Analysis of the Vibrations of a Damped Visco-Elastic Right Triangle Plate Whose Thickness Varies Linearly in One Direction

Manu Gupta[1], Prachi[2]

[1]Professor, Department of Mathematics J.V. Jain College Saharanpur (UP) 247001, India

[2]Research Scholar, Department of Mathematics J.V. Jain College Saharanpur (UP) 247001, India

Abstract: ***A lot of work in vibration analysis involves both fundamental and particular analysis of plates of various forms. Vibration analysis is employed in many industries, including the nuclear energy, construction, automotive, petroleum, and sports sectors. Numerous characteristics, such as non-uniformity, variable thickness, viscoelastic effect, elastic basis, etc., are used in plate analysis. The vibration analysis of a right triangular plate with linearly variable thickness will be covered in this paper. taking into account the damping parameter and the visco-elastic effect. CSS and thickness variations in the x-direction are the problem's boundary conditions. The separation of variables and the Rayleigh Ritz technique have been used to solve the governing differential equation. Using the Gram Schmidt Orthogonalization technique, a two-term deflection function is produced. The logarithmic decrement, time period, frequency parameter, and deflection at various locations of the first two modes of vibration are calculated for a range of values of taper constants, damping parameters, and aspect ratio.***

Keywords: Vibration, Rayleigh-Ritz technique, logarithmic decrement, orthogonal functions, Edge conditions

1. Introduction

Numerous studies have been conducted on the vibrations of rectangular plates with varying thicknesses [1-3], but none have been conducted on visco-elastic plates. Sobotka [4] has examined the free vibrations of uniform visco-elastic orthotropic rectangular plates. Bhatnagar and Gupta [5-6] conducted a study on the impact of thermal gradient on vibration of visco-elastic circular and elliptic plates of varying thickness. An excellent monograph on triangular plates was published in 1969 by Leissa [7] and also flexural vibration studies on different shapes and configurations of plates are well documented. Research into triangular plate vibration is quite limited However, the triangular configuration of panels is common in many industries, so this area needs more research. One of the disadvantages of analyzing these problems is the difficulty of creating changes in two related variables to explain the triangle. Triangular plates with different configurations usually involve a lot of work and structural analysis, so more research is needed in this area.

The concept of free vibration in a structure, plate or system is hypothetical because damping is always present in all forms. The literature review also shows that problem of damping decisions has not been studied much. This research, which incorporates damping into plate vibration studies, will benefit fields such as seismology, Nuclear structure design and the design of dam and bridge. The main problem in analyzing a triangular plate is the amount of work involved in implicitly constructing the transformation function to describe the triangle. As technology advances, researchers use different methods to study triangles. F.E.M is a particular example of the approach by [8,10] Gorman [11,12] proposed an analysis method to determine the location of the building structure in a triangle without vibration. He also worked on right angled triangles for vibration analysis under different boundary conditions. The main purpose of this study is to investigate the effect of constant x direction taper on the vibration of visco elastic right angled triangular plate with CSS boundary conditions of the edges. Assumptions of small deflections and linear, isotropic viscoelastic properties are made. The Viscoelasticity of the plate is assumed to be Kelvin type. Figures are made of norma metal "Duralium" material. The taper constant and damping parameter values for different aspect ratios, deviation of the first two vibration modes of time difference, frequency, time period and logarithmic decrement are calculated and the results are presented in tabular and graphical form.

2. Mathematical Model for the Problem

We obtain mathematical model for equation of isotropic right triangular plate with variable thickness by introducing K as the damping parameter and assume that the damping forces are proportional to velocity, then model equation given by Leissa [7] is transformed and equation of motion is given as

$$\widehat{D}[\tilde{d}(\frac{\partial^4\widetilde{w}}{\partial x^4}+2\frac{\partial^4\widetilde{w}}{\partial x^2\partial y^2}+\frac{\partial^4\widetilde{w}}{\partial y^4})+2\frac{\partial\tilde{d}}{\partial x}(\frac{\partial^3\widetilde{w}}{\partial x^3}+\frac{\partial^3\widetilde{w}}{\partial x\partial y^2})+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3\widetilde{w}}{\partial y^3}+\frac{\partial^3\widetilde{w}}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2\widetilde{w}}{\partial x^2}+\upsilon\frac{\partial^2\widetilde{w}}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\widetilde{w}}{\partial y^2}+\upsilon\frac{\partial^2\widetilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\widetilde{w}}{\partial x\partial y}]+K\frac{\partial\widetilde{w}}{\partial t}+\rho h\frac{\partial^2\widetilde{w}}{\partial t^2}=0 \quad (1)$$

