

Signal Flow Graph and Mason's Gain Formula in Analysis of Nonlinear Electrical Network

A. K. Baruah¹, T. Ali², B. C. Phukan³

¹Retd. Faculty for Department of Mathematics Dibrugarh University, Dibrugarh 786004, Assam, India
Email: [arun\[at\]dibru.ac.in](mailto:arun[at]dibru.ac.in)

²Faculty for Department of Mathematics Dibrugarh University, Dibrugarh 786004, Assam, India
Email: [tazid\[at\]dibru.ac.in](mailto:tazid[at]dibru.ac.in)

³Faculty for Department of Mathematics Jorhat Engineering College, Jorhat 785007, Assam, India
Email: [bcphukan.jec\[at\]gmail.com](mailto:bcphukan.jec[at]gmail.com)

Abstract: *The transfer function of an electrical network can be calculated using various techniques, including Kirchhoff's laws, two-point network theory, nodal analysis, and the time constant method. Another approach for algebraically determining the transfer function is through the use of the Signal Flow Graph (SFG). An SFG consists of a directed, weighted graph made up of nodes and branches. In this context, we will create a signal flow graph of a nonlinear electrical network to analyze it using Mason's gain formula and the Signal Flow Graph's reduction rules.*

Keywords: Transfer function, Mason's formula, Signal Flow Graph, Nonlinear Electrical Network

1. Introduction

One of the most significant and fascinating areas of mathematics is graph theory. This subfield studies networks of points connected by lines [1]. Graph theory [3] originated from number games and recreational math puzzles but has since evolved into a substantial area of mathematical study, with applications in computer science, social science, operational research, chemistry, and various types of networks [2] [17].

A graph is defined as an ordered pair $G = (V, E)$, where V is a finite, non-empty set of vertices, and E is the set of edges. In graphical representations, the vertices are depicted as dots, while the edges are illustrated as line segments [2]. Theoretical properties of graphs allow us to analyse networks and provide solutions to various problems.

An SFG, or signal flow graph [4], is a directed, weighted graph in which signals are applied to nodes and functions are applied to edges to represent a system's signal flow. The transfer function of electrical circuits can be calculated using a number of techniques, including Kirchhoff's rules [5], two-port network theory [6], the nodal analysis approach [7], and the time constant method [10]. These techniques typically require a lot of computation and can be time-consuming. Additionally, the Signal Flow Graph may be used to create a common graphical model that connects the state variable (parameters) and the transfer function and helps one grasp the intricate workings of a network. With the aid of mesh rules, node rules, and Ohm's equations, a signal flow graph can be created. This presentation enables a more accurate result.

Furthermore, the signal flow graph can be used to create a common graphical model that connects state variables (parameters) to the transfer function, helping to clarify the complex workings of a network. By utilizing mesh rules, node rules, and Ohm's law, one can construct a signal flow graph more accurately.

Signal Flow Graphs (SFG) have greatly benefited from the work of numerous scholars. Samuel Jefferson Mason initially used the phrase to refer to a specialized flow graph before Claude Shannon created the Signal Flow Graph, often known as the Mason graph. Shannon [1942] first applied the Single Flow Graph concept [11] to dealing with analogue computers. Mason [1953] [1956] deserves the most credit for creating signal flow graphs since he demonstrated how to apply the method to tackle certain challenging electronic issues relatively easily. The phrase "signal flow graph" was chosen since it was first applied to electronic problems [12] involving electronic signals and system flow charts. Signal Flow Graphs (SFG) have greatly benefitted from the contributions of many scholars. Samuel Jefferson Mason was the first to use the term to describe a specialized type of flow graph. Later, Claude Shannon developed what is now commonly known as the Signal Flow Graph, or Mason graph [8-9]. In 1942, Shannon applied the concept of Signal Flow Graphs to solve problems related to analog computers. However, it is Mason, in his works from 1953 and 1956, who deserves the most recognition for developing signal flow graphs. He demonstrated how this method could effectively tackle complex electronic issues with relative ease. The term "signal flow graph" was chosen because it was initially applied to electronic problems that involved electronic signals and system flow charts.

