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Designing Smarter Transit: A Hybrid Optimization Framework for Congestion-Aware Public Transport Networks

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Abstract: ***We present an integrated mathematical framework to design and optimize urban public-transport routes and service frequencies while accounting for dynamic road congestion and stochastic passenger demand. The model couples a Transit Network Design Problem (TNDP) - formulated as a mixed-integer program that selects routes and frequencies to minimize a weighted sum of operator cost and passenger generalized travel cost - with a dynamic traffic model (Cell Transmission Model / Lighthill–Whitham–Richards family) to capture congestion-dependent travel times. Passenger demand is treated stochastically (Poisson or time-varying origin–destination rates) and assigned using a user-equilibrium / capacity-constrained assignment. Solution approaches combine exact MILP for small instances and heuristics/metaheuristics (route-generation + genetic algorithms / local search + simulation-based evaluation) for large networks. We describe model formulation, numerical solution strategy, and evaluation metrics; and provide a case study plan and recommended data sources. The framework balances operator cost, passenger waiting and in-vehicle times, and system robustness to demand uncertainty.***

Keywords: Urban transport planning, transit network design, dynamic traffic modeling, heuristic optimization, stochastic demand

1.Introduction

Urban public-transport planning faces two coupled difficulties: combinatorial route/frequency design and strongly time-varying travel times due to road congestion. Designing routes in isolation can yield suboptimal passenger outcomes when congestion increases in response to vehicle routing and road-user behavior. To address this, we propose a combined optimization–simulation framework that (1) chooses a set of transit lines (sequences of stops) and service frequencies, (2) assigns passenger demand to the transit network (and optionally to private modes), and (3) evaluates network performance under a dynamic traffic model so that travel times used in planning reflect realistic congestion feedbacks.

Key foundations for dynamic traffic modelling include the Lighthill–Whitham–Richards (LWR) kinematic wave framework and its discrete numerical counterpart, the Cell Transmission Model (CTM). These models allow translating vehicle densities into travel-time dynamics and shock-wave propagation on links, which is crucial when bus speeds and road capacities vary with congestion.

2.Literature review (brief)

- **Transit Network Design:** The Transit Route Network Design Problem (TRNDP/TNDP) is a long-studied combinatorial optimization problem; surveys document a mix of exact methods and heuristics to generate route sets and frequencies. Recent reviews recommend hybrid approaches that combine candidate-route generation with optimization over frequencies and passenger assignment.
- **Dynamic Traffic Models:** The LWR model (first-order conservation law) and Daganzo's Cell Transmission Model are widely used to model link-level dynamics and network interactions in traffic, and are amenable to incorporating transit vehicles (buses) as mobile bottlenecks or as part of multi-commodity flows.
- **Integrated Transit Design + Congestion:** Recent work integrates route/frequency optimization with link capacity constraints and dynamic assignment; mixed-integer linear programming (MILP) formulations and simulation-based optimization are common. Newer 2020s literature emphasizes data-driven route design and joint optimization of routes and timetables.

3.Problem statement and objectives

Given:
- Urban road network G=(V,E)G=(V,E)G=(V,E) with link lengths LeL_eLe, free-flow travel times te0t_e^0te0, capacities CeC_eCe.
- Candidate stops S⊂VS\subset VS⊂V and an OD (origin–destination) demand matrix Dod(t)D_{od}(t)Dod(t) (time-dependent, possibly stochastic).
- A fleet and cost structure (vehicle fixed cost, per-km cost).

Decide:
- A set of transit lines L\mathcal{L}L. Each line ℓ∈L\ell \in \mathcal{L}ℓ∈L is an ordered sequence of stops and an associated frequency fℓf_\ellfℓ (vehicles per hour).
- Vehicle allocation and schedule (or headway approximation).
- Assignment of passengers to routes (including transfers) and possibly private modes.

Objectives (multi-objective / single scalarized):

Minimize
Z=α·OperatorCost(L,f)+β·PassengerCost(L,f,travel times)+γ·UnreliabilityPenaltyZ = \alpha \cdot \text{OperatorCost}(\mathcal{L},f) + \beta \cdot \text{PassengerCost}(\mathcal{L},f,\text{travel times}) + \gamma \cdot \text{UnreliabilityPenalty}Z=α·OperatorCost(L,f)+β·PassengerCost(L,f,travel times)+γ·UnreliabilityPenalty

where passenger cost includes expected waiting time, in-vehicle time, transfers, and access/egress times.

Constraints:

- Fleet size and vehicle-hour limits.
- Line design constraints (max/min route length, stop spacing).
- Link capacity constraints (through traffic model).
- Service frequency bounds.

4.Mathematical formulation (high-level)

4.1 Decision variables

- xℓ∈{0,1}x_{\ell} \in \{0,1\}xℓ∈{0,1}: whether route ℓ\ellℓ is operated (we work from a generated candidate set of feasible routes).
- fℓ≥0f_{\ell} \ge 0fℓ≥0: frequency for route ℓ\ellℓ (vehicles/hour).
- yp,ℓ∈{0,1}y_{p,\ell} \in \{0,1\}yp,ℓ∈{0,1} or continuous: assignment of passenger path ppp to route(s) ℓ\ellℓ (depending on model granularity).

4.2 Operator cost

OperatorCost=∑ℓ(cfixed xℓ+cvh fℓ τℓ)\text{OperatorCost} = \sum_{\ell} \big(c_{\text{fixed}}\; x_{\ell} + c_{\text{vh}}\; f_{\ell}\; \tau_{\ell}\big)OperatorCost=ℓ∑(cfixedxℓ+cvhfℓτℓ)

where cvhc_{\text{vh}}cvh is cost per vehicle-hour, τℓ\tau_{\ell}τℓ is round-trip time (depends on congested link travel times from the traffic model).