For solution of equation (1) the following form, which is taken in the form of products of two functions is assumed as

$$\widetilde{w}(x,y,t)=\widetilde{W}(x,y)\widetilde{T}(t) \quad (2)$$

Substituting the value from (2) in (1) we get the following transformed equations i.e. (3) and (4)

$$\frac{\partial^2\widetilde{T}}{\partial t^2}+\frac{K}{\rho h}\frac{\partial\widetilde{T}}{\partial t}+\mathcal{P}^2\widehat{D}\widetilde{T}=0 \quad (3)$$

and $\tilde{d}(\frac{\partial^4 \tilde{w}}{\partial x^4}+2\frac{\partial^4 \tilde{w}}{\partial x^2 \partial y^2}+\frac{\partial^4 \tilde{w}}{\partial y^4})+2\frac{\partial \tilde{d}}{\partial x}(\frac{\partial^3 \tilde{w}}{\partial x^3}+\frac{\partial^3 \tilde{w}}{\partial x \partial y^2})+2\frac{\partial \tilde{d}}{\partial y}\left(\frac{\partial^3 \tilde{w}}{\partial y^3}+\frac{\partial^3 \tilde{w}}{\partial y \partial x^2}\right)+\frac{\partial^2 \tilde{d}}{\partial x^2}\left(\frac{\partial^2 \tilde{w}}{\partial x^2}+\upsilon\frac{\partial^2 \tilde{w}}{\partial y^2}\right)+\frac{\partial^2 \tilde{d}}{\partial y^2}\left(\frac{\partial^2 \tilde{w}}{\partial y^2}+\upsilon\frac{\partial^2 \tilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 \tilde{d}}{\partial x \partial y}\frac{\partial^2 \tilde{w}}{\partial x \partial y}-\rho h \mathcal{P}^2 \tilde{w}=0$ (4)

Thus Equation (3) and (4) are the required differential equation of motion for the plate and time function of free and damped vibration having variable thickness respectively. We shall now separately solve above two equations.

Time function of variable of plates

Time functions of free vibrations of visco-elastic plates are defined by the general ordinary differential Equation (3). Their form depends on visco-elastic operator $\widehat{D}$.For Kelvin's model, one can have $\widehat{D} \equiv \{1+(\frac{\eta}{G})(\frac{d}{dt})\}$ (5) Using equation(5) in equation (3) ,one obtains $\ddot{\tilde{T}}+(\frac{K}{\rho h}+\frac{\mathcal{P}^2\eta}{G})\dot{\tilde{T}}+\mathcal{P}^2\tilde{T}=0$ (6)

Equation (6) is a differential eqation of order two for time function T. The Solution of equation (6) is established as $\tilde{T}(t)=e^{\alpha t}\{\tilde{c_1}cos\beta t+\tilde{c_2}sin\beta t\}$ (7)

Where $\alpha=-(\frac{K}{2\rho h}+\frac{\mathcal{P}^2\eta}{2G})$ and $\beta=\mathcal{P}\sqrt{1-\frac{\alpha^2}{\mathcal{P}^2}}=\mathcal{P}_0$, here $\mathcal{P}$ is natural frequency, $\mathcal{P}_0$ is angular frequency and $\tilde{c_1}$, $\tilde{c_2}$ are arbitrary but fixed which are calculated from the primary restriction of the plate. For the present study initial conditions as $\tilde{T}=1$ and $\dot{\tilde{T}}=0$ at t=0 (8)

Using equation (8) in equation (7), one obtains $\tilde{c_1}$=1 and $\tilde{c_2}$= $-\frac{\alpha}{\beta}$ (9)

Using equation (9) in equation (7), one have $\tilde{T}(t)=e^{\alpha t}\{cos\beta t-\frac{\alpha}{\beta}sin\beta t\}$ (10)

For damped transverse vibration of the triangular plate amplitude ($\tilde{w}$) can be expressed as
$\tilde{w}(x,y,t)=\tilde{W}(x,y)[e^{\alpha t}\left\{cos\beta t-\frac{\alpha}{\beta}sin\beta t\right\}]$ (11)

