r\rho_iR_i+\sum_{j=1}^t\lambda_jk_j+1\right)}\frac{1}{(2\pi\omega)^r}\int_{\Im_1\ldots}\int_{\Im_r}\xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^r\psi(R_i)z_i^{R_i}dR_i\right\}$$

$$\Rightarrow\frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}}\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{(-h_1)g_1k_1}{k_1!}\cdots\frac{(-h_t)g_tk_t}{k_t!}A[h_1,k_1;\ldots;h_t,k_t]\times$$

$$\left\{\frac{x_1}{(4uv+w)^{\delta_1}}\right\}^{k_1}\cdots\left\{\frac{x_1}{(4uv+w)^{\delta_t}}\right\}^{k_t}\frac{\Gamma\left(\delta+\sum_{i=1}^r\rho_iR_i+\sum_{j=1}^t\lambda_jk_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^r\rho_iR_i+\sum_{j=1}^t\lambda_jk_j+1\right)}\times$$

$$\frac{1}{(2\pi\omega)^r}\int_{\Im_1\ldots}\int_{\Im_r}\xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{i=1}^r\psi(R_i)\left\{z_i\left((4uv+w)^{-\delta_i}\right)\right\}z_i^{R_i}dR_i\right\}$$

From the interpretation of equations (4) and (15), we derive the desired result.

$$\Rightarrow\frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}}S^{g_1,g_2,\ldots,g_t}_{h_1,h_2,\ldots,h_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right]\times$$

$$\Gamma^{M,N+1;m_1,n_1;\ldots;m_r,n_r}_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c}z_1(4uv+w)^{-\rho_1}\\ \vdots \\ z_r(4uv+w)^{-\rho_r}\end{array}\left|\begin{array}{l}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^t\lambda_jk_j\right);\sum_{i=1}^r\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^t\lambda_jk_j\right);\sum_{i=1}^r\rho_i\right\}\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{array}\right]$$

It follows that theorem 1 is verified.

Where $-\delta>\sum_{j=1}^t\lambda_jk_j$

Theorem 2: If we set $u>0,v>0;(4uv+w)>0;\delta\geq 0$ and $\lambda_i>0,\rho_i>0\,(\forall i=1,2,\ldots,r)$ then the following integration expressions are satisfied;

$$\int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} S^{g_1,g_2,\ldots,g_t}_{h_1,h_2,\ldots,h_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$$\gamma^{M,N;m_1,n_1;\ldots;m_r,n_r}_{P,Q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{array}\left|\begin{array}{l}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{array}\right]dy$$

$$=\frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}}S^{g_1,g_2,\ldots,g_t}_{h_1,h_2,\ldots,h_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right]\times$$

$$\gamma^{M,N+1;m_1,n_1;\ldots;m_r,n_r}_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c}z_1(4uv+w)^{-\rho_1}\\ \vdots \\ z_r(4uv+w)^{-\rho_r}\end{array}\left|\begin{array}{l}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^t\lambda_jk_j\right);\sum_{i=1}^r\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^t\lambda_jk_j\right);\sum_{i=1}^r\rho_i\right\};\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{array}\right] \tag{19}$$

Provided that the conditions of incomplete H-function $\gamma[z_1,z_2,\ldots,z_r]$ in (7) are satisfied and the above integral will be convergence for condition (10), (11) and (12).

Proof: L.H.S. of equation (19)

$$\int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} S^{g_1,g_2,\ldots,g_t}_{h_1,h_2,\ldots,h_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$$\gamma^{M,N;m_1,n_1;\ldots;m_r,n_r}_{P,Q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{array}\left|\begin{array}{l}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{array}\right]dy$$

By substituting the definitions of Srivastava's polynomial (15) and the incomplete H-function of several complex variables (7), we can write the following expression

$$\Rightarrow \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$
$$x_1^{k_1}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1 k_1} \ldots x_t^{k_t}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t k_1} \times$$
$$\frac{1}{(2\pi\omega)^r}\int_{\mathfrak{I}_1}\ldots\int_{\mathfrak{I}_r} \xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{i=1}^r \psi(R_i) z_i^{R_i}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_i R_i} dR_i\right\} dy$$

