A Sustainable Economic Production Quantity (EPQ) Inventory Model for Deteriorating Products with Circular Economic Indicator and Product Greening Level Dependent Demand Rate

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Abstract: ***Circular economy basically minimizes waste and maximizes resources. Natural resources are limited and need to be recollected and reproduced and reused. In this way natural resources are saved for future generations. Reproduction and reuse of products minimize waste and hence the release of carbon emission is controlled through the production and supply process. In the modern era, environmental concerns are widely observed by the customers and demand of green products increases. The present work proffers a profit maximization inventory production model where products are produced based on variable circularity index and greening level. Customer's awareness about environmental impact of products and social responsibilities in the present era, customer's demand and gross profit of the manufacturer are influenced by both circularity index and greening level of products. These outcomes are explored by linear form of demand and gross profit. Since operations involved production and supply chain management, environmental issues have become more of a concern in the inventory modelling system. Moreover, carbon emission produced due to activities like holding inventory, deterioration process and setup operations are considered to be sources of carbon emission. The objective of the present study is to maximize producer's profit jointly with optimal circularity, greening level and production level of product under reduction of carbon emission to protect environment. Analytical results are explored in determining optimal profit and reduction of carbon emission. A numerical example is presented to investigate model applicability and to help managers in deciding parameters' behavioral patterns on the optimality strategies, an analysis is conducted.***

Keywords: Deterioration, EPQ, Circular economy indicator, Carbon Emissions and Customers Environmental Awareness (CEA)

1. Introduction

Global marketing influencing the economy as well as the environment and thus has become a challenging task for model developers. The global impact of various activities on earth has been observed for many decades challenging economic growth and obstructing environmental objectives. Global warming is one of the most concern policy makers and of researchers while developing inventory models. Mostly, the economy of local and global market depends on the supply chain management system involved with production policy. Excessive use of natural resources for production is burdening waste material along with carbon emission globally impacting climate. Being affected by global changes impacting human lives, now customers have become aware of using green products that release less carbon emission resulting in a clean and green environment pressurizing manufacturers to produce and supply more green products (Du et al. 2015; Kannan et al.,2020). As a result, researchers are focused on developing reverse supply chain models, reverse logistic models, inventory models producing green products and reducing greenhouse gases (Carbon di oxide, Sulphur di oxide, Nitrogen di oxide etc.) emissions. Researchers (Darbari et al., 2019; Kumar et al., 2019; Jaber, Glock, and El Saadany2013) have incorporated all these issues in their research study.

After industrial revolution, emission concentration of greenhouse gases has been increased, and 30-40% and 50-60% of carbon pollutions are emitted due to manufacturing and transportation activities respectively (Zheng, Yang, and Jiang 2015). The figure indicates urgent attention on controlling pollutions by different means of operations like imposing tax and promotion for green production and consumption.

Green practices are required around the world in both developed and developing nations. Geng, Mansouri, and Aktas 2017 have described in their study that Green Supply Chain Management (GSCM) practices will led in three facets: environmental, economic and operational. Wu, Geng, and Liu (2017) have outlined that per capita footprints in BRICS countries have increased rapidly during the year 1995 to 2008.

Jaber, Glock, and El Saadany (2013) in their studies have pointed that European Union Emissions Trading System (EU ETS) is first emission trading scheme launched in 2005 to control emissions in 31 countries including all 28 EU countries indicating grave concern towards green practices. There are many possibilities for reduction of emission such as manufacturing industries may reduce by minimizing waste from used substances and harmful materials and by recycling defective products which are collected from primary market. Environmental Awareness among customers may lead towards use of products having higher greening level and keen to pay high prices. This behavior of customers attracted industries and manufactures to incline to produce green products.

Also, the policy of recollecting, recycling and reuse of products helps in minimizing waste and controlling use of resources which is economical to the industries. Traditionally, processed products from raw materials are dumped into the environment after losing economic value and burdening ecosystem not limited with waste materials but also increases emission level. Sebatjane (2024) develop models for deteriorating items with imperfect quality under circular economic production system and environmental issues. They have considered demand and gross unit profit as a function of deteriorating products' circular index and source of carbon emission are production process, setup activities, storage process and deteriorated products are considered. However, various carbon regulations policies are adopted to control the release of carbon emission in their study. Various demand combinations are used in investigating gross unit profit of business.

Circular economy (CE) is systematic process for production and consumption to push economy and reuse of resources. Linear processing system in supply chain reduces resources and economy of industries with increasing dump by waste. Circular economy helps in increasing money and tackling environmental damage. Based on business structures, Lewandowski (2016) proposed a comprehensive frame to recognize features of the Circular Economy. The principles of Circular economy in a scientific manner with possible challenges of sustainable environment has been demonstrated by Korhonen et al. (2018). De Angelis et al. (2018) have studied drawbacks of linear economy practices and suggested implementation of Circular supply Chain system. In recent years Rabta (2020) first time, studied the impact of product circularity level on the demand structure in an economic order quantity inventory model to retailer's perspective. Further, Md. Khan et al. (2023) investigated the impact of Circularity level on the produced product perspective to manufactures. In their research development, they have studied the manufacturers' best production and circularity index policies along with sustainable environmental policy to control emissions level. In the modern era, customers are more concerned with eco-friendly products and environmental issue while there is also, necessity of minimum utilization of natural resources with maximum utilization along with strategic plans of reducing emissions, pollution and waste produced during manufacturing. While considering implementation of circular economy index policy in production policy in modelling, decay of products cannot be ignored. As decay is a natural phenomenon which happens with all kinds of raw materials and finished products ready for use. Deterioration means loss of originality in a freshly produced product after a certain period or at instant when it enters business such as fruits, vegetables, bakery products and many more eatables' products.

Economy and environment are both major concerns of researchers with a global perspective and need to be incorporated while studying these issues in supply chain management, at this stage, research questions arise: How does Circular Economy Index affect the total profit of manufacture along with Customer Environmental Awareness? And what is the impact on gross unit profit of manufacture when customers' demand is jointly dependent on product greenness quality?

The present study focused on manufactures' business policy by incorporating the best production, circularity index and customers' environmental awareness policies along with sustainable production system for deteriorating products. Merely, researchers have studied manufactures' production policies by integrating factors required to be involved for a sustainable environment and health point of view.

Present research work outlined after framed in the following manner: Based on literature review in Section 2; subsection 2.1 highlights research gap and contributions. In section 3, the hypothesis to design the manufacturing system, along with assumptions and notations, a brief description of problems under study have been extended. Section 4 bestowed production procedure for optimization. Section 5 is equipped with an analytical derivation of optimal solutions for incorporated linear demand and linear per unit gross profit along with optimal policies adopted for the proposed production system model with circularity index and customers' awareness for greenness level of products. Numerical examples are illustrated to validate models developed in Section 6 and Section 7 consist of graphical representations showing concavity of gross unit profit function for both models and variation of gross unit profit function and other function depending on different parameters. Behavioral impact of parameters on the proposed manufacturers' optimal policies is described in section 8 along with managerial insights based on manufactures' ecofriendly and green product policies. Finally, outcomes and future extensions are described in the conclusion section marked as section 9.