Substituting the above value from (10) in (4) and equating the coefficient sine and cosine term we establish the equation for deflection function as
$\tilde{d}(\frac{\partial^4 \tilde{W}}{\partial x^4}+2\frac{\partial^4 \tilde{W}}{\partial x^2 \partial y^2}+\frac{\partial^4 \tilde{W}}{\partial y^4})+2\frac{\partial \tilde{d}}{\partial x}(\frac{\partial^3 \tilde{W}}{\partial x^3}+\frac{\partial^3 \tilde{W}}{\partial x \partial y^2})+2\frac{\partial \tilde{d}}{\partial y}\left(\frac{\partial^3 \tilde{W}}{\partial y^3}+\frac{\partial^3 \tilde{W}}{\partial y \partial x^2}\right)+\frac{\partial^2 \tilde{d}}{\partial x^2}\left(\frac{\partial^2 \tilde{W}}{\partial x^2}+\upsilon\frac{\partial^2 \tilde{W}}{\partial y^2}\right)+\frac{\partial^2 \tilde{d}}{\partial y^2}\left(\frac{\partial^2 \tilde{W}}{\partial y^2}+\upsilon\frac{\partial^2 \tilde{W}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 \tilde{d}}{\partial x \partial y}\frac{\partial^2 \tilde{W}}{\partial x \partial y}-\frac{k^2\tilde{W}}{4\rho h}-\rho h P^2 \tilde{W}=0$ (12)

Assuming that the thickness variation of the plate in x direction, the flexural rigidity of the plate $\tilde{d}$ is written as (assuming possion's ratio υ is constant)
$\tilde{d}=\frac{E{h_0}^3}{12(1-\upsilon^2)}(1+\beta^*\frac{x}{a})^3$. , $\tilde{d}=\tilde{d}_0(1+\beta^*\frac{x}{a})^3$ Where $\tilde{d}_0=\frac{E{h_0}^3}{12(1-\upsilon^2)}$ (13)

substituting (13) in (12) and introducing non dimensional variables X'=x/a and Y'=y/b and simplifying one gets
$a^4(1+\beta^*X')^4\left(\frac{\partial^4\tilde{W}}{\partial X'^4}+2\frac{\partial^4\tilde{W}}{\partial X'^2\partial Y'^2}+\frac{\partial^4\tilde{W}}{\partial Y'^4}\right)+2a^4(1+\beta^*X')\frac{\partial}{\partial X'}(1+\beta^*X')^3\left(\frac{\partial^3\tilde{W}}{\partial X'^3}+\frac{\partial^3\tilde{W}}{\partial X'\partial Y'^2}\right)+2a^4(1+\beta^*X')\frac{\partial(1+\beta^*X')^3}{\partial Y'}\left(\frac{\partial^3\tilde{W}}{\partial Y'^3}+\frac{\partial^3\tilde{W}}{\partial Y'\partial X'^2}\right)+a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial X'^2}\left(\frac{\partial^2\tilde{W}}{\partial X'^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial Y'^2}\right)+a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial Y'^2}\left(\frac{\partial^2\tilde{W}}{\partial Y'^2}+\upsilon\frac{\partial^2\tilde{W}}{\partial X'^2}\right)+2(1-\upsilon)a^4(1+\beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial X'\partial Y'}\frac{\partial^2\tilde{W}}{\partial X'\partial Y'}-\frac{k^2a^4\tilde{W}}{4\rho h_0\,\tilde{d}_0}-\frac{\rho h_0 a^4P^2}{\tilde{d}_0}(1+\beta^*X')^2\tilde{W}=0$ (14) Replacing $\frac{4\rho h_0\,\tilde{d}_0}{a^4}$ by $\frac{{K_0}^2}{\mu^2}$ and $\frac{\rho h_0 a^4P^2}{\tilde{d}_0}$ by Λ^2.We shall now find the deflection function $\tilde{W}$ using orthogonal plate function.

Orthogonal Plate Function

The deflection function for a vibrating triangular plate may be defined by a set of two dimensional orthogonal plate function $\tilde{W}(X',Y')=A_1\Phi_1+A_2\Phi_2$ (15)
where Φ_1 and Φ_2 are orthogonal plate function $\Phi_1(X',Y')$ is so chosen for triangular plate in our study such that it at least satisfy the geometrical boundary conditions of the plate and a better approximation and convergence is achieved if $\Phi_1(X',Y')$ it also satisfy the natural boundary conditions.