The SFG (Signal Flow Graph) method, developed by J. A. Brzozowski and E. J. McCluskey Jr. in 1963, is designed to characterize sequential circuits using regular expressions [14]. It has been shown that state diagrams of sequential circuits can be accurately interpreted using methodologies from signal flow graph theory. After 1990, Dalibor Biolek and Viera Biolkova utilized signal flow graphs to explain linear circuits. In their 2017 work [15-16], Feim Ridvan Rasim and Sebastian M. Sattler evaluated the reliability of the SFG method [13] [18]. A signal flow graph can be validated by generating the graph's identity.

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2. Preliminary

Table 1: Basic definitions

Electrical Network	A closed electrical network connects various electrical components, including transformers, resistors, inductors, capacitors, and both voltage and current sources.
Circuit	A circuit is a closed electric network.
Graph	A graph is defined as a pair $G = (V, E)$, where V is a collection of nodes or vertices and E is a collection of edges or branches.
Node	A node is a point that symbolizes a signal or variable.
Branch	A branch is a piece of a line that connects two nodes.
Walk	A walk is a finite alternating sequence of vertices and edges that starts and ends with vertices, characterized by each edge being incident to the vertices immediately before and after it.
Path	A path is a continuous route where no vertex appears more than once.
Loop	A loop is a closed path that starts at a vertex and ends at the same vertex or node, never passing any node more than once.
Self-loop	One branch makes up this feedback loop.
Loop gain	The gain achieved by following a path that starts and ends at the same node is referred to as loop gain.
Non-touching loops	Non-touching loops are defined as loops that do not have any nodes in common.
Forward path	If a path starts with an initial node, continues to another node, and ends at an output node, it is known as a forward path.
Feedback path	A "feedback path" is a path that begins and ends at the same node.
Forwarded path gain	The total of gains found by tracing a path from an input node to an output node in a signal flow graph is referred to as the forwarded path gain.
Digraph	A directed graph, also referred to as a digraph, is a graph where each edge has a specific direction.
Transfer function	The system's transfer function $T(s)$ is defined as $C(s)/R(s)$, where $R(s)$ represents the system's input as a single flow graph. The system's output, denoted by the signal flow graph, is $C(s)$.

3. Methodology

3.1 Signal Flow Graph

A Signal Flow Graph (SFG) is a directed and weighted graph that describes a system's signal flow. A Signal Flow Graph (SFG) offers a set's graphical representation.

3.1.1. Reduction Rule of Signal Flow Graph (SFG) (Modification of Signal Flow Graph):

Rule 1:

(a)

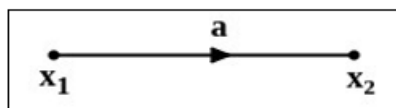


Figure 1

If "input" is represented as " x_1 " and "output" as " x_2 ," with "a" denoting the gain, then $x_2 = ax_1$

(b)

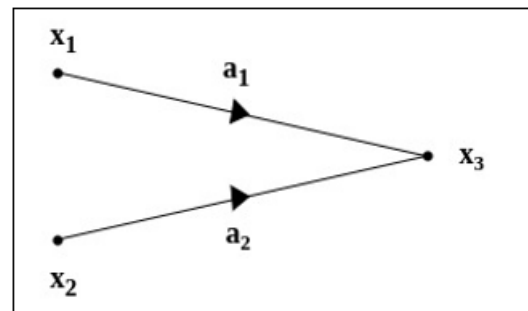


Figure 2

If x_1 and x_2 are input signals with gains a_1 and a_2 respectively, then the output signal can be written as $x_3 = a_1x_1 + a_2x_2$

Rule2:

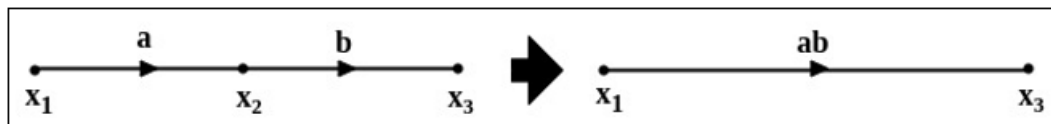


Figure 3

If $x_1, x_2 \wedge x_3$ are nodes where x_1 is input and x_3 is output with gains a and b . After making the adjustments, we have $x_2 = ax_1$, $x_3 = bx_2 = abx_1$

Rule3:

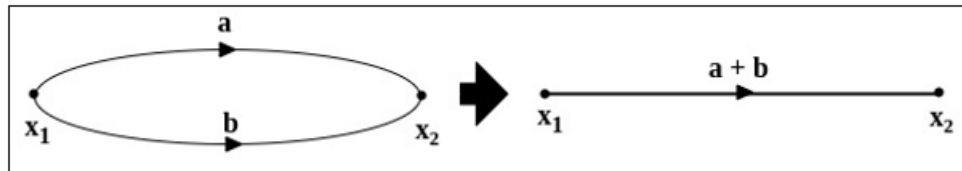


Figure 4

If input is represented by x_1 and output by x_2 , with gains a and b for the respective branches, then the resulting output will be $x_2 = (a + b)x_1 = ax_1 + bx_1$

Rule4:

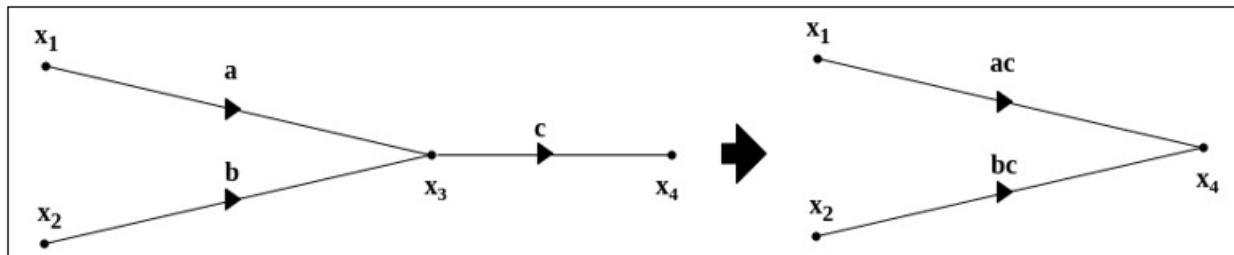


Figure 5

If x_1, x_2, x_3 and x_4 are nodes, and a, b , and c represent the gains of branches, then the relationships can be established. Therefore, we can conclude $x_3 = ax_1 + bx_2$ and $x_4 = cx_3 = c(ax_1 + bx_2)$. Therefore $x_4 = acx_1 + bcx_2$

Rule5:

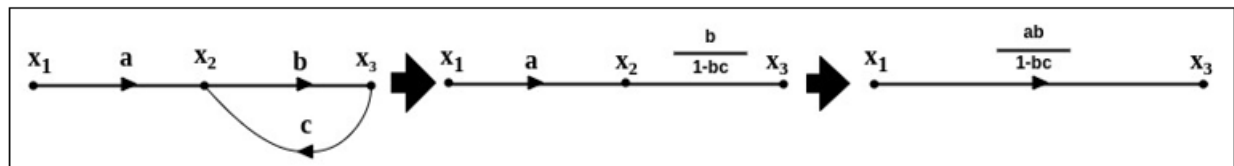


Figure 6

If x_1, x_2 and x_3 are nodes with a, b , and c as gains of branches, as shown in the figure, then we have $x_2 = ax_1$ and $x_3 = \left(\frac{b}{1-bc}\right)x_2 = \left(\frac{b}{1-bc}\right)ax_1 = \frac{ab}{1-bc}x_1$

Rule6:

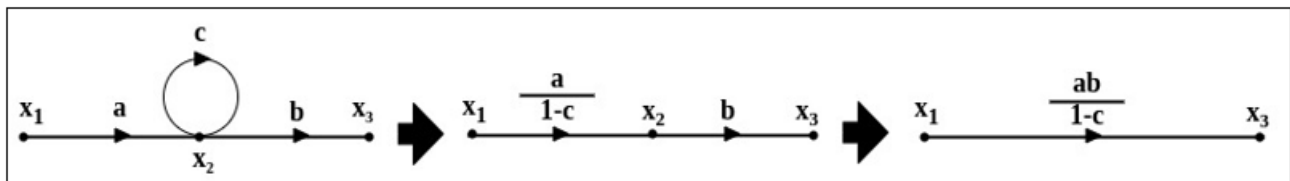


Figure 7

If x_1, x_2 and x_3 are nodes with a, b (reflexive or self-referential), and c are gains of branches as shown in the figure, then $x_2 = \frac{a}{1-c}x_1$ and $x_3 = bx_2 = b\frac{a}{1-c}x_1 = \frac{ab}{1-c}x_1$