2. Literature Review

Some recent papers (e.g. Guo, Cheng, and Liu 2020) consider green quality as a decision variable, none of them investigate the mutually effects of green quality enhancement and Circularity index in presence of Customer Environmental Awareness (CEA) as conflicting decisions. With respect to previous models, the proposed model contributes to an analysis of the channel coordination issue in the presence of consumer environmental awareness; at particular level of green quality value of optimal circularity index and optimal quantity produced are calculated. In the proposed model, the product's green quality influences the market demand; a green quality cost sharing agreement, along with a revenue sharing agreement, is designated to encourage the manufacturer to enhance the greening level and circularity index in order to stimulate demand.

In the present era, the European Commission is looking over the sequel of product level into a Circular Economy practice (EC 2015). To design a transparent circularity, index a considerable number of ventures at the product level have been executed (For instance Franklin-Johnson et al. 2016, Elia et al. 2017, Nunez-Cacho et al. 2018). Saidani et al. 2019 has provided a supportive selection procedure on the reliable metric based on the decision maker requirement by comparing a set of 55 circularity indices. The amount that may be put back in the production system to reuse, is presented by Rabta (2020) based on percentages of circularity index in his developed model. Several researchers have incorporated various sources of carbon emissions in their modelling such as Tiwari et al. (2018) formulated a sustainable integrated production system for imperfect items by considering source of emissions from transporting, storing and deterioration activities. Teleizadeh et al. (2018) developed a sustainable economic production model considering carbon emission to explore its impact on the optimum production policy.

The first time Rabta (2020) embraced the influence of product's circularity level on demand structure in inventory modelling, later on, linear decreasing demand structure in terms of circularity level is studied by Thomas and Mishra (2022) in a two-echelon supply chain model. Different demand patterns with different combinations is studied by Rabta (2020) along with unit gross profit in his model. Thomas and Mishra (2022) investigated impacts of linear, exponential and logistic per unit gross profit functions under linear demand pattern depending on circularity index.

Jaber et al. (2024) presented a review of mathematical modeling (1967-1922) based on Reverse Logistics for Economic Order Quantity (EOQ)/Economic Production Quantity (EPQ). Since many decades ago, natural resources have been used by humans because natural resources available on earth will never vanish. But rapid increase in populations on earth and changing lifestyles resulted in many problems and environmental issues. Finite resources diminishing day by day so the concept of "cyclic ecological system" was followed and later it was taking a boost in 1990s along with reveres logistic (RL). Reverse Logistic is a concept of repair, refurbishing, recycling and reuse of product and disposal activities. Similar concepts are adopted in Circular Economy in the broader sense covering economic, social and environmental sustainability. Jaber et al. (2024) aims to provide a concise review of the mathematical modeling of inventory models for RL. The scope focuses on those works that focus on producing, selling, collecting, and recovering (reusing, repairing, refurbishing, remanufacturing, and recycling). Of course, reviewing the mathematics of all those works will be impossible for one paper to consider. So, they focus on those works that used the economic order/production quantity (EOQ/EPQ) model as it has been the edifice and the first scientific approach reported in inventory theory and management and continue to be celebrated by many researchers (Andriolo et al.2014, Bushuev et al. 2015).

Soleymanfar et al. (2021) investigated sustainable lot-sizing/pricing problem in respect of retailer and supplier in two echelon supply systems. They focused on various social, economic and environmental issues. Thenarasu et al. (2025) developed inventory models considering closed loop supply chain for electronic manufacturers. They found that two policies Fixed Order quantity and Closed loop supply chain. Increases profitability of supply chain as compared to other models and aimed to make manufacturing industries profitable under recovery processes.

Daniel Heged and Dora Longauer (2023) investigated the implementation of circular supply chain models by using reusable components in multiple product generations. They focused on reusability of components and raw materials in manufacturing industries. Authors observed from study that the company's production rate and set-up costs influence the company's optimal strategy, while the inventory cost has a smaller influence on the optimal strategy. Based on our results, in addition to pure strategies, a mixed strategy is also a possible optimal strategy for the company. As a result of the company reusing previously manufactured components, the amount of generated waste is reduced, and the extraction of new raw materials and energy consumption are also reduced. Rung-Hung Su (2023) have investigated Optimal Circular Economy and Process Maintenance Strategies for an Imperfect Production–Inventory Model with Scrap Return. Heydari et al. (2021) investigated the impact of green quality products under customer environmental awareness and developed models for balancing price and greenness of product in the market. They have discussed different coordination policies for profit optimization. Demand is considered to be a function of selling price and green quality of product for retailers' and manufacturers' business perspective.

For long decay, emission reduction is other major concerns of researchers in the field of sustainable environment in supply chain management (Govindan et al., 2020; Govindan et al., 2020a). Compared to the above reviewed studies, this study contributes to the literature by providing an analytical approach to address environmental issues by producing green products under circular economic conditions and in the presence of CEA and the ability of the manufacturer to enhance, with investments, the product's green quality. The closely related research is represented in Table 1.

2.1 Research Gap and Our contribution

Circular Economy is the most demanded area of research for the last few decayed as to maximum utilization of natural resources and minimum production of waste. Researchers have focused on remanufacturing products using waste materials collected back from the market. Deterioration is one of the natural phenomena that occurs during production and storage of inventory and carbon emissions releases form various activities involved in inventory management system. Researchers have developed inventory models EOQ and EQP models incorporating circular economy indicators that indicate how products are utilized in remanufacturing based on circularity index. To the best of knowledge of the authors and form comparative Table-1, authors concluded that no one has developed EQP model in combination with circularity indicator and greening level of product for deteriorating products. In the present era of global warming and climate change, customers are attracted towards the use of green products that minimize the amount of carbon emissions responsible for global warming and impact human lives. The present work has introduced an economic production model incorporating economic circularity index and greenness of product for deteriorating products. This study analyzes the impact of circularity index and greening level of product on gross unit profit of manufacturers and reduction of carbon emission.