For present problem the required function in equation (15) is $\Phi_i(X',Y')=\prod_{k=1}^{3}\Theta_k(X',Y')$ (16)

The Π denotes the product of terms, Θ_k are the edge functions which can be easily formulated if the edge support condition of the plate is known. The function $\Theta_k(X',Y')$ for different support conditions are summarized below

(i)For simply supported edge $\Theta(X',Y')=\begin{cases} X'-c\text{ , at edge } X'=c \\ Y'-d\text{ , at edge } Y'=d \\ Y'-tX'-e\text{ , at edge } Y'=tX'+e \end{cases}$ (17)

(ii) for clamped edge $\Theta(X',Y')=\begin{cases} (X'-c)^2\text{ , at edge } X'=c \\ (Y'-d)^2\text{ , at edge } Y'=d \\ (Y'-tX'-e)^2\text{ , at edge } Y'=tX'+e \end{cases}$ (18)

For $\Phi_2(X',Y')=f_2(X',Y')\Phi_1(X',Y')-a_{2,1}\Phi_1(X',Y')$, $\iint\Phi_i(X',Y')\Phi_j(X',Y')dX'dY'=\begin{cases}0\text{ , } if\ i\neq j\\1\text{ , } if\ i=j\end{cases}$ (By gram Schmidt orthogonalization definition)

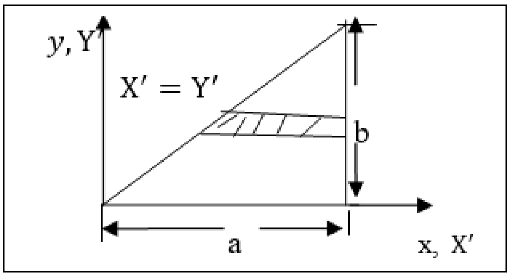
Where $a_{mi}=\frac{\iint f_m(X',Y')\Phi_1(X',Y')\Phi_i(X',Y')dX'dY'}{\iint\Phi_i(X',Y')\Phi_i(X',Y')dX'dY'}$ (19)

Where $f_m(X',Y')$ is the generating function , r=[$\sqrt{m-1}$] and t = (m-1)-r^2 If t is even, thens=t/2;0≤ $s\leq r$ => f_m

$(X', Y')=X'^{r}$ If t is odd, then s= (t-1)/2; $0 \le s \le r-1$ => f_m $(X', Y')=X'^{r}Y'^{s}$ Where [ ] is denotes the greatest integer function and f_m (X', Y') is calculated by deciding the parameters

Boundary condition	Generating functions	Plate functions
CSS	$f_1(X', Y')=1$	$\Phi_1=Y'^2(X'-1)(Y'-X')$
	$f_2(X', Y')=X'$	$\Phi_2=X'\Phi_1(X', Y')-a_{2,1}\Phi_1(X', Y')$

Method of analysis: The plate geometry for thin triangular plate is given in fig. (a) and approximate solution is derived using Rayleigh's principle, which states that



$\widetilde{V}_{max}=\widetilde{T}_{max}$ (20)

Here $\widetilde{V}_{max}$ is max. Strain energy and $\widetilde{T}_{max}$ is max. Kinetic energy

Equation of motion:- The expression for K.E. (T) and S.E. (V) are $\widetilde{T}_{max}=\frac{1}{2}ab\Lambda^2\iint[((1+\beta^*X')^2+(\frac{K}{K_0})^2]\widetilde{W}^2((X',,Y')dX'dY'$ (21)

$\widetilde{V}_{max}=\frac{1}{2}ab\iint\{(1+\beta^*X')^4\left(\frac{\partial^2\widetilde{W}}{\partial X'^2}\right)^2+2\upsilon\alpha'^2(1+\beta^*X')^4\left(\frac{\partial^2\widetilde{W}}{\partial X'^2}\frac{\partial^2\widetilde{W}}{\partial Y'^2}\right)+\alpha'^4(1+\beta^*X')^4\left(\frac{\partial^2\widetilde{W}}{\partial Y'^2}\right)^2+2(1-\upsilon)(1+\beta^*X')^4\alpha'^2\left(\frac{\partial^2\widetilde{W}}{\partial X'\partial Y'}\right)^2\}dX'dY'$ (22)

Solution and frequency equation

$\frac{\partial}{\partial A_i}(\widetilde{V}max-\widetilde{T}max)$ (23)

Which leads to the governing Eigen value equation $\sum[K_{ij}-\Lambda^2 M_{ij}]c_i=0$ (24)

$K_{ij}=P_{ij}+\alpha'^4Q_{ij}+\alpha'^2\upsilon(R_{ij}+S_{ij})+2(1-\upsilon)\alpha'^2T_{ij}$

$M_{ij}=\iint[(1+\beta^*X')^2+(\frac{K}{K_0})^2]\Phi_i(X', Y')\Phi_j(X', Y')dX'dY'$,

$P_{ij}=\iint(1+\beta^*X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial X'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial X'}dX'dY'$,