Rule 7: Loop reduction:

(a) **Signal loop:** The transmission (T_{ij}) from an independent variable x_i (source node) to a dependent variable x_j (internal or sink node) In a signal flow graph containing only one loop and one path, equals

$$T_{ij} = \frac{P_{ij}}{1-L} \dots \dots \dots (1)$$

Where P_{ij} is the forward path transmission from x_i and x_j and L is the transmission of the loop.

(b) **Multi-loop with non-touching loops:** With a cascade of non-touching (loops possessing no common nodes), the overall transmission is the product of the individual transmissions as given by equation (1) (single loop)

$$T = \frac{P_{ij}}{1-L_j} \cdot \frac{P_{jk}}{1-L_k} \dots \dots \dots \frac{P_{(n-1)n}}{1-L_n} \dots \dots \dots (2)$$

(c) Multiloop with touching loops:

For the case that the two loops do share at least one node, as shown in the figure below, the overall transmission is given by

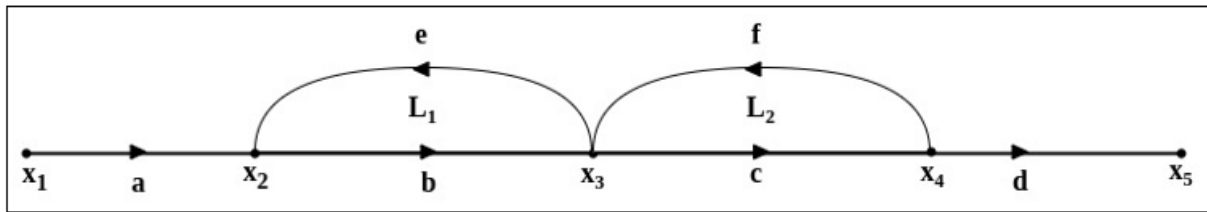


Figure 8

Here, $x_1, x_2, x_3, x_4 \wedge x_5$ are nodes with a, b, c, d, e, and f are gains of branches as shown in the figure, then the transfer function between $x_1 \wedge x_5$ we have as follows:

$$T_{15} = \frac{x_5}{x_1} = \frac{abcd}{1-be-cf} = \frac{P_{15}}{1-L_1-L_2} \dots\dots\dots (3)$$

$$x_5 = \frac{abcd}{1-be-cf} x_1$$

3.2 Mason's Gain Formula (MGF)

The Mason's Gain Formula (MGF) is a formula for calculating the transfer function of a linear signal flow graph. Let $R(S)$ be the system's input in the signal flow graph model. And $C(S)$ represents the system's output as shown by a signal flow graph.

The system's transfer function is given by $T(S) = \frac{C(S)}{R(S)}$

$$\text{Overall gain } T = T(S) = \sum_{k=1}^N \frac{P_k \Delta_k}{\Delta}$$

where,

N Number of paths from input to output

P_k Forward path gain of the k^{th} forward path

$\Delta = 1 - (\text{Sum of individual loop gains}) + (\text{sum of two non touching loops gain})$

$- (\text{sum of three non-touching loops gain}) + (\text{sum of four non-touching loops gain})$

$\dots\dots\dots$

$\Delta_k = 1 - (\text{loops gain which are not touching the } k^{\text{th}} \text{ forward path})$

4. An electrical Network illustration:

Graphs are useful for representing electrical networks, where each network [13] [16 17] node can be considered as a vertex and each network branch can be seen as an edge. When dealing with simpler circuits, it is straightforward to determine the loop currents and node voltages. However, as the complexity of the circuit increases, it can become challenging to solve. In such cases, using a graph to represent the electrical network can simplify the process. A network's graph is essential for understanding the circuit. Below is an illustration of the signal flow graph for an electrical network:

4.1 A Non-linear Electrical Circuit

In this section, we will analyze a non-linear electrical circuit by calculating the transfer function using a Signal Flow Graph (SFG). A diode serves as an example of a non-linear device because the current flowing through it is not directly proportional to the voltage across it; its voltage-current (V-I) characteristic is not a straight line.