3. Assumptions and Notations

3.1 Assumptions

The various hypotheses based on which model is proposed are described as follows:

1) An infinite time horizon is adopted by manufacturers for production system.
2) Single green products are considered for production and selling.
3) Manufacturers produce products and sell them directly in the market.
4) Produced products are stocked and hence there is a holding charge incurred by manufacturers for providing an essential environment so that the rate of deterioration be minimum. Holding charge is independent of circularity and greenness of product.
5) Considered Product is of deteriorating nature and deterioration starts instant when product enters the business hence constant rate of deterioration is considered.
6) Manufacturers produce and supply products as per customer requirements and hence consider them to be deterministic therefore shortages are not considered.
7) Demand of customers linearly depends on the greening level (ability of emitting low carbon) due to Customer Environmental Awareness and circularity index of product. Symbolically demand is defined as $D(\varphi\ \omega) = d + a\,\varphi + \tau\,\omega$ where a and $\tau > 0$ are constant influencing demand and d is constant demand of customers.
8) Circularity level and greenness of product directly affect the market demand and thus gros unit profit of manufacturers is also affected and thus is considered as dependent on both factors and is symbolically defined as $\Omega\,(\varphi\ \omega) = (p + b\,\varphi + c\,\omega)$ where $p, c > 0$ and $b < 0$ are constant influencing gross unit profit and p is constant profit of manufacturer.
9) Customers Environmental Awareness influenced demand depending on greening level of product and manufacturers invest in producing green product with greening cost (Heydari et al. 2021)
10) For each production a setup cost is incurred and is not included in the holding cost.
11) Since every activity involved in manufacturing and storing products uses various kinds of energy and product deterioration releases different kinds of hazardous gases and thus carbon emission forms these activities are taken into consideration and to control its amount government imposes charges in the form of carbon tax which is borne by manufacturers.
12) The sensitivity of customers to circular index is unknown and is assured to be measured. Furthermore, the CEA level of customer's community is to be measured previously using survey approaches (e.g. see Jafari, Heydari, and Keramati 2017).

3.2 Notations

The following symbols are used throughout the model development to articulate the production system:

S	Fixed setup cost for each production activity (In $/Setup)
χ	Total Quantity of products produced in one setup (Units)
T	Business length of manufacture (Unit of time)
P	Production rate of manufacturer (Units/unit of time)
h	Holding cost (In $/unit/unit of time)
θ	Rate of deterioration (Per unit/unit of time)
π	Deterioration cost (In $ per unit)
δ	Carbon emissions from storage operations during complete business length (ton/unit of time)
σ	Carbon emissions from deteriorated products during complete business length (ton/unit of time)
ψ	Carbon emission from setup operations of the production system during each business length (ton/setup)
ρ	Carbon emissions due to production (ton/unit)
t	Carbon tax imposed by government agency ($/ton)
φ	Circularity index of product such that $\varphi \in [0\ \ 1]$
ω	Greenness level of product (quality having low emitting carbon) such that $\omega \in [0\ \ 1]$
g	Coefficient influencing greening cost in case of demand function (positive constant)
a	Scaling parameter to circularity index in case of demand function (Positive constant)
b	Scaling parameter to circularity index in case of unit gross profit function (Negative constant)
c	Customer environmental awareness level (CEA)
$D(\varphi\ \omega)$	Customers' demand per business length (Units)
$\Psi\,(\chi\ \varphi\ \omega)$	Gross unit profit per business (In $)
$I(t)$	Inventory level at any time throughout business length (units)
Decision variables	
χ	Quantity required to be produced (Unit)
φ	Circularity index of product
ω	Greening level of product

Development of Economic Production Quantity Model (EPQ) for Deteriorating Products with the Circularity Index and Green Level of Products

In this section, an EPQ model for manufacturer's gross unit profit under stated assumptions for single deteriorating product is formulated under circularity index. Under linear demand assumption depending upon circularity index and greening level of product manufacturer produces product and then supply directly to the customers. Manufacturers produce and sell χ unit of products with per unit gross profit per business cycle. The Gross profit from χ unit is given by

$$\Psi\,(\chi\ \varphi\ \omega) * \chi$$

Other associated costs in production and supply system are described as

Setup cost$=\frac{D(\varphi\ \omega)}{\chi}$

Products holding cost$=\frac{1}{2}\,h\,\chi\left(1-\frac{D(\varphi\ \omega)}{P}\right)T$

Deterioration cost$=\pi\,\theta\,\frac{1}{2}\,\chi\left(1-\frac{D(\varphi\ \omega)}{P}\right)T$

The manufacturer bears carbon tax imposed by governmental authority on the carbon emission produced during inventory management system and is given as

$$t\left\{\psi\,\frac{D(\varphi\ \omega)}{\chi}+\rho\,D(\varphi\ \omega)T\right\}+\ \chi\,t\,(\delta+\sigma)\,\left(1-\frac{D(\varphi\ \omega)}{P}\right)T$$

Due to Customer Environmental Awareness and customers demand influenced by the greening level of product and thus manufacturers invest some cost in manufacturing green product and hence bears a greening cost. Greening cost beard by manufacturer is

$$g\,\omega^2$$

Finally, total manufacturer's profit per unit of time is given by

$$\Psi\,(\chi\ \varphi\ \omega\ ) = \ D(\varphi\ \omega)\ \Omega\,(\varphi\ \omega) - \frac{S\ D(\varphi\ \omega)}{\chi} - \frac{1}{2}\ h\ \ \chi\ \left(1 - \frac{D(\varphi\ \omega)}{P}\right) - \pi\ \theta\ \frac{1}{2}\ \chi\left(1 - \frac{D(\varphi\ \omega)}{P}\right) - g\ \omega^2 - t\ \left\{\psi \frac{D(\varphi\ \omega)}{\chi} + \rho\ D(\varphi\ \omega)\right\} - \ \chi\ \ t\ (\delta + \sigma)\ \left(1 - \frac{D(\varphi\ \omega)}{P}\right) \qquad (1)$$

Now, the objective of the model is to determine maximum profit for manufacturer based on the optimal values of decision variables, therefore, to find optimal value of decision variables φ^* and χ^*, we differentiate equation (1) with respect to φ and χ and equating to zero. Consequently, following maximizing problems comes into existence

Max $\Psi\,(\chi\ \varphi\ \omega\ )$: $(\chi\ \varphi\ \omega\ ) \mathcal{E}\ Z$ (2)

Table 1: Representing comparative study for assessment of related research work under circular economy indicator

Author(s)	Type of model	Circular Economy Indicator	Damand			Per unit gross profit		Source of Carbon emission				Deterioration rate	Objective Function as
			Constant	Linear in	Non-linear in	Constant	Depend on	Setup/ ordering	Storage	Production	Deterioration		
Tiwari et al. (2018)	SC	✓	✓	✓	✓	✓	Price	✓	✓	✓	✓	✓	Cost
Wang et al. (2018)	EPQ	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	Profit
Taleizadeh et al.(2018)	EPQ	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	Profit
Zhang et al. (2019)	SC	✓	✓	P & EQ	✓	✓	Price	✓	✓	✓	✓	✓	Profit
Wang et al. (2019)	EOQ	✓	✓	P & GL	✓	✓	Price	✓	✓	✓	✓	✓	Profit
Rabta (2020)	EOQ	✓		CI	C I (L & L Forms)	✓	CI (Linear, E & L forms)	✓	✓	✓	✓	✓	Profit
Heydari et al. (2021)	SC	✓	✓	P&GL	✓	✓	Price	✓	✓	✓	✓	✓	Profit
Mana et al. (2021)	EPQ	✓	✓	WP&CE	✓	✓	✓	✓	✓	✓	✓	✓	Profit
Yadav et al. (2021)	EOQ	✓	✓	CE	✓	✓	✓	✓	✓	✓	✓	✓	Cost
		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	
Thomas & Mishra (2022)	SC	✓	✓	CI	✓	✓	CI (Linear, E & L forms)	✓	✓	✓	✓	✓	Profit
Wani &Mishra (2022)		✓	✓	✓	✓	✓	✓	✓	✓		✓	constant	
Khan et al. (2023)	EPQ	✓	✓	CI	C I (L & L Forms)	✓	CI (Linear & L forms)	Setup	✓	✓	✓	✓	Profit
John & Mishra (2023a)		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
John & Mishra (2023b)		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Sebatjane M. (2024)	EPQ	✓	✓	✓	✓	✓	✓	Ordering/ Setup	✓	✓	✓	✓	Profit
Present work	EPQ	✓	✓	✓	✓	✓	✓	Setup	✓	✓	✓	✓	Profit