$Q_{ij}=\iint(1+\beta^*X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial Y'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial Y'\partial Y'}dX'dY'$,

$R_{ij}=\iint(1+\beta^*X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial Y'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial X'}dX'dY'$,

$S_{ij}=\iint(1+\beta^*X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial X'}\frac{\partial^2\Phi_j(X',Y')}{\partial Y'\partial Y'}dX'dY'$,

$T_{ij}=\iint(1+\beta^*X')^4\frac{\partial^2\Phi_i(X',Y')}{\partial X'\partial Y'}\frac{\partial^2\Phi_j(X',Y')}{\partial X'\partial Y'}dX'dY'$

$F_{ij}=K_{ij}-\Lambda^2 M_{ij}$, i,j=1,2

On simplifying (24) one gets $F_{i1}A_1+F_{i2}A_2=0$, i=1,2 (25)

Where F_{i1}, F_{i2} (i=1,2) involve parametric constant and the frequency parameter.

For a non trivial solution, the determinant of the coefficient of equation (25) must be zero. So one gets the frequency equation as $\begin{vmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{vmatrix}$=0 (26)

From equation (34), one can obtains a quadratic equation in Λ^2 from which the two values of Λ^2 can found .After determining A_1 and A_2 from equation (26), one can obtain deflection function $\widetilde{W}$.

Choosing $A_1=1$,one obtains $A_2=-\frac{b_{11}}{b_{12}}$ and then $\widetilde{W}$ comes out as $\widetilde{W}=\Phi_1+(-\frac{b_{11}}{b_{12}})\Phi_2$ (27)

Thus deflection $\widetilde{w}$ may be expressed, by using equation (27) and (10) in equation (2), to give

$\widetilde{w}(x,y,t)=(\Phi_1+(-\frac{b_{11}}{b_{12}})\Phi_2)[e^{\alpha t}\left\{\cos\beta t-\frac{\alpha}{\beta}\sin\beta t\right\}]$ (28)

Time period of the vibration of the plate is given by $\widetilde{K}=\frac{2\Pi}{P}$, Where P is angular frequency

Logarithmic decrement of the vibrations given by the standard formula $\wedge=\frac{1}{N}log_e(\frac{\widetilde{w}_1}{\widetilde{w}_{N+1}})$ (29)

Numerical evaluations: $E=7.08\times10^{10}N/M^2$, $G=2.632\times10^{10}N/M^2$, $\eta=14.612\times10^{5}Ns/M^2$, $\rho=2.80\times10^{3}kg/M^3$, $\upsilon=0.3$, h=0.01m.

Result and discussion: Table1 constitutes 1st and 2nd frequency modes which are evaluated for the clamped simply supported visco elastic right triangular boundary condition for different values of damping parameter K/K_0 and taper parameter β^* for Poisson ratio $\upsilon=0.3$, thickness of plate h=0.01 and aspect ratio a/b=5/2. Projecting table1 through graph fig.1 shows the behavior of frequency parameter Λ with the increasing value of taper parameter (β^*) for any fixed but arbitrary value of damping parameter viz. K/K_0 (0.0, 0.2, 0.4, 0.6, 0.8,).

Table 1: Frequency Parameter of a CSS Visco Elastic Triangular Plate For Different Values of Taper Constant, A Damping Factor and A Constant Aspect Ratio A/B=5/2

β^*	0.0		0.2		0.4		0.6		0.8	
$\frac{K}{K0}$	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.0	284.24	706.25	329.63	748.17	375.84	795.10	422.51	845.81	469.41	899.43
0.2	278.72	692.53	324.84	736.44	371.62	784.84	418.71	836.68	465.96	891.19
0.4	263.91	655.73	311.64	704.33	359.71	756.30	407.88	810.96	456.03	867.77
0.6	243.73	605.60	292.81	659.04	342.17	714.97	391.56	772.94	440.81	832.56
0.8	221.95	551.49	271.39	608.18	321.45	667.07	371.68	727.72	421.84	789.80