To describe an electrical network, we can utilize a graph, with nodes represented as vertices and branches represented as edges. In this discussion, we will focus on this electrical network.

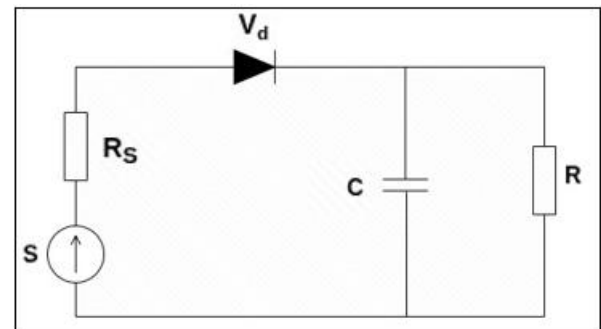


Figure 9

4.2 The graphical representation of the electrical network (Fig.1) is illustrated using vertices and edges that cover the entire network.

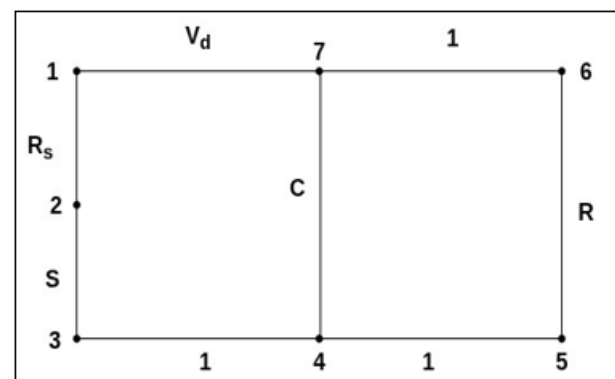


Figure 10

4.3 A digraph representation

The electrical network's graph can be depicted as a digraph by using vertices and edges, with specific directions for the edges.

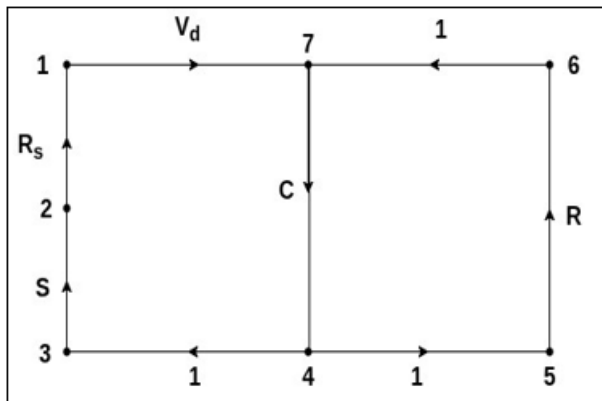


Figure 11

4.4 Signal Flow Graph Representations

To represent the electrical networks outlined in the directed graph model, we use a signal flow graph. This graph will include vertices and directed, weighted edges that encompass the entire network. The signal flow graph corresponding to the above diagram is illustrated in Fig.12 below.

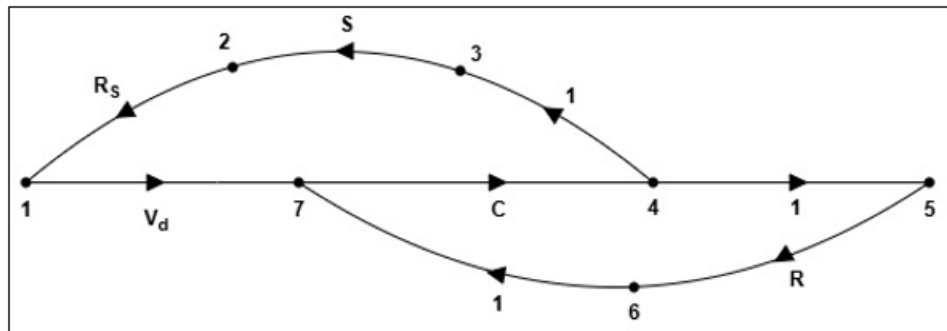


Figure 12

Nodes 2, 3, and 6 have been removed using Rule 2.