Description: EPQ-Economic Production Quantity Model, EOQ-Economic Ordered Quantity Model, SC-Supply Chain, P &EQ- Price & Environmental Quality, P & GL-Price & Green level, C I (L & L Forms) – Circularity index (Logistic & Logarithmic form), such that $Z \equiv \{\Psi\,(\chi\ \varphi\ \omega\ ): 0 < \chi < \infty\ and\ \ 0 \le \varphi \le 1\ with\ fixed\ \omega: 0 < \omega < 1\}$

Profit function

With Greening Level of Product

To account for the effect of circularity index and greening level of product on demand as well as selling prices and costs, authors will proceed with the profit maximization instead of cost minimization as in the EPQ model. If authors consider no impact of circularity index and greening level of product on demand and profit function (e.g. $D(\varphi\ \omega) = d\ \Omega\ (\varphi\ \omega) = p$) then the demand and unit gross profit functions are constant, and this model coincides with classical EPQ. If we include the impact of circularity index and greening level of product, then the following cases are distinguishing:

Case-1: If we consider, both demand and per unit gross profit are increasing (or one is increasing and other is constant) with respect to circularity index (At a constant value of greening level) then the optimal solution of eq. (2) is $\varphi^* = 1$ and $\chi^* = \sqrt{\frac{2\ D(1\ \omega)\ (S + t\ \psi)}{\left(1 - \frac{D(1\ \omega)}{P}\right)(h + \pi\ \theta) + t(\delta + \sigma\theta)}}$

Case-2: If we consider, both demand and unit per gross profit are decreasing (or one is decreasing and other is constant) with respect to circularity index (At a constant value of greening level) then the optimal solution of eq. (2) is $\varphi^* = 0$ and $\chi^* = \sqrt{\frac{2\,D(0\,\omega)\,(S+t\,\psi)}{\left(1-\frac{D(0\,\omega)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}}$

Case-3: If we consider, one of the demands and per unit gross profit is increasing and the other is decreasing with respect to circularity index (At a constant value of greening level) then the optimal solution of eq. (2) might be different as compared to Case-1 and Case-2. In the present scenario the optimal solution is determined with the help of Lagrange's Multiplier Technique (LMT). The Lagrange's function for the problem developed in eq (2) is defined as follows:

$$Ұ\,(\chi,\varphi,\omega,\alpha_1,\alpha_2) = D(\varphi\,\omega)\,\Omega\,(\varphi\,\omega) - \frac{D(\varphi\,\omega)\,S}{\chi} - \frac{1}{2}\,h\,\chi\left(1-\frac{D(\varphi\,\omega)}{P}\right) - \pi\,\theta\,\frac{1}{2}\,\chi\left(1-\frac{D(\varphi\,\omega)}{P}\right) - g\,\omega^2 - t\left\{\tau\frac{D(\varphi\,\omega)}{P} + \rho\,D(\varphi\,\omega)\right\} - \chi\,t\,(\delta+\sigma)\left(1-\frac{D(\varphi\,\omega)}{P}\right) - \alpha_1(1-\varphi) + \alpha_2\varphi; \quad (3)$$

Without Greening level of Product

Case-1: If we consider, both demand and per unit gross profit are increasing (or one is increasing and other is constant) with respect to circularity index then the optimal solution of eq. (2) is $\varphi^* = 1$ and

$$\chi^* = \sqrt{\frac{2\,D(1)\,(S+t\,\psi)}{\left(1-\frac{D(1)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}} \quad (4)$$

Case-2: If we consider, both demand and unit per gross profit are decreasing (or one is decreasing and other is constant) with respect to circularity index (At a constant value of greening level) then the optimal solution of eq. (2)

$$\text{is } \varphi^* = 0 \text{ and } \chi^* = \sqrt{\frac{2\,D(0)\,(S+t\,\psi)}{\left(1-\frac{D(0)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}} \quad (5)$$

Case-3: If we consider, one of the demands and per unit gross profit is increasing and the other is decreasing with respect to circularity index (At a constant value of greening level) then the optimal solution of eq. (2) might be different as compared to Case-1 and Case-2. In the present scenario the optimal solution is determined with the help of Lagrange's Multiplier Technique (LMT). The Lagrange's function for the problem developed in eq (2) is defined as follows:

$$Ұ\,(\chi,\varphi,\alpha_1,\alpha_2) = D(\varphi)\,\Omega\,(\varphi) - \frac{D(\varphi)\,S}{\chi} - \frac{1}{2}\,h\,\chi\left(1-\frac{D(\varphi)}{P}\right) - \pi\,\theta\,\frac{1}{2}\,\chi\left(1-\frac{D(\varphi)}{P}\right) - t\left\{\tau\frac{D(\varphi)}{P} + \rho\,D(\varphi)\right\} - \chi\,t\,(\delta+\sigma)\left(1-\frac{D(\varphi)}{P}\right) - \alpha_1(1-\varphi) + \alpha_2\varphi; \quad (6)$$

4. Optimality Conditions

For an optimal solution, the necessary conditions under Karush Kuhn Tucker (KKT) method are given as

$$\frac{\partial\,Ұ\,(\chi,\varphi,\omega,\alpha_1,\alpha_2)}{\partial\chi} = \frac{D(\varphi\,\omega)\,(S+t\,\psi)}{\chi^2} - \frac{1}{2}\left((h+\pi\,\theta)+t(\delta+\sigma\theta)\right)\left(1-\frac{D(\varphi\,\omega)}{P}\right) = 0; \quad (7)$$

and

$$\frac{\partial\,Ұ\,(\chi,\varphi,\omega,\alpha_1,\alpha_2)}{\partial\varphi} = \frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\Omega\,(\varphi\,\omega) + \frac{\partial\,\Omega\,(\varphi\,\omega)}{\partial\,\varphi}D(\varphi\,\omega) - t\,\rho\,\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi} + \left\{\frac{((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P} - \frac{(S+t\,\psi)}{\chi}\right\}\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi} - \alpha_1 + \alpha_2 = 0; \quad (8)$$

$\alpha_1(1-\varphi) = 0;$ (9)

$\alpha_2\varphi = 0;$ (10)

and $\alpha_1, \alpha_2 \geq 0;$ (11)

Conditions given in equation (9) and (10) provide the following three candidate solutions of the optimization problem (2)