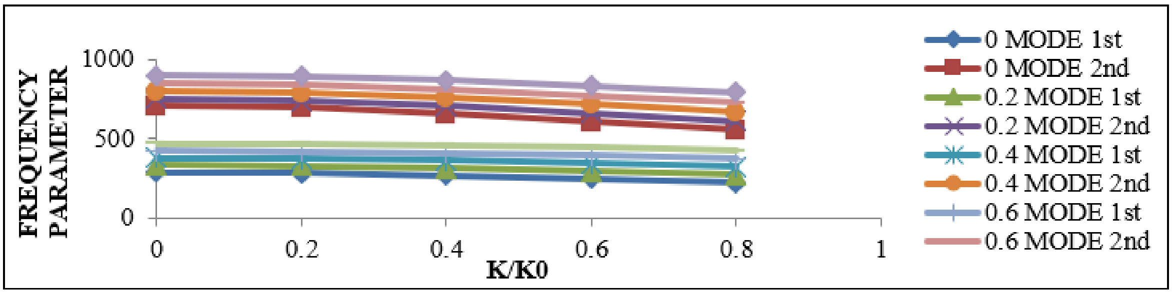


Figure 1: Frequency Parameter of a CSS Visco Elastic Triangular Plate for Different Values of Taper Constant, A Damping Factor and a Constant

Frequency parameter ʎ shows increments in its value with increase in value of taper parameter for the both modes of vibration w.r.t distinct value of damping parameter. Also table 1 and figure 1, provide us the inference of damping parameter K/K_0 on frequency parameter for two modes of vibration, if we take any fixed but arbitrary value of taper parameter β* (0.0,0.2,0.4,0.6,0.8) that there is decrement in value of ʎ with the increase of damping parameter K/K_0 and this decrement is linear in nature. Table 2 and Figure 2 explain the importance of frequency parameter for the first two modes of Vibration at different a/b ratios in the following three cases:

(i) β*=0.2, $\frac{K}{K_0}$=0.2,(ii) β*=0.6, $\frac{K}{K_0}$=0.6 ,(iii) β*=0.8, $\frac{K}{K_0}$=0.8 we observe that as value of a/b increases for different β*(0.0,0.2,0.4,0.6,0.8) these is parabolic increment in frequency parameter ʎ.

Table 2: Frequency Parameter of a CSS Visco Elastic Right Triangular Plate for Different Aspect Ratio (a/b)

a/b	β*=0.2, K/K_0=0.2		β*=0.6, K/K_0=0.6		β*=0.8, K/K_0=0.8	
	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.5	85.6522	223.5754	101.1533	279.3438	107.8517	304.6686
1	123.2963	264.2706	145.1654	316.6385	154.3210	341.0806
1.5	183.8450	354.9676	212.1673	403.7523	227.0462	425.1705
2	242.1672	518.6366	293.2695	556.5276	315.7926	574.6746
2.5	324.8414	736.4421	391.5597	772.9385	421.8376	789.8044

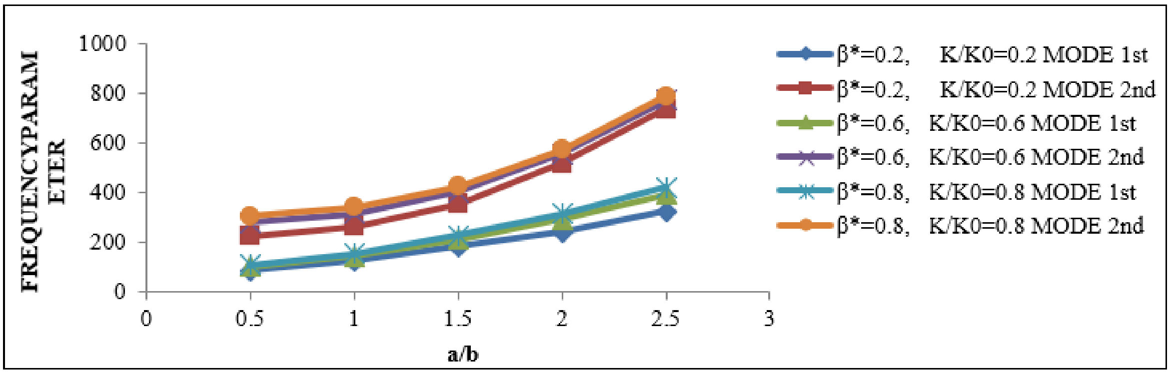


Figure 2: Frequency Parameter of a CSS Visco Elastic Right Triangular Plate For n different Aspect Ratio (a/b)

From table 3 and figure 3, for aspect ratio a/b=5/2 the time period $\widetilde{K}$ have been computed for CSS right triangular plate for two modes of vibrations with different edge restrictions for variation in different value of taper constant β* and damping parameter K/K_0. It can be seen from table 3 that as taper constant increases for any fixed but arbitrary value of damping parameter K/K_0(0.0,0.2,0.4,0.6,0.8),the time period decreases.Table3 and figure 3 also provide the result that if we take any fixed value of taper parameter β* and value of damping parameter K/K_0 increases frequency parameter increases and the increment is parabolic for 1st mode and linear for 2nd mode