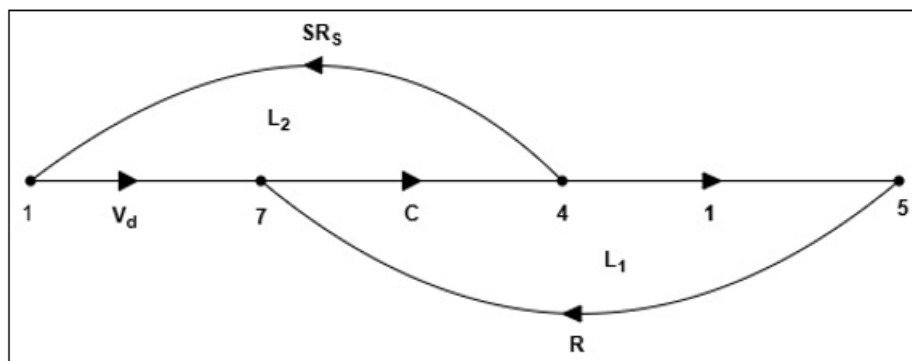


Figure 13

5. Calculation of Transfer Function by Reduction of the Signal Flow Graph:

In this section, we consider the loops defined as follows:

Loop 1 $L_1 = C.1. R = CR$

Loop 2 $L_2 = V_d C. S. R_s$

All loops are touching loops that share a common node, specifically nodes 4 and 7 in the figure above. To determine the transmittance between node 1 and node 5, we will apply Rule 7 (c) of loop reduction, which applies to multi-loops with touching loops. The resulting equation is given by:

$$T_{15} = \frac{P_{15}}{1 - L_1 - L_2} = \frac{cv_d}{1 - CR - CSR_s V_d} \dots \dots \dots (1)$$

5.1 Calculation of the Transfer Function using Mason's Gain Formula from the Signal Flow Graph (SFG) Representation

We know Mason's formula for the Transfer Function.

$$T(S) = \frac{1}{\Delta} \sum_k F_k \Delta_k$$

Where $T(s)$ = Transfer function of the system

F_k = forward path gain of the k^{th} forward path

$\Delta = 1 - (\text{Sum of individual loops gain}) + (\text{Sum of two non-touching loops gain}) - (\text{Sum of three non-touching loops gain}) + \dots \dots \dots$

$\Delta_k = 1 - (\text{loop gain, which is not touching the } k^{\text{th}} \text{ forwarded path})$

In this particular problem, there is only one forward path, and it is as follows from Fig.13:

$$F_1 = v_d \cdot c \cdot 1 = cv_d$$

Secondly, we have two separate loops here, each with its own gains.

$$L_1 = C \cdot 1 \cdot R = CR$$

$$L_2 = V_d \cdot C \cdot S \cdot R_s = CSR_s V_d$$

Thirdly, we need to identify the gain of non-touching loops. In this case, there are no non-touching loops present. Thus, we can express the relationship as follows:

$$\Delta = 1 - (\text{Sum of individual loops gain}) + (\text{Sum of two non-touching loops gain}) - (\text{Sum of three non-touching loops gain}) + \dots$$

$$\text{There fore } \Delta = 1 - CR - CSR_s V_d$$

Now, $\Delta_1 = 1 - (\text{loop gain, which is not touching the } k^{\text{th}} \text{ forwarded path})$
 $= 1 - 0 = 1$

The Transfer function between nodes 1 and 5 $\frac{C(S)}{R(S)} = T_{15}$

$$\text{Therefore } T(S) = T_{15} = \frac{F_1 \Delta_1}{\Delta}$$

$$= \frac{CV_d}{1 - CR - CSR_s V_d} \dots \dots \dots (2)$$

We observe that equations (1) and (2) are equal.

We can calculate the transfer function, or transmittance, between any two nodes in the same nonlinear electrical circuit by using the reduction rules of signal flow graphs and Mason's gain formula. In this paper, we demonstrate that we can derive the same transfer function between nodes 1 and 5, as shown in Figure 13, through both the reduction of the signal flow graph (illustrated in equation (1)) and Mason's gain formula (shown in equation (2)). Notably, both equations yield the same result.

6. Calculation of the Transfer Function of the same Nonlinear circuit (Fig.9) by considering the direction of current reverse in the circuit:

6.1 A Digraph Representation

A digraph model is utilized to represent a graph using vertices and directed, weighted edges that encompass the entire network. Below is the digraph model of the circuit illustrated in Fig.9. In this model, the direction of the circuit is reversed, except for the section between node 1 and node 7. This exception is due to the presence of a diode, which is a nonlinear circuit element, between these two nodes.

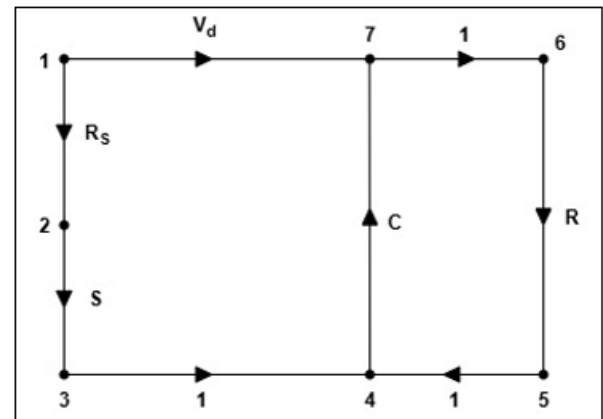


Figure 14

6.2. Signal Flow Graph (SFG) Representation

A signal flow graph (SFG) is a model that represents a directed graph using vertices and weighted edges to depict the entire network. Below is the SFG model of the network mentioned above.

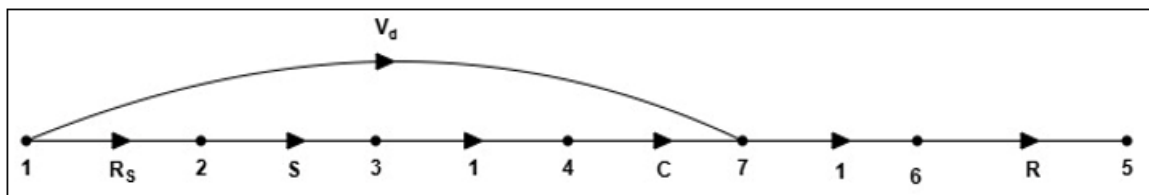


Figure15

Nodes 2, 3, and 4 have been removed using Rule 2.

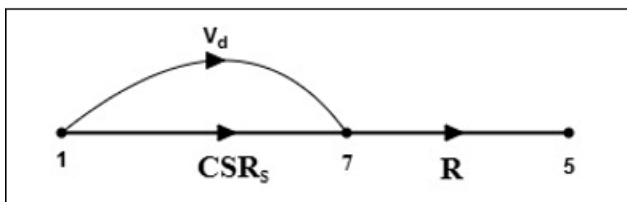


Figure 16

By using Rule-3

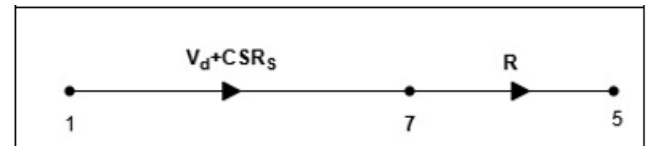


Figure 17

Node 7 is taken out by using Rule 2

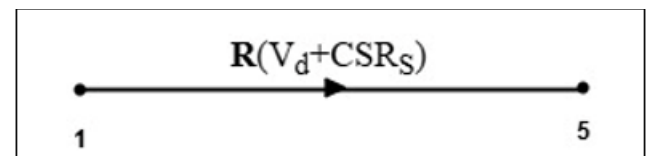


Figure 18

Transmittance between the nodes 1 and 5

$$T_{15} = RV_d + CSRR_s \dots\dots\dots (A)$$

6.3. Calculation of the Transfer Function Using Mason's Gain Formula from Signal Flow Graph (SFG) Representation:

We know Mason's formula for the transfer function.