1) $\alpha_1 = 0$ and $\varphi = 0$. Then $\chi^* = \sqrt{\frac{2\,D(0\,\omega)\,(S+t\,\psi)}{\left(1-\frac{D(0\,\omega)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}}$ (14) and

$$\alpha_2 = t\,\rho\,\frac{\partial\,D(0\,\omega)}{\partial\,\varphi} + \left\{\frac{(S+t\,\psi)}{\chi} - \frac{((h+\pi\,\theta)+t(\delta+\sigma\,\theta))\chi}{2\,P} - \Omega\,(0\,\omega)\right\}\frac{\partial\,D(0\,\omega)}{\partial\,\varphi} - D(0\,\omega)\frac{\partial\,\Omega\,(0\,\omega)}{\partial\,\varphi} \quad (15)$$

2) $\alpha_2 = 0$ and $\varphi = 1$. Then $\chi^* = \sqrt{\frac{2\,D(1\,\omega)\,(S+t\,\psi)}{\left(1-\frac{D(1\,\omega)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}}$ (16) and

$$\alpha_1 = \frac{\partial\,D(1\,\omega)}{\partial\,\varphi}\,\Omega\,(1\,\omega) + \frac{\partial\,\Omega\,(1\,\omega)}{\partial\,\varphi}D(1\,\omega) - t\,\rho\,\frac{\partial\,D(1\,\omega)}{\partial\,\varphi} + \left\{\frac{((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P} - \frac{(S+t\,\psi)}{\chi}\right\}\frac{\partial\,D(1\,\omega)}{\partial\,\varphi} \quad (17)$$

When $\alpha_1 = 0$ and $\alpha_2 = 0$ Therefore, from equation (6) we can find

$$\chi^* = \sqrt{\frac{2\,D(\varphi\,\omega)\,(S+t\,\psi)}{\left(1-\frac{D(\varphi\,\omega)}{P}\right)(h+\pi\,\theta)+t(\delta+\sigma\theta)}} \quad (18)$$

and thereafter from equation (7) we can find

$$\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\Omega\,(\varphi\,\omega) + \frac{\partial\,\Omega\,(\varphi\,\omega)}{\partial\,\varphi}D(\varphi\,\omega) - t\,\rho\,\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi} + \left\{\frac{((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P} - \frac{(S+t\,\psi)}{\chi}\right\}\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi} = 0$$

After rearranging the above equation, we rewrite the same as

$$\left(\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\Omega\,(\varphi\,\omega) + \frac{\partial\,\Omega\,(\varphi\,\omega)}{\partial\,\varphi}D(\varphi\,\omega)\right)^2 = \frac{\left(\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\right)^2(S+t\,\psi)((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2P}\left\{\frac{D(\varphi\,\omega)}{P-D(\varphi\,\omega)} + \frac{P-D(\varphi\,\omega)}{D(\varphi\,\omega)} - 2\right\} \quad (19)$$

Now, expressing equation (19) as $\Phi\,(\varphi) = 0$ with

$$\Phi(\varphi) = \left(\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\Omega\,(\varphi\,\omega) + \frac{\partial\,\Omega\,(\varphi\,\omega)}{\partial\,\varphi}D(\varphi\,\omega)\right)^2 - \frac{\left(\frac{\partial\,D(\varphi\,\omega)}{\partial\,\varphi}\right)^2(S+t\,\psi)((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2P}\left\{\frac{D(\varphi\,\omega)}{P-D(\varphi\,\omega)} + \frac{P-D(\varphi\,\omega)}{D(\varphi\,\omega)} - 2\right\} \quad (20)$$

Now to find optimal and feasible candidate solution equation (20) can be expressed as $\Phi(\varphi) = 0$ (21)

Equation (21) can be solved using various numerical techniques for the circular index φ. Since the above equation is non-linear. Therefore, more than one feasible solution in interval [0 1] may be found. All the contender solutions must maintain all the necessary feasibility conditions defined as $0 < \chi < \infty$ *and* $0 \le \varphi \le 1$ with fixed ω: $0 < \omega < 1$ along with condition defined in equation (8). KKT conditions ensure an optimal solution when the manufacturer's profit $\Psi(\chi\,\varphi\,\omega)$ is concave. The concavity of the $\Psi(\chi\,\varphi\,\omega)$ is investigated for the case considered as linear demand and linear unit gross profit and is mathematically discussed in Appendix-A.

5. Solution Algorithm and Numerical Examples

To find an optimal and feasible solution the following steps are involved for two models developed in section 4.0.

5.1 Solution Algorithm

Case-1: First, input all parameters value. Calculate χ^* using equation (14) and value of $\alpha_2 \neq 0$ using equation (15) at different greening level or product and then input obtained values of decision variables in equation (3) yielding profit value.

Case-2: First, input all parameters value. Calculate χ^* using equation (16) and value of $\alpha_1 \neq 0$ using equation (17) at different greening level or product and then input obtained values of decision variables in equation (3) yielding profit value.

Case-3: First, input all parameters value. At $\alpha_1 = 0$ & $\alpha_2 = 0$ Calculate φ & χ using equation (21) at different greening level of product and then input obtained values of decision variables in equation (3) yielding profit value.

Similarly, we can calculate optimal and feasible values of candidate solutions for the three

Cases of models without greening products are described through equations (4), (5) & (6).

5.2 Numerical Examples

Numerical Example-1: Fixed setup cost S=180; $h = 0.2$; $P = 69000$; a=8500; b=0.25; g=0.001; $\sigma = 0.25$; d=60000; $\theta = 0.15$; $\rho = 0.03$; $\psi = 0.13$; t=1.5; c=0.10; p=2; $\tau = 0.15$; $\pi = 0.3$; $\delta = 0.21$.

Table 2: Representing candidates' solutions for numerical example-1 at different greening levels of product

Case-1				
When $\varphi = 0$; $\alpha_1 = 0$ & $\alpha_2 \neq 0$				
ω	α_2^*	χ^*	$\Psi^*(\chi\,\varphi\,\omega)$	Carbon Emission (CE)
0.1	16401.6	-1.39424*10^7	116185	2329.96
0.2	16401.7	-1.39424*10^7	116785	2329.96
0.3	16401.7	-1.39424*10^7	117385	2329.96
0.4	16401.7	-1.39424*10^7	117985	2329.96
0.5	16401.7	-1.39424*10^7	118585	2329.96
0.6	16401.7	-1.39425*10^7	119185	2329.96
0.7	16401.7	-1.39425*10^7	119785	2329.96
0.8	16401.7	-1.39425*10^7	120385	2329.96
0.9	16401.8	-1.39425*10^7	120985	2329.95

Table 3: Representing candidates' solutions for numerical example 1 at different greening levels of product

Case-2				
When $\varphi = 1$; $\alpha_2 = 0$ & $\alpha_1 \neq 0$				
ω	α_1^*	χ^*	$\Psi^*(\chi\,\varphi\,\omega)$	Carbon Emission (CE)
0.1	254.15	74353.1	117045	2188.47
0.2	339.189	74354.3	117731	2188.47
0.3	424.29	74355.4	118416	2188.46
0.4	509.268	74357.5	119101	2188.46
0.5	594.307	74357.6	119786	2188.46
0.6	679.346	74358.8	120471	2188.46
0.7	764.386	74359.9	121156	2188.46
0.8	849.425	74361.0	121848	2188.46
0.9	934.464	74362.1	122526	2188.45
1.0	254.15	74353.1	117045	2188.47

Table 4: Representing candidates' solutions for numerical example 1 at different greening levels of product

Case-3				
When $\varphi \neq 0$; $\alpha_1 = 0$ & $\alpha_2 = 0$				
$\omega = 0.1$	φ^*	χ^*	$\Psi^*(\chi\,\varphi\,\omega)$	Carbon Emission (CE)
	0.249041	19082.9	116666	2335.08
	0.626262	26775.3	116989	2313.14
	0.984225	65960.3	117031	2201.30
$\omega = 0.2$				
	0.266711	19317.9	117314	2334.97
	0.655858	27794.5	117652	2309.03
	0.976957	62939.0	117709	2206.36
$\omega = 0.3$				
	0.284319	19559.7	117964	2334.80
	0.687164	29000.2	118319	2304.14
	0.968041	59733.4	118386	2212.33
$\omega = 0.4$				
	0.301863	19808.8	118616	2334.55
	0.721041	30486.3	118990	2298.11
	0.956619	56257.0	119060	2219.39
$\omega = 0.5$				
	0.319339	20065.0	119270	2334.22
	0.759375	32458.7	119666	2290.25
	0.940810	52302.0	119731	2228.25
$\omega = 0.6$				
	0.336743	20329.6	119926	2333.84
	0.808579	35618.5	120351	2278.18
	0.914199	47167.1	120395	2241.28
$\omega = 0.7$				
	0.354074	20602.2	120583	2333.36
$\omega = 0.8$				
	0.371326	20883.4	121242	2332.82
$\omega = 0.9$				
	0.388496	21173.6	121903	2332.19

When no green products are considered

Numerical Example-2: Fixed setup cost S=180; $h = 0.2$; $P = 69000$;a=8500; b=0.25; g=0.001; $\sigma = 0.25$;d=60000; $\theta = 0.15$; $\rho = 0.03$; $\psi = 0.13$; t=1.5; c=0.10; p=2; $\tau = 0$; $\pi = 0.3$; $\delta = 0.21$.

Table 5: Representing candidates' solutions for numerical example 2 at different greening level of product

Case-1				
When $\varphi = 0$; $\alpha_1 = 0$ & $\alpha_2 \neq 0$				
ω	α_2^*	χ^*	$\Psi^*(\chi\,\varphi\,\omega)$	Carbon Emission (CE)
0	16401.6	-1.39430*10^7	119079	2329.96

Table 6: Representing candidates' solutions for numerical example 2 at different greening level of product

Case- 2				
When $\varphi = 1;\ \alpha_2 = 0\ \&\ \alpha_1 \neq 0$				
ω	α_1^*	χ^*	Ψ^* $(\chi\ \varphi\ \omega\ )$	Carbon Emission (CE)
0	169.111	74352.0	116360	2188.47

Table 7: Representing candidates' solutions for numerical example 2 at different greening level of product

Case-3				
When $\varphi \neq 0;\ \alpha_1 = 0\ \&\ \alpha_2 = 0$				
$\omega = 0$	φ^*	χ^*	Ψ^* $(\chi\ \varphi\ \omega\ )$	Carbon Emission (CE)
	0.231311	18854.4	116019	2335.11
	0.597900	25890.4	116329	2316.63
	0.903170	68855.9	116352	2196.48

6. Graphical representation of models

6.1 3D Graph representing Profit function with Greening Lev product and Circularity index

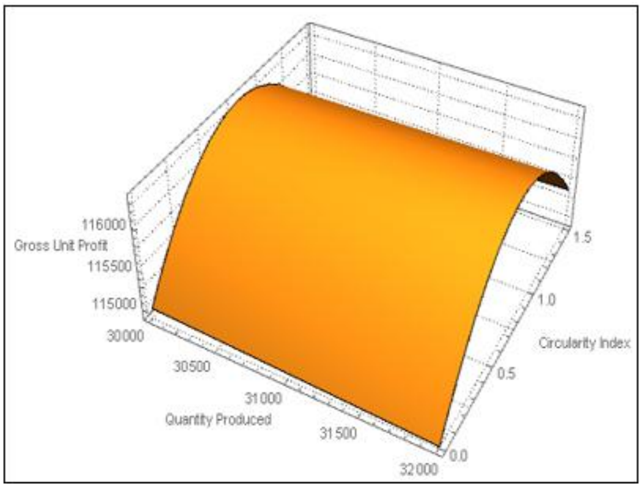

Figure 1: Circularity Index and Product Quantity Produced Vs Profit Function

6.2 2D Graph representing different functions and other functions based on different parameter

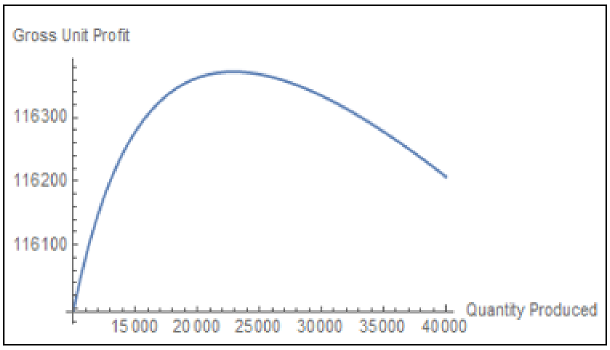

Figure 2: Product Quantity Produced Vs Profit Function (At constant Greening Level and Circularity Index)

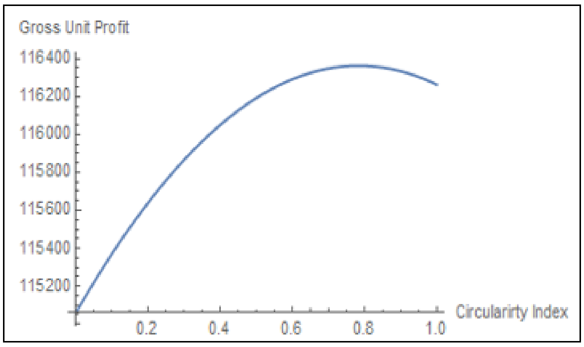

Figure 3: Circularity Index Vs Profit Function (At constant Greening Level and Product Quantity Produced)