Table 3: Time Period ($\widetilde{K}\times10^{-5}$) of a CSS Visco Elastic Triangular Plate for Different Values of Taper Constant, A Damping Factor and a Constant Aspect Ratio a/b=5/2

β^*	0.0		0.2		0.4		0.6		0.8	
K/K_0	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.0	145.26	58.46	125.26	55.19	109.86	51.93	97.73	48.82	87.96	45.91
0.2	148.14	59.62	127.11	56.07	111.11	52.61	98.61	49.35	88.61	46.33
0.4	156.45	62.96	132.49	58.62	114.79	54.59	101.23	50.91	90.54	47.58
0.6	169.40	68.18	141.01	62.65	120.67	57.75	105.45	53.42	93.67	49.59
0.8	186.03	74.87	152.14	67.89	128.45	61.89	111.09	56.74	97.88	52.28
1.0	205.43	82.68	165.36	74.08	137.81	66.85	117.95	60.74	103.04	55.53

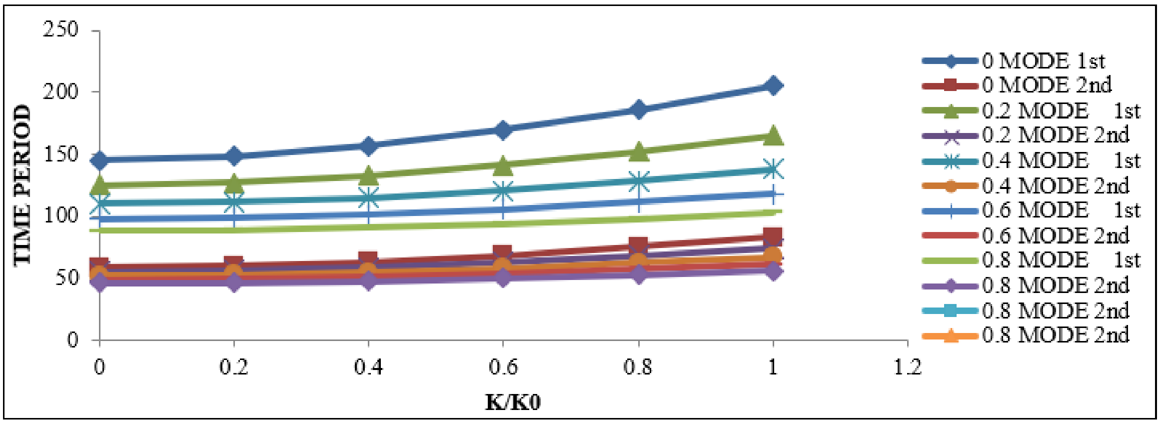


Figure 3: Time Period of a CSS Visco Elastic Triangular Plate for Different Values of Taper Constant, A Damping Factor and a Constant Aspect Ratio a/b=5/2

Table 4: Time Period ($\widetilde{K}\times 10^{-5}$) of a CSS Visco Elastic Right Triangular Plate for Different Aspect Ratio (a/b)

a/b	β^*=0.2 K/K_0=0.2		β^*=0.6 K/K_0=0.6		β^*=0.8 K/K_0=0.8	
	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.5	482.08	184.68	408.20	147.81	382.85	135.53
1	334.89	156.24	284.44	130.40	267.56	121.06
1.5	224.59	116.33	194.61	102.27	181.86	97.16
2	170.50	79.61	140.79	74.19	130.75	71.85
2.5	127.11	56.07	105.45	53.42	97.88	52.27

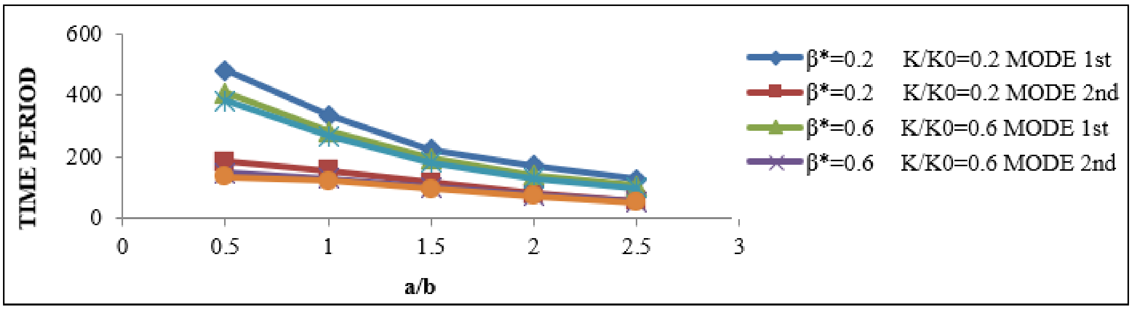


Figure 4: Time Period of a CSS Visco Elastic Right Triangular Plate For Different Aspect Ratio (a/b)