$$T(S) = \frac{1}{\Delta} \sum_k F_k \Delta_k$$

Where $T(s)$ = Transfer function of the system

F_k = forward path gain of the k^{th} forward path

$\Delta = 1 - (\text{Sum of individual loops gain}) + (\text{Sum of two non-touching loops gain}) - (\text{Sum of three non-touching loops gain}) + \dots\dots\dots$

$\Delta_k = 1 - (\text{loop gain, which is not touching the } k^{\text{th}} \text{ forwarded path})$

As illustrated in Fig.14, we have gains along the forward path.

$$F_1 = R_s \cdot S \cdot 1 \cdot C \cdot 1 \cdot R = R_s SCR$$

$$F_2 = V_d \cdot 1 \cdot R$$

Secondly, we do not have any individual loops or gains.

Thirdly, we must identify the gains from non-touching loops. However, there are no non-touching loops present. This is the issue.

$\Delta = 1 - (\text{Sum of individual loops gain}) + (\text{Sum of two non-touching loops gain}) - (\text{Sum of three non-touching loops gain}) + \dots\dots\dots$

$$= 1 - 0 + 0 - 0$$

$$= 1$$

$\Delta_1 = 1 - (\text{loops gain which does not touch the forward path } F_1)$

$$= 1 - 0$$

$$= 1$$

$\Delta_2 = 1 - (\text{loops gain which does not touch the forward path } F_2)$

$$= 1 - 0$$

$$= 1$$

The Transfer function between nodes 1 and 5 = $\frac{c(s)}{R(s)} = T_{15}$

$$\text{Therefore } T(s) = T_{15} = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta}$$

$$= \frac{(CSRR_s) \cdot 1 + (R \cdot V_d) \cdot 1}{1}$$

$$T_{15} = RV_d + CSRR_s \dots\dots\dots (B)$$

From relations (A) and (B), we have seen that both are equal.

Observation: We have noted that different digraphs of the same electrical circuit yield varying results (transfer functions) between the same nodes. In this paper, we have analyzed two digraph models, illustrated in Fig.11 and Fig.14, for the nonlinear electrical circuit shown in Fig.9. This analysis involves reversing the direction of current in the circuit, except for the edge between nodes 1 and 7, as there is a diode present between these nodes.

7. Transfer Function (TF) in Electrical Networks

In an electrical system, the Transfer Function (TF) is defined as the ratio of the output to the input. The Transfer Function provides valuable insights into the circuit's gain, frequency response, stability, and the range of input values for which the output remains stable. It also describes the circuit's behavior under different input types, whether alternating current (AC) or direct current (DC), along with its key characteristics. By determining the Transfer Function of a system or circuit, we can analyze many of its parameters and predict how the system will respond to various inputs. Additionally, Transfer Functions allow us to represent the entire system as a single edge between any two nodes, simplifying the study of complex systems.

The Signal Flow Graph (SFG) is an effective tool for analysing electrical networks. This method serves as an alternative to traditional approaches, such as Kirchhoff's rules and nodal analysis. While calculating the transfer function through the reduction of a signal flow graph can sometimes be challenging, Mason's Gain Formula provides a valuable solution for complex network circuits. Specifically, Mason's Gain Formula offers a systematic and algebraic method for determining the transfer function, which can be particularly advantageous when traditional techniques are impractical. Moreover, signal flow graphs and their reductions provide graphical representations that enhance visualization and deepen understanding of the network's behaviour.

8. Conclusion

This paper aimed to contribute to the field of electrical network analysis by introducing the use of Mason's Gain Formula and Signal Flow Graph (SFG) to determine the transfer function of electrical networks. The Signal Flow Graph is essential for analyzing these networks. By creating a graphical representation of the electrical network, we could construct a Signal Flow Graph, which was then used to find the transmittance, or transfer function, between the vertices. To do this, we needed to determine the forward path gains and the various loops within the graph. A distinct edge, referred to as a one-edge (I-edge), can illustrate the transfer function or transmittance of the electrical network. Mason's Gain Formula offers a systematic and algebraic method for calculating the transfer function. Additionally, the Reduction Rules of the Signal Flow Graph provide a quicker and more efficient alternative for network analysis. This method holds significant potential and offers various academic opportunities. However, further research is still required to explore the Signal Flow Graph approach.

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