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# On Leap Zagreb Polynomials of Generalized Transformation Graphs

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**Abstract:** ***In 2017, Naji et al. introduced the concept of leap Zagreb indices of a graph based on the second degree of vertices [1]. For a graph G, the leap first Zagreb polynomial is defined as:*** $LM_1(G,x) = \sum_{uv\in E(G)} x^{d_2(u)+d_2(v)}$***,where*** $d_2(v)$ ***is*** $d_2$***-distance degree of vertex*** $v \in V(G)$***. In this paper leap first, second, hyper leap first and second Zagreb polynomials of some generalized transformation graphs*** $G^{xy}$, $\overline{G^{xy}}$, $G^{xyz}$ ***and*** $\overline{G^{xyz}}$ ***are studied in triangle with pendant edge graph and path graph*** $P_4$***.***

**Keywords:** Hyper leap Zagreb polynomial, generalized transformation graph, leap degree, leap Zagreb polynomial, line vertex, path graph, point vertex and triangle with pendant edge graph

## 1. Introduction

Let G = (V, E) be a graph with vertex set V(G) and edge set E(G). The degree of a vertex u∈V(G) is denoted by $d_G(u)$ and is the number of vertices adjacent to u. The edge connecting the vertices u and v is denoted by uv [2-3]. All graphs considered here are finite, undirected and simple. A topological index is a numerical parameter mathematically derived from the graph structure. In graph theory the leap degree or second degree of a vertex refers to the number of vertices that are at a distance of two from that vertex. Leap degree (second degree) is the denoted by $d_2(v)$. A transformation graph is a general term that refers to a graph obtained from another graph G by some transformation, such as: line graph L(G), total graph T(G), complement graph $\overline{G}$ and subdivision S(G) etc. The procedure of obtaining a new graph from given graph using adjacency (or non-adjacency) and incidence (non-incidence) relationship between elements of a graph is known as transformation graph. There are four transformations of $G^{xy}$ as $G^{++}$, $G^{+-}$, $G^{-+}$, $G^{--}$ and their complements: $\overline{G^{++}}$, $\overline{G^{+-}}$, $\overline{G^{-+}}$ and $\overline{G^{--}}$. For three variables x,y,z there are eight distinct 3-permutations of {+,-} so has eight corresponding graph transformations. The generalized transformation graph $G^{xy}$ is a graph whose vertex set is V(G)⋃ E(G) and α, β ∈V($G^{xy}$). Then α and β are adjacent in $G^{xy}$ if and only if (a) and (b) holds:

a) α, β ∈V(G),α, β are adjacent in G if x = + and α, β not adjacent in G if x = -.

b) α ∈ V(G) and β ∈ E(G),α, β are incident in G if y = + and α, β not incident in G if y = -.

The complement or inverse of a graph G is a graph H on the same vertices such that two distinct vertices of H are adjacent if and only if they are not adjacent in G. That is to generate the complement of a graph, one fills all the missing edges required to form a complete graph and removes all the edges that were previously there. Therefore $\overline{G}$ has n vertices and $\binom{n}{2}$ - m edges. The degree of a vertex v in $\overline{G}$ is $d_{\overline{G}}(v) = n - 1 - d_G(v)$.

We use the following lemma for defining $d_G(e)$.
Lemma: Let G be a graph with u,v∈V(G) and e = uv∈E(G) then $d_G(e) = d_e = d_u + d_v - 2$.