By changing the order of integration, we get

$$\Rightarrow \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \frac{1}{(2\pi\omega)^r} \times$$
$$\int_{\mathfrak{I}_1}\ldots\int_{\mathfrak{I}_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^r \psi(R_i) z_i^{R_i} dR_i\right\} \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1-\sum_{i=1}^r \rho_i R_i - \sum_{j=1}^t \lambda_j k_j} dy$$

$$\Rightarrow \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\sum_{i=1}^r \rho_i R_i+\sum_{j=1}^t \lambda_j k_j+\frac{1}{2}}} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$
$$\frac{\Gamma\left(\delta+\sum_{i=1}^r \rho_i R_i+\sum_{j=1}^t \lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^r \rho_i R_i+\sum_{j=1}^t \lambda_j k_j+1\right)} \frac{1}{(2\pi\omega)^r}\int_{\mathfrak{I}_1}\ldots\int_{\mathfrak{I}_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^r \psi(R_i) z_i^{R_i} dR_i\right\}$$

$$\Rightarrow \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$
$$\left\{\frac{x_1}{(4uv+w)^{\delta_1}}\right\}^{k_1} \cdots \left\{\frac{x_1}{(4uv+w)^{\delta_t}}\right\}^{k_t} \times$$
$$\frac{\Gamma\left(\delta+\sum_{i=1}^r \rho_i R_i+\sum_{j=1}^t \lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^r \rho_i R_i+\sum_{j=1}^t \lambda_j k_j+1\right)} \frac{1}{(2\pi\omega)^r}\int_{\mathfrak{I}_1}\ldots\int_{\mathfrak{I}_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^r \psi(R_i) z_i^{R_i} dR_i\right\}$$

From the interpretation of equations (7) and (15), we derive the desired result.

$$\Rightarrow \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right] \times$$
$$\gamma_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}^{M,N+1;m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c} z_1(4uv+w)^{-\rho_1} \\ \vdots \\ z_r(4uv+w)^{-\rho_r} \end{array}\middle| \begin{array}{l} \left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^t \lambda_j k_j\right);\sum_{i=1}^r \rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^t \lambda_j k_j\right);\sum_{i=1}^r \rho_i\right\} \end{array}\right.$$
$$\left.\begin{array}{c} \left(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)} \end{array}\right]$$

It follows that theorem 2 is verified.

Where $-\delta > \sum_{j=1}^t \lambda_j k_j$

Theorem 3: If we set $u>0, v>0; (4uv+w)>0; \delta\geq 0$ and $\lambda_i>0, \rho_i>0 (\forall i=1,2,\ldots,r)$ then the following integration expressions are satisfied;

$$\int_0^\infty \frac{1}{y^2}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right] \times$$
$$\Gamma_{P,Q;p_1,q_1;\ldots;p_r,q_r}^{M,N;m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r} \end{array}\middle| \begin{array}{l} \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)}; \end{array}\right.$$
$$\left.\begin{array}{c} \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)} \end{array}\right] dy$$
$$= \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right] \times$$
$$\Gamma_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}^{M,N+1;m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c} z_1(4uv+w)^{-\rho_1} \\ \vdots \\ z_r(4uv+w)^{-\rho_r} \end{array}\middle| \begin{array}{l} \left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^t \lambda_j k_j\right);\sum_{i=1}^r \rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^t \lambda_j k_j\right);\sum_{i=1}^r \rho_i\right\}; \end{array}\right.$$
$$\left.\begin{array}{c} \left(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)} \end{array}\right] \tag{20}$$

Provided that the conditions of incomplete H-function $\Gamma[z_1,z_2,\ldots,z_r]$ in (4) are satisfied and the above integral will be convergence for condition (10), (11) and (12).

Proof: L.H.S. of equation (20)

$$\int_0^\infty \frac{1}{y^2}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} \times$$
$$S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right] \times$$
$$\Gamma_{P,Q;p_1,q_1;\ldots;p_r,q_r}^{M,N;m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r} \end{array}\middle| \begin{array}{l} \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta\right); \\ \left(f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)}\right)_{(1,q)}; \end{array}\right.$$
$$\left.\begin{array}{c} \left(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\ldots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)} \\ \left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)} \end{array}\right] dy$$

By substituting the definitions of Srivastava's polynomial (15) and the incomplete H-function of several complex variables (4), we can write the following expression

$$\Rightarrow \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$
$$x_1^{k_1}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1 k_1} \ldots x_t^{k_t}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t k_1} \times$$
$$\frac{1}{(2\pi\omega)^r}\int_{\mathfrak{I}_1}\ldots\int_{\mathfrak{I}_r} \xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{i=1}^r \psi(R_i) z_i^{R_i}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_i R_i} dR_i\right\} dy$$

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By changing the order of integration, we get

$$\Rightarrow \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \frac{1}{(2\pi\omega)^r} \times$$

$$\int_{\Im_1} \cdots \int_{\Im_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^{r}\psi(R_i) z_i^{R_i} dR_i\right\} \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1-\sum_{i=1}^{r}\rho_i R_i-\sum_{j=1}^{t}\lambda_j k_j} dy$$

$$\Rightarrow \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}}} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$

$$\frac{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+1\right)} \frac{1}{(2\pi\omega)^r} \int_{\Im_1} \cdots \int_{\Im_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^{r}\psi(R_i) z_i^{R_i} dR_i\right\}$$

$$\Rightarrow \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}}$$

$$\left\{\frac{x_1}{(4uv+w)^{\delta_1}}\right\}^{k_1} \cdots \left\{\frac{x_1}{(4uv+w)^{\delta_t}}\right\}^{k_t} \times$$

$$\frac{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+1\right)} \frac{1}{(2\pi\omega)^r} \int_{\Im_1} \cdots \int_{\Im_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^{r}\psi(R_i) z_i^{R_i} dR_i\right\}$$

From the interpretation of equations (4) and (15), we derive the desired result.

$$\Rightarrow \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right] \times$$

$$\Gamma_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}^{M,N+1;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1(4uv+w)^{-\rho_1} \\ \vdots \\ z_r(4uv+w)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta);(e_j,E_j^{(1)},\ldots,E_j^{(r)})_{(2,p)};(a_j^{(1)},A_j^{(1)})_{(1,p_1)};\ldots;(a_j^{(r)},A_j^{(r)})_{(1,p_r)} \\ (f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)})_{(1,q)};(b_j^{(1)},B_j^{(1)})_{(1,q_1)};\ldots;(b_j^{(r)},B_j^{(r)})_{(1,q_r)}\end{matrix}\right]$$

It follows that theorem 3 is verified.

Where $-\delta > \sum_{j=1}^{t}\lambda_j k_j$

Theorem 4: If we set $u>0, v>0; (4uv+w)>0; \delta \geq 0$ and $\lambda_i > 0, \rho_i > 0 (\forall i = 1,2,\ldots,r)$ then the following integration expressions are satisfied;

$$\int_0^\infty \frac{1}{y^2}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right] \times$$

$$\gamma_{P,Q;p_1,q_1;\ldots;p_r,q_r}^{M,N;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{matrix}\left|\begin{matrix}(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta); \\ (f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)})_{(1,q)};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)})_{(2,p)};(a_j^{(1)},A_j^{(1)})_{(1,p_1)};\ldots;(a_j^{(r)},A_j^{(r)})_{(1,p_r)} \\ (b_j^{(1)},B_j^{(1)})_{(1,q_1)};\ldots;(b_j^{(r)},B_j^{(r)})_{(1,q_r)}\end{matrix}\right] dy$$

$$= \frac{\sqrt{\pi}}{2u(4uv+w)^{\delta+\frac{1}{2}}} S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[\frac{x_1}{(4uv+w)^{\lambda_1}},\frac{x_2}{(4uv+w)^{\lambda_2}},\ldots,\frac{x_t}{(4uv+w)^{\lambda_t}}\right] \times \gamma_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_r}^{M,N+1;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1(4uv+w)^{-\rho_1} \\ \vdots \\ z_r(4uv+w)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(e_j,E_j^{(1)},\ldots,E_j^{(r)},\eta);(e_j,E_j^{(1)},\ldots,E_j^{(r)})_{(2,p)};(a_j^{(1)},A_j^{(1)})_{(1,p_1)};\ldots;(a_j^{(r)},A_j^{(r)})_{(1,p_r)} \\ (f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)})_{(1,q)};(b_j^{(1)},B_j^{(1)})_{(1,q_1)};\ldots;(b_j^{(r)},B_j^{(r)})_{(1,q_r)}\end{matrix}\right] \tag{21}$$

Provided that the conditions of incomplete H-function $\gamma[z_1,z_2,\ldots,z_r]$ in (7) are satisfied and the above integral will be convergence for condition (10), (11) and (12).

Proof: L.H.S. of equation (21)

$$\int_0^\infty \frac{1}{y^2}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} \times$$

$$S_{h_1,h_2,\ldots,h_t}^{g_1,g_2,\ldots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\ldots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]$$

$$\gamma_{P,Q;p_1,q_1;\ldots;p_r,q_r}^{M,N;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1} \\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{matrix}\left|\begin{matrix}(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)},\eta); \\ (f_j,F_j^{(1)},F_j^{(2)},\ldots,F_j^{(r)})_{(1,q)};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(e_j,E_j^{(1)},E_j^{(2)},\ldots,E_j^{(r)})_{(2,p)};(a_j^{(1)},A_j^{(1)})_{(1,p_1)};\ldots;(a_j^{(r)},A_j^{(r)})_{(1,p_r)} \\ (b_j^{(1)},B_j^{(1)})_{(1,q_1)};\ldots;(b_j^{(r)},B_j^{(r)})_{(1,q_r)}\end{matrix}\right] dy$$

By substituting the definitions of Srivastava's polynomial (15) and the incomplete H-function of several complex variables (7), we can write the following expression

$$\Rightarrow \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1} \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \times$$

$$x_1^{k_1}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1 k_1} \cdots x_t^{k_t}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t k_1}$$

$$\frac{1}{(2\pi\omega)^r} \int_{\Im_1} \cdots \int_{\Im_r} \xi(R_1,R_2,\ldots,R_r,\eta)\left\{\prod_{i=1}^{r}\psi(R_i) z_i^{R_i}\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_i R_i} dR_i\right\} dy$$

By changing the order of integration, we get

$$\Rightarrow \sum_{k_1=0}^{\frac{h_1}{g_1}} \cdots \sum_{k_t=0}^{\frac{h_t}{g_t}} \frac{(-h_1)_{g_1 k_1}}{k_1!} \cdots \frac{(-h_t)_{g_t k_t}}{k_t!} A[h_1,k_1;\ldots;h_t,k_t] \frac{1}{(2\pi\omega)^r} \times$$

$$\int_{\Im_1} \cdots \int_{\Im_r} \xi(R_1,\ldots,R_r,\eta)\left\{\prod_{i=1}^{r}\psi(R_i) z_i^{R_i} dR_i\right\} \int_0^\infty \left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1-\sum_{i=1}^{r}\rho_i R_i-\sum_{j=1}^{t}\lambda_j k_j} dy$$

$$\Rightarrow \frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}}}\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{\left(-h_1\right)_{g_1k_1}}{k_1!}\cdots\frac{\left(-h_t\right)_{g_tk_t}}{k_t!}A[h_1,k_1;\dots;h_t,k_t]\times$$

$$\frac{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+1\right)}\frac{1}{\left(2\pi\omega\right)^r}\int_{\mathfrak{I}_1}\cdots\int_{\mathfrak{I}_r}\xi\left(R_1,\dots,R_r,\eta\right)\left\{\prod_{i=1}^{r}\psi\left(R_i\right)z_i^{R_i}dR_i\right\}$$

$$\Rightarrow \frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}\sum_{k_1=0}^{\frac{h_1}{g_1}}\cdots\sum_{k_t=0}^{\frac{h_t}{g_t}}\frac{\left(-h_1\right)_{g_1k_1}}{k_1!}\cdots\frac{\left(-h_t\right)_{g_tk_t}}{k_t!}A[h_1,k_1;\dots;h_t,k_t]\times$$

$$\left\{\frac{x_1}{\left(4uv+w\right)^{\delta_1}}\right\}^{k_1}\cdots\left\{\frac{x_1}{\left(4uv+w\right)^{\delta_t}}\right\}^{k_t}\times$$

$$\frac{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+\frac{1}{2}\right)}{\Gamma\left(\delta+\sum_{i=1}^{r}\rho_i R_i+\sum_{j=1}^{t}\lambda_j k_j+1\right)}\frac{1}{\left(2\pi\omega\right)^r}\int_{\mathfrak{I}_1}\cdots\int_{\mathfrak{I}_r}\xi\left(R_1,\dots,R_r,\eta\right)\left\{\prod_{i=1}^{r}\psi\left(R_i\right)z_i^{R_i}dR_i\right\}$$

From the interpretation of equations (7) and (15), we derive the desired result.

$$\Rightarrow \frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[\frac{x_1}{\left(4uv+w\right)^{\lambda_1}},\frac{x_2}{\left(4uv+w\right)^{\lambda_2}},\dots,\frac{x_t}{\left(4uv+w\right)^{\lambda_t}}\right]\times$$

$$\gamma_{P+1,Q+1;p_1,q_1;\dots;p_r,q_t}^{M,N+1;m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\left(4uv+w\right)^{-\rho_1}\\ \vdots \\ z_r\left(4uv+w\right)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(e_j,E_j^{(1)},\dots,E_j^{(r)},\eta\right);\left(e_j,E_j^{(1)},\dots,E_j^{(r)}\right)_{(2,p)};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\dots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)}\\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(f_j,F_j^{(1)},F_j^{(2)},\dots,F_j^{(r)}\right)_{(1,q)};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\dots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

It follows that theorem 4 is verified.

Where $-\delta>\sum_{j=1}^{t}\lambda_j k_j$

5. Special Cases:

Corollary 1:

On substituting M=N=P=Q=0 in equation (18) and (19) the following new result is obtained

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\dots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$$\prod_{i=1}^{r}\Gamma_{p_i,q_i}^{m_i,n_i}\left[z_i\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_i}\left|\begin{matrix}\left(a_j^{(i)},A_j^{(i)}\right)_{(1,p_i)}\\ \left(b_j^{(i)},B_j^{(i)}\right)_{(1,q_i)}\end{matrix}\right.\right]dy=\frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}$$

$$S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[\frac{x_1}{\left(4uv+w\right)^{\lambda_1}},\frac{x_2}{\left(4uv+w\right)^{\lambda_2}},\dots,\frac{x_t}{\left(4uv+w\right)^{\lambda_t}}\right]\times\Gamma_{1,1;p_1,q_1;\dots;p_r,q_t}^{0,1;m_1,n_1;\dots;m_r,n_r}$$

$$\left[\begin{matrix} z_1\left(4uv+w\right)^{-\rho_1}\\ \vdots \\ z_r\left(4uv+w\right)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\dots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)}\\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\dots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

(22)

and

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\dots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$$\prod_{i=1}^{r}\gamma_{p_i,q_i}^{m_i,n_i}\left[z_i\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_i}\left|\begin{matrix}\left(a_j^{(i)},A_j^{(i)}\right)_{(1,p_i)}\\ \left(b_j^{(i)},B_j^{(i)}\right)_{(1,q_i)}\end{matrix}\right.\right]dy=\frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}$$

$$S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[\frac{x_1}{\left(4uv+w\right)^{\lambda_1}},\frac{x_2}{\left(4uv+w\right)^{\lambda_2}},\dots,\frac{x_t}{\left(4uv+w\right)^{\lambda_t}}\right]\times\gamma_{1,1;p_1,q_1;\dots;p_r,q_t}^{0,1;m_1,n_1;\dots;m_r,n_r}$$

$$\left[\begin{matrix} z_1\left(4uv+w\right)^{-\rho_1}\\ \vdots \\ z_r\left(4uv+w\right)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(a_j^{(1)},A_j^{(1)}\right)_{(1,p_1)};\dots;\left(a_j^{(r)},A_j^{(r)}\right)_{(1,p_r)}\\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(b_j^{(1)},B_j^{(1)}\right)_{(1,q_1)};\dots;\left(b_j^{(r)},B_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

(23)

Similarly, from the equation (20) and (21), we can get new results.

Corollary 2:

On substituting $E_j^{(i)}=F_j^{(i)}=A_j^{(i)}=B_j^{(i)};(\forall i=1,2,\dots,r)$ in equation (18) and (19) the Incomplete H-function reduce to Incomplete Meijer G-function and we get

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\dots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$${}^{(\Gamma)}G_{P,Q;p_1,q_1;\dots;p_r,q_t}^{M,N;m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1}\\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{matrix}\left|\begin{matrix}\left(e_j,\eta\right)_{(1,p)};\left(a_j^{(1)}\right)_{(1,p_1)},\dots,\left(a_j^{(r)}\right)_{(1,p_r)}\\ \left(f_j\right)_{(1,Q)};\left(b_j^{(1)}\right)_{(1,q_1)},\dots,\left(b_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]dy$$

$$=\frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[\frac{x_1}{\left(4uv+w\right)^{\lambda_1}},\frac{x_2}{\left(4uv+w\right)^{\lambda_2}},\dots,\frac{x_t}{\left(4uv+w\right)^{\lambda_t}}\right]\times$$

$${}^{(\Gamma)}G_{P+1,Q+1;p_1,q_1;\dots;p_r,q_t}^{M,N+1;m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\left(4uv+w\right)^{-\rho_1}\\ \vdots \\ z_r\left(4uv+w\right)^{-\rho_r}\end{matrix}\left|\begin{matrix}\left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(e_j,\eta\right)_{(1,p)};\left(a_j^{(1)}\right)_{(1,p_1)};\dots;\left(a_j^{(r)}\right)_{(1,p_r)}\\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\};\left(f_j\right)_{(1,Q)};\left(b_j^{(1)}\right)_{(1,q_1)};\dots;\left(b_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]$$

and

$$\int_0^\infty\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\delta-1}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[x_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_1},\dots,x_t\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\lambda_t}\right]\times$$

$${}^{(\gamma)}G_{P,Q;p_1,q_1;\dots;p_r,q_t}^{M,N;m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_1}\\ \vdots \\ z_r\left\{\left(uy+\frac{v}{y}\right)^2+w\right\}^{-\rho_r}\end{matrix}\left|\begin{matrix}\left(e_j,\eta\right)_{(1,p)};\left(a_j^{(1)}\right)_{(1,p_1)};\dots;\left(a_j^{(r)}\right)_{(1,p_r)}\\ \left(f_j\right)_{(1,Q)};\left(b_j^{(1)}\right)_{(1,q_1)};\dots;\left(b_j^{(r)}\right)_{(1,q_r)}\end{matrix}\right.\right]dy$$

$$=\frac{\sqrt{\pi}}{2u\left(4uv+w\right)^{\delta+\frac{1}{2}}}S_{h_1,h_2,\dots,h_t}^{g_1,g_2,\dots,g_t}\left[\frac{x_1}{\left(4uv+w\right)^{\lambda_1}},\frac{x_2}{\left(4uv+w\right)^{\lambda_2}},\dots,\frac{x_t}{\left(4uv+w\right)^{\lambda_t}}\right]\times$$

$$
{}^{(\gamma)}G^{M,N+1;m_1,n_1;\ldots;m_r,n_r}_{P+1,Q+1;p_1,q_1;\ldots;p_r,q_t}\left[\begin{matrix} z_1\left(4uv+w\right)^{-\rho_1} \\ \vdots \\ z_r\left(4uv+w\right)^{-\rho_r}\end{matrix}\middle| \begin{matrix} \left\{\left(\frac{1}{2}-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\}; \\ \left\{\left(-\delta-\sum_{j=1}^{t}\lambda_j k_j\right);\sum_{i=1}^{r}\rho_i\right\}; \end{matrix}\right. \tag{24}
$$

$$
\left.\begin{matrix} \left(e_j,\eta\right)_{(1,p)};\left(a_j^{(1)}\right)_{(1,p_1)};\ldots,\left(a_j^{(r)}\right)_{(1,p_r)} \\ \left(f_j\right)_{(1,Q)};\left(b_j^{(1)}\right)_{(1,q_1)};\ldots;\left(b_j^{(r)}\right)_{(1,q_r)} \end{matrix}\right]
$$

Similarly, from the equation (20) and (21), we can get new results.

6. Conclusion

The incomplete H-function emerges as a fundamental structure within the theory of special functions. By varying its parameters, one can recover many well-known functions such as the incomplete Meijer's G-function, incomplete Fox–Wright functions, and incomplete generalized hypergeometric functions. Its adaptability further allows the formulation of unified integral relations, highlighting its significance in advancing mathematical investigations.

References

[1] Ashiq Hussain Khan, Neelam Pandey, Integrals Involving H-function of Several Complex Variables, International Journal of Scientific and Research Publications, Volume 7, Issue 2, February 2017.

[2] Bateman Project, (1953) Higher Transcendental Function, Vol. I.

[3] Gradshteyn, I.S. and Ryzhik, I.M., (2007), "Table of Integrals, Series and Products", Edited by A. Jeffrey, Daniel Zwillinger, Academic Press in an imprint of Elsevier,

[4] Mathai, A. M. and Saxena, R. K. "The H-function with Application in Statistics and Other Disciplines", Wiley Eastern Limited, New Delhi, Bangalore, Bombay, (1978).

[5] Srivastava, H. M. and Panda, R., "Some expansion theorems and generating relations for the H -function of several complex variables. I and II, Comment. Math. Univ. St. Paul. 24, fasc. 2, 119-137 (1975); ibid. 25, fasc. 2, 167-197.

[6] Panda, R. & Srivastava, H. (1976). Some bilateral generating functions for a class of generalized hypergeometric polynomials. Journal für die Reine und angewandte Mathematic, 976 (283-284), 265-274. https://doi.org/10.1515/crll.1976.283-284.265

[7] Rainville, E. D., (1971), "Special Functions", Chelsea Publication Company, Bronx, New York. [8] Srivastava, H. M., K.C. Gupta and S. P. Goyal, (1982), "The H-Functions of One and Two Variables with Applications", South Asian Publishers, New Delhi, Madras. https://doi.org/10.1002/asna.2113060427.

[8] Srivastava, H.M., (1972), "A contour integral involving fox's H-function", Indian J. Math.

[9] M.K. Bansal, D. Kumar, I. Khan, J. Singh and K. S. Nisar, Certain Unified Integrals Associated with Product of *M*-Series and Incomplete *H*-functions, *Mathematics*, 7(12), 1191, (2019).

[10] Bansal, M.K.; Choi, J. A Note on Pathway Fractional Integral Formulas Associated with the Incomplete *H*-functions. *Int. J. Appl. Comput. Math.* **2019**, *5*, 133. [CrossRef]

[11] Bansal, M.K.; Jolly, N.; Jain, R.; Kumar, D. An integral operator involving generalized Mittag-Leffler function and associated fractional calculus results. *J. Anal.* **2019**, *27*, 727–740. [CrossRef]

[12] Chaurasia, V.B.L.; Kumar, D. The integration of certain product involving special functions. *Sci. Ser. A Math. Sci.* **2010**, *19*, 7–12.

[13] Srivastava, H.M.; Chaudhary, M.A.; Agarwal, R.P. The Incomplete Pochhammer Symbols and Their Applications to Hypergeometric and Related Functions. *Integral Transform. Spec. Funct.* **2012**, *23*, 659–683. [CrossRef]

[14] Srivastava, H.M.; Saxena, R.K.; Parmar, R.K. Some Families of the Incomplete *H*-functions and the Incomplete *H*−functions and Associated Integral Transforms and Operators of Fractional Calculus with Applications. *Russ. J. Math. Phys.* **2018**, *25*, 116–138. [CrossRef]

[15] Kilab's, A.A.; Srivastava, H.M.; Trujillo, J.J. *Theory and Applications of Fractional Differential Equations*; (North-Holland Mathematical Studies); Elsevier (North Holland) Science Publishers: Amsterdam, The Netherland; London, UK; New York, NX, USA, 2006; p. 204.

[16] Mathai, A.M.; Saxena, R.K. *The H-Function with Applications in Statistics Other Disciplines*; Wilex Eastern: New Delhi, India; Wilex Halsted: New York, NX, USA, 1978.

[17] Mathai, A.M.; Saxena, R.K.; Haubold, H.J. *The H-function: Theory and Applications*; Springer: New York, NX, USA, 2009.

[18] Srivastava, H.M.; Gupta, K.C.; Goxal, S.P. *The H-Functions of One and Two Variables with Applications*; South Asian Publishers: New Delhi, India, 1982