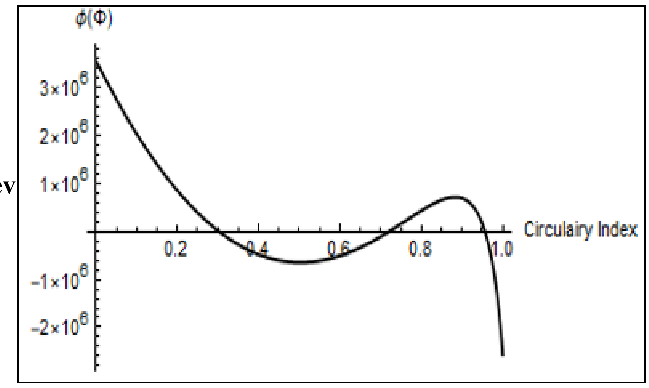

Figure 4: Behavior of $\Phi(\varphi)$ when $0 < \varphi < 1$

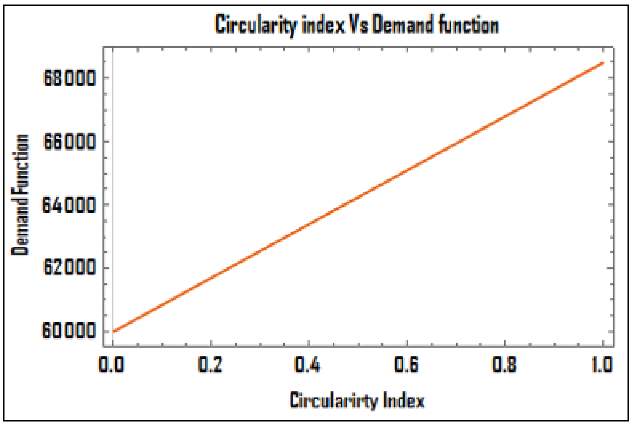

Figure 5: Circularity Index Vs Demand Function

6.3 3D Graph representing Profit function without Greening Level of product and with Circularity index

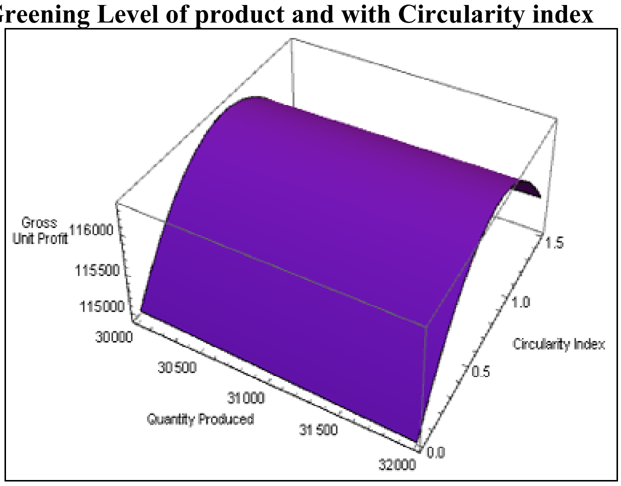

Figure 6: Circularity Index and Product Quantity Produced Vs Profit Function

6.4 2D Graph representing different functions and other functions based on different parameters

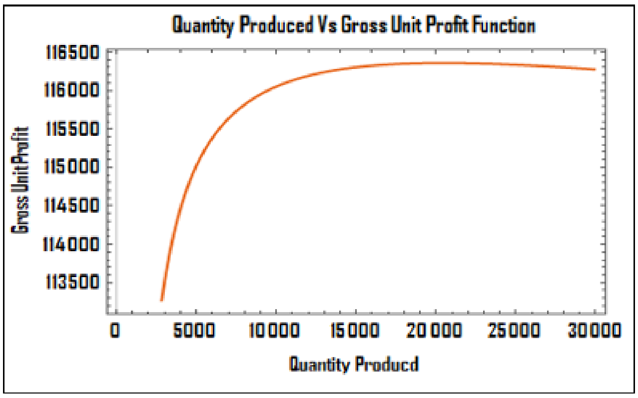

Figure 7: Product Quantity Produced Vs Profit Function

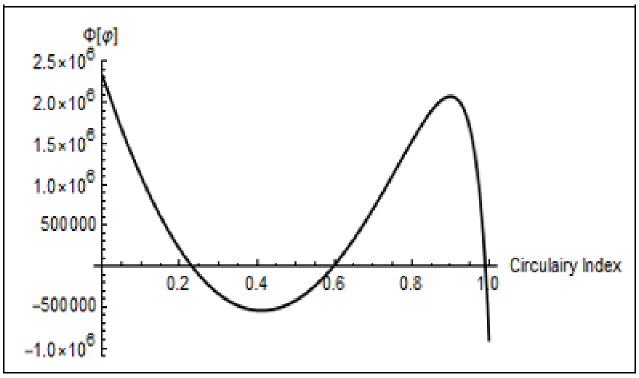

Figure 8: Behavior of $\Phi(\varphi)$ when $0 < \varphi < 1$

7. Observations and Managerial Insights

7.1Observations

Demand is a linear function of circularity index with additional demand a and greening level ω which increases depending upon the circularity index of the product and unit gross profit is function of both with unit premium factor $-b$. The optimal candidate solutions in two cases of models, one with greening level of product and other without greening level of product are discussed here: -

With greening level of product and circularity index

1) From Table-4, the optimal candidate solution and gross unit profit of manufacturer is $\varphi^* = 0.759375$ $\chi^* = 32458.7$ unit with annual profit $\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$119666$ and carbon emission $= 2290.25$/year at greening level of product $\omega = 0.5$.

The comparison with two trivial candidate solutions given in Table-2 and Table-3:
$\varphi^* = 0$ and $\chi^* = 16401.7$ unit with annual profit $\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$118585$ and Carbon emission= 2329.96 kg/year at greening level of product $\omega = 0.5$.
$\varphi^* = 1$ and $\chi^* = 74357.6$unit with annual profit
$\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$119786$ and Carbon emission=
2188.46 kg/year at greening level of product $\omega = 0.5$.

Without greening level of product and circularity index

2) From Table-7, the optimal candidate solution and gross unit profit of manufacturer is $\varphi^* = 0.597900$ and $\chi^* = 25890.4$. unit with annual profit $\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$116329$ and carbon emission $= 2316.63$ kg/year.

The comparison with two trivial candidate solutions given in Table-5 and Table-6:
$\varphi^* = 0$ and $\chi^* = 16401.6$ unit with annual profit $\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$119079$ and Carbon emission= 2329.96 kg/year.
$\varphi^* = 1$ and $\chi^* = 74352.0$ unit with annual profit
$\text{₽}^* (\chi\ \varphi\ \omega\ ) = \$116360$ and Carbon emission= 2188.47 kg/year.

In case without greening the level of product manufacturers' profit is lower as compared to model with greening level of products due to decrease in the cutomers' demand.

From above observations, authors conclude that gross unit profit is maximum at feasible optimal solution when $\varphi^* = 0.721041$ $\chi^* = 30486.3$ unit with carbon emission= 2290.25 kg/year as compared to feasible solutions to all other trivial cases and model without greening level of products.

Trend for the increase in the gross unit profit is seen from Table-2, Table-3 and Table-4, as the greening level increases, the value of produced carbon emissions decreases when optimally feasible candidate solution is considered depending upon the greening level of products. After certain value of circularity index there is only a single feasible candidate solution obtained with lower profit and high carbon emissions.

From Figure-1 & Figure-6, it is observed that models are concave with respect to quantity produced and circularity index at fixed greening level of product for both models.