Table 4 and figure4 depicts value of time period $\widetilde{K}$ for first two modes of vibration for different values of aspect ratio a/b for the following three cases: (i) β^* =0.2, $\frac{K}{K_0}$ =0.2, (ii) β^*=0.6, $\frac{K}{K_0}$=0.6, (iii) β^* =0.8, $\frac{K}{K_0}$=0.8

It is important to observe that when aspect ratio increases, time period reduces in the preceding three situations for both modes of vibration. This reduction is again parabolic in nature.

Table 5: Logarithmic Decrement If a CSS Visco Elastic Triangular Plate for Different Values of Taper Constant, A Damping Factor and A Constant Aspect Ratio a/b=5/2

β	0.0		0.2		0.4		0.6		0.8	
K/K_0	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.0	-0.754	-1.874	-0.875	-1.986	-0.997	-2.110	-1.121	-2.245	-1.246	-2.387
0.2	-1.996	-3.094	-2.025	-3.118	-2.069	-3.166	-2.125	-3.234	-2.189	-3.317
0.4	-3.213	-4.253	-3.154	-4.196	-3.121	-4.174	-3.109	-4.179	-3.114	-4.207
0.6	-4.416	-5.377	-4.267	-5.239	-4.158	-5.147	-4.079	-5.091	-4.026	-5.066
0.8	-5.615	-6.490	-5.374	-6.268	-5.186	-6.104	-5.040	-5.985	-4.927	-5.904

Table 5 contains the results of the logarithmic decrement of the ratio a / b = 5/2 for the first two types of vibration for different values of the taper constant and damping parameter, respectively. It is observed from the table that the logarithmic decrement decreases as the damping parameter increases.

Table 6: Deflection ($\widetilde{w}$) of a CSS Right Triangular Plate for Different Values of X′ and Y′, A Constant Aspect Ratio (a/b=5/2) and β^*=0.4, K/K_0=0.4 and Time $\widetilde{T}$ =0$\widetilde{K}$ AND 5$\widetilde{K}$

X′	0.2		0.4		0.6	
Y′	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd	MODE 1st	MODE 2nd
0.2	0 **0**	0 **0**	0.000514283 **0.857955×** $\mathbf{10^{-10}}$	0.035374471 **0.305718×** $\mathbf{10^{-10}}$	0.00417777645 **14.0197×** $\mathbf{10^{-10}}$	0.022253429914 **0.386865×** $\mathbf{10^{-10}}$
0.4	0.01122541 **8.39392×** $\mathbf{10^{-10}}$	-0.288319 **-1.11688×** $\mathbf{10^{-10}}$	0 **0**	0 **0**	0.00835555291 **28.0394×** $\mathbf{10^{-10}}$	0.04450685982 **0.773731×** $\mathbf{10^{-10}}$
0.6	0.05051438 **37.7727×** $\mathbf{10^{-10}}$	-1.2974129 **-5.02585×** $\mathbf{10^{-10}}$	-0.00462854 **-7.72159×** $\mathbf{10^{-10}}$	-0.318370247 **-2.75146×** $\mathbf{10^{-10}}$	0 **0**	0 **0**
0.8	0.134705024 **100.727×** $\mathbf{10^{-10}}$	-3.4597677 **-13.4023×** $\mathbf{10^{-10}}$	-0.01645706 **-27.4546×** $\mathbf{10^{-10}}$	-1.131983103 **-9.78297×** $\mathbf{10^{-10}}$	-0.0334222116 **-112.158×** $\mathbf{10^{-10}}$	-0.1780274393 **-3.09492×** $\mathbf{10^{-10}}$

Table6 contains the deflection function values (i.e. the amplitude of the vibration modes)for the first two vibration modes with aspect ratios a/b = 5/2, respectively, for distinct values of X′ and Y′.The following parameter are used to calculate $\widetilde{w}$.
Table 6: β^*=0.4, K/K_0=0.4, time $\widetilde{T}$ =0$\widetilde{K}$, time $\widetilde{T}$=5$\widetilde{K}$ and values in bold indicate deflection at $\widetilde{T}$ =5$\widetilde{K}$

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