4.3 Passenger cost (expected)

Using standard transit choice models, approximate expected waiting time as 1/(2fℓ)1/(2f_{\ell})1/(2fℓ) (Poisson arrivals) and in-vehicle time as sum of congested link travel times along ℓ\ellℓ. For passenger class kkk:

PassengerCost=∑k∑p∈Pkλk,p(waitp+invehiclep+θ·transfersp)\text{PassengerCost} = \sum_{k}\sum_{p\in P_k} \lambda_{k,p}\big(\text{wait}_p + \text{invehicle}_p + \theta \cdot \text{transfers}_p \big)PassengerCost=k∑p∈Pk∑λk,p(waitp+invehiclep+θ·transfersp)

Stochastic demand: λk,p\lambda_{k,p}λk,p may be expected demand or a distribution; robust/objective can minimize expected or CVaR cost.

4.4 Congestion coupling (dynamic)

Travel time on link eee at time ttt, Te(t)T_e(t)Te(t), is produced by a dynamic traffic model. We adopt the Cell Transmission Model (CTM) for discrete-time link evolution:

ρen+1=ρen+ΔtLe(qin,en−qout,en)\rho_e^{n+1} = \rho_e^{n} + \frac{\Delta t}{L_e} \big(q_{in,e}^n - q_{out,e}^n \big)ρen+1=ρen+LeΔt(qin,en−qout,en)

with flow demands and supplies computed via sending/receiving functions derived from the fundamental diagram; bus vehicles contribute to link usage and reduce effective capacity when stopped/boarding. See Daganzo (1994) for CTM formulation.

4.5 Integrated MILP (sketch)

When link travel times are approximated as piecewise-linear functions of flow (or precomputed scenarios), the model can be linearized and written as a Mixed-Integer Linear Program:

- Linear objective (operator + passenger travel-time costs via linear approximations),
- Integer route activation variables,
- Continuous frequencies and passenger flows,
- Capacity constraints approximated per time-slice.

For exact dynamic coupling, the problem becomes a mixed-integer nonlinear problem (MINLP) or bilevel formulation (upper level: route design; lower level: dynamic assignment/traffic), often solved by decomposition (outer-loop search over route sets; inner-loop simulation for travel times).

Demand uncertainty and stochastic modeling

- Model arrival processes at stops with Poisson processes; waiting-time expectation 1/(2f)1/(2f)1/(2f) holds under Poisson arrival assumptions.
- Incorporate day-to-day demand variation by scenario sampling: evaluate candidate network on a set of demand scenarios, and optimize for expected or risk-averse objective (e.g., CVaR).
- Optionally treat bus travel-time variability by adding stochastic perturbations to link free-flow times or arrival rates and evaluate robustness.

5.Solution approach

1. **Candidate route generation**: generate feasible route pool using heuristics (k-shortest paths with stop spacing rules, corridor-based generation).
2. **Frequency discretization**: restrict frequencies to discrete levels to reduce search space.
3. **Master MILP / heuristic search**:
   - For small networks: solve MILP with off-the-shelf solvers (CPLEX/Gurobi).
   - For city-scale: use metaheuristics - genetic algorithms, large neighborhood search, or simulated annealing - each solution evaluated by a traffic simulator (CTM-based) that returns link travel times and passenger costs. Recent works use data-driven and hybrid methods for scalability.
4. **Simulation-based evaluation**: run CTM (or microscopic simulator if required) to compute congested travel times and vehicle round-trip times; iterate until convergence or budget exhaustion.

6. Validation & performance metrics

- **System metrics:** total operator cost, total passenger generalized travel cost, vehicle-hours, fleet utilization, system-wide delay.
- **Passenger metrics:** average waiting time, average in-vehicle time, average transfers, % demand served within X minutes.
- **Network metrics:** link-level travel times, queue lengths, congestion hotspots.
- **Robustness metrics:** objective variance across scenarios, CVaR at 90% level.

Use cross-validation withhold-out demand days or bootstrapped demand realizations.

Case study plan (suggested workflow)

1. **Data collection:** road graph (OSM), bus stop locations, historical OD matrices (smartcard/Census/survey), link capacities, fleet costs.
2. **Preprocessing:** generate candidate routes, discretize time-of-day into intervals (e.g., 15-min).
3. **Modeling:** implement CTM for dynamic link travel times; implement MILP/heuristics in Python + Gurobi / OR-Tools.
4. **Calibration:** calibrate fundamental diagram parameters using observed speed–flow data.
5. **Experiments:** baseline network vs. optimized networks; stress test under peak demand and incident scenarios.
6. **Policy analysis:** explore trade-offs (operator subsidy vs. passenger time reduction), evaluate targeted improvements (priority lanes, stop consolidation). Relevant methodologies and examples are documented in TNDP surveys and practical works.

7. Discussion

The main challenge is the computational coupling between combinatorial route choice and nonlinear, time-dependent congestion. Practical deployments rely on candidate-route sets plus simulation-based evaluation and heuristics. Incorporating realistic passenger behavior (non-Poisson arrivals, smartcard variability), operator constraints (driver shifts, depot locations), and multimodal integration (walk, bike, microtransit) improves realism but increases complexity. Data availability (high-resolution OD and link counts) markedly improves solution quality; recent data-driven frameworks scale TNDP solutions to city-scale problems.

8. Conclusion

We proposed an integrated modeling framework that couples a mixed-integer transit network design model with dynamic traffic (CTM/LWR) and stochastic passenger demand. The approach is flexible: MILP/heuristic solution pipelines can be tailored to network size and data availability. Future extensions include explicit timetable optimization, real-time adaptive routing, and inclusion of priority measures (bus lanes) as decision variables.

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