Figure-2 & Figure-7 shows that as quantity produced increases; gross unit profit decreases due to an increase in the cost component of the production system.

Figure 3 shows that after a certain level of circularity index at a fixed greening level of product, gross unit profit starts decreasing.

Behavior of $\Phi(\varphi)$ when $0 < \varphi < 1$ is shown in Figure-4 & Figure-8 and Figurer-5 shows that as when circularity index of product increases, demand increases linearly as assumed.

7.2 Managerial Insights

The present research work reveals that the model developed is more beneficial to the managers involved in the inventory management system. As compared to other models incorporating circular economy index this model has incorporated production of green products emitting less carbon as compared to model without green product consideration. Managers may make decisions in production of products with required level of green quality of a product in combination with minimum investment in producing green products emitting low carbon emission with maximum profit. The impact of circular economic practices can be studied by the managers with the customers' response to circularity index and green level of product that helps them in deciding production of product quality. It is not possible to make products that are completely circular ($\varphi = 1$) and completely green ($\omega = 1$) due to technological constraints therefore, managers may impose an upper bound in for both constraints while deciding for production. Here it is possible to simulate the adoption of circular and green products at a given target circularity and greening level. Additionally, managers can fix a limit on circularity indicator and greening level to prevent company from operating at a lower level. If base customers' demand declines the managers may advise the manufacturer to shrink total number of productions for each cycle. In addition, due to lower customers' demand, total carrying cost of company increases and profit decreases, hence manager may try to increase demand by improving circularity index and greening level of product to increase profit. Managers can plan to reduce carbon emissions by adjusting production along with greening level of product and circularity index for better and sustainable environment.

8. Conclusion and Future Extensions

In this paper, an Economic Production Quantity inventory model with circular economy indicator and greening level of product is presented. It is assumed that product can be manufactured with a variable level of circularity (measured by an index between zero to one) and greening level of product (measured by index between zero to one) that has impact on the demand, gross unit profit. In addition to demand and gross unit profit circularity and greening level index has also an impact on the quantity produced to fulfil demand of the market. Besides the simplistic nature of EPQ models, important insights can be derived from them along with effective and transparent measures to motivate, monitor and manage the transition towards circularity and greening level index of product. The present study has considered two models: one demand and gross unit profit depending upon circularity and greening level of product and other only depending upon circularity index. Optimal parameters are determined analytically, and numerical examples are provided for two different models. Presently a linear combination of model with demand and gross unit profit has been developed. Models are solved using Mathematica-11.3 Software. The gross unit profit for two different models at a fixed greening level of product ($\omega = 0.5$) has been calculated and authors found that model with green product has approx. 2.79% more profit and approx. 0.57% less carbon emissions as compared to model without green product. This indicates that developed model with circularity and greening level index are profitable and environmentally sustainable. Further, manufacturer's gross unit profit increases as greening level of product increases in all three cases of trivial and feasible solutions for model with greening level index. Also, model with optimal and feasible candidate solution has more profit as compared to trivial solutions obtained at circularity index which. The product with circularity index ($\varphi = 1$) has little higher profit but with high carbon emissions and model without economic circular indicator has approx. 0.90% less profit with optimal feasible candidate solutions.

The present work may be extended with different combinations of demand and gross unit functions with non-linear variants such as logarithmic, logistic and exponential. Also, present work may be extended considering various carbon cap policies imposed by governmental agencies to restrict production of carbon emissions for a sustainable environment.

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Appendix-A

Since demand and unit profit function both are linear and are $D(\varphi\ \omega) = d + a\,\varphi + \tau\,\omega$ where $a \text{ and } \tau > 0$ and $\Omega\,(\varphi\ \omega) = (p + b\,\varphi + c\,\omega)$ where $p, c > 0 \text{ and } b < 0$

In this case

$$\frac{\partial^2\, \text{Ᵽ}(\chi\,\varphi\,\omega)}{\partial\,\chi^2} = -2\,\frac{\big((S+t\,\psi)+t(\delta+\sigma\theta)\big)(d+a\,\varphi+\tau\,\omega)}{\chi^3}$$

$$\frac{\partial^2\, \text{Ᵽ}(\chi\,\varphi\,\omega)}{\partial\chi\,\partial\,\varphi} = \frac{\partial^2\, \text{Ᵽ}(\chi\,\varphi\,\omega)}{\partial\,\varphi\partial\chi} = \frac{a\,\big((S+t\,\psi)+t(\delta+\sigma\theta)\big)}{\chi^2} + \qquad \frac{a\,\big((h+\pi\,\theta)+t(\delta+\sigma\theta)\big)}{2\,P}$$

And $\frac{\partial^2\, \text{Ᵽ}(\chi\,\varphi\,\omega)}{\partial\,\varphi^2} = 2\,d\,p < 0$

This indicated that the Hassian Matrix of $\text{Ᵽ}\,(\chi\,\varphi\,\omega)$ is given by

$$\text{Ꜧ} = \begin{pmatrix} \text{Ꜧ}_{11} & \text{Ꜧ}_{12} \\ \text{Ꜧ}_{21} & \text{Ꜧ}_{22} \end{pmatrix}$$

$Ꞙ_{11} = -2\, \frac{((S+t\,\psi)+t(\delta+\sigma\theta))(d+a\,\varphi+\tau\,\omega\,)}{\chi^3}$

$Ꞙ_{12} = Ꞙ_{21} = \frac{a\,((S+t\,\psi)+t(\delta+\sigma\theta))}{\chi^2} + \frac{a\,((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P}$

$Ꞙ_{22} = 2\,d\,p$

To ensure concavity of the Ꝕ $(\chi\,\varphi\,\omega\,)$ function the Hessian matrix must be Negative Semi definite. Moreover,

$$\text{Det}(Ꞙ) = 4ab\,\frac{((S+t\,\psi)+t(\delta+\sigma\theta))(d+a\,\varphi+\tau\,\omega\,)}{\chi^3} - \left(\frac{a\,((S+t\,\psi)+t(\delta+\sigma\theta))}{\chi^2} + \frac{a\,((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P}\right)^2$$

Therefore, the corresponding Ꞙ is Negative Semi definite if Det $(Ꞙ) > 0$ and

Det $(Ꞙ) > 0 \Leftrightarrow\ Ꞙ_{11}Ꞙ_{22} - Ꞙ_{12}Ꞙ_{21} > 0$

$$\Leftrightarrow\ (d + a\,\varphi + \tau\,\omega\,) > \frac{a\,\chi^3}{4b((S+t\,\psi)+t(\delta+\sigma\theta))}\left(\frac{((S+t\,\psi)+t(\delta+\sigma\theta))}{\chi^2} + \frac{a\,((h+\pi\,\theta)+t(\delta+\sigma\theta))}{2\,P}\right)^2.$$

Under the above conditions possible, feasible and optimal candidate solutions under the possible conditions are described in Section- 4.

Paper ID: SR251005185647 DOI: https://dx.doi.org/10.21275/SR251005185647