The transformation graph $G^{xy}$ is just the semi-total point graph G which was introduced by Sampathkumar et al. [4]. The Zagreb polynomials $M_1(G,x)$-$M_5(G,x)$ and $M_{a,b}(G,x)$-$M'_{a,b}(G,x)$ was defined and studied for $HAC_5C_6C_7[p,q]$ in [5]. Distance based topological indices of generalized transformation graphs were studied in [6]. The Zagreb indices and Zagreb polynomials of transformation graphs was studied by [7-9]. Wu and Meng generalized the concept of total graph to a total transformation graph $G^{xyz}$ with x,y,z∈{-,+}[10]. Sombor index of generalized transformation graphs $G^{xy}$ and their complements were computed by H.S.Ramane et al. [11]. Leap reduced reciprocal Randic and leap reduced second Zagreb indices of some graphs were investigated in [12]. Some degree based topological indices of generalized transformation graphs and their complements were computed in [13]. The forgotten indices and their complements of transformation graphs are investigated in [14-15]. Some degree based topological indices of transformation graphs [16-21] and topological polynomials of generalized transformation graphs were obtained in [22-24]. Eccentricity based topological indices of transformation graph were discussed by S.M.Hosamani [25]. Study on basic properties of transformation graphs was found in [26-27]. Some leap indices of graphs are defined and studied by Kulli [28].Leap Zagreb indices of generalized xyz-point-line transformation graphs $T^{xyz}(G)$ when z = 1 were discussed with transformation graphs in [29].Gourava first, second indices, hyper first and second Gourava indices were computed by [30].In $HAC_5C_6C_7[p,q]$,where p is the number of pentagons in one row and q is the number of periods in the whole lattice. The edge partition in $HAC_5C_6C_7[p,q]$ is $|E_{2,3}|$=4p,$|E_{1,3}|$=2p and

$|E_{3,3}|$=24pq-6p [31]. The leap first and second Zagreb polynomials are defined as

$LM_1(G,x) = \sum_{uv\in E(G)} x^{d_2(u)+d_2(v)}$. (1)

$LM_2(G, x)=\sum_{uv\in E(G)} x^{d_2(u)\times d_2(v)}$. (2)

Hyper leap-Zagreb polynomials are defined as [32]

$HLM_1(G, x) =\sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]^2}$. (3)

$HLM_2(G, x) =\sum_{uv\in E(G)} x^{[d_2(u)\times d_2(v)]^2}$. (4)

For a figure of triangle with pendant edge graph and its transformation graphs we refer to [13] and path graph $P_4$ to [15, 29]. The symbols and notations are standard, taken from books of graph theory [33-35]. The leap first, second, hyper leap first and second Zagreb polynomials are obtained for triangle with pendant edge graph and path graph $P_4$ in generalized transformation graphs $G^{xy}, \overline{G^{xy}}$ and $G^{xyz}, \overline{G^{xyz}}$.

## 2. Materials and methods

There are four transformations of a graph $G^{xy}$ and four for their complements $\overline{G^{xy}}$.For three variables x,y,z there are eight distinct 3-permutations of {+,-} so eight corresponding graph transformations in $G^{xyz}$ and $\overline{G^{xyz}}$. The $d_2$-distance degree for point vertices and line vertices along with edge partitions in transformation graphs $G^{xy}, \overline{G^{xy}}$ and $G^{xyz}, \overline{G^{xyz}}$ of triangle with pendant edge graph [13] and path graph $P_4$ [15, 29] are determined to compute leap and hyper leap Zagreb polynomials. The triangle with pendant edge graph is shown in figure (1) and path graph $P_4$ in (2).

## 3. Results and discussion

The vertex v of $G^{xy}$ corresponding to a vertex v of G is referred to as a point vertex and vertex e of $G^{xy}$ corresponding to an edge e of G is referred to as a line vertex.

**Triangle with pendant edge graph**

Proposition 1.1. Let G be a graph with n vertices and m edges. Let u∈ V(G) and e∈ E(G).Then the $d_2$-distance degree of point and line vertices in $G^{xy}$ are

(i) $d_{2(G^{++})}(u)$ = (n+m-1)-2$d_G(u)$ and $d_{2(G^{++})}(e)$ = n+m-3.

(ii) $d_{2(G^{+-})}(u)$ = (n+m-1)-2$d_G(u)$ and $d_{2(G^{+-})}(e)$ = m+1.

(iii) $d_{2(G^{-+})}(u)$ = m and $d_{2(G^{-+})}(e)$ = n+m-3.

(iv) $d_{2(G^{--})}(u)$ = 2$d_G(u)$ and $d_{2(G^{--})}(e)$ = m+1.

Proposition1.2. Let G be a graph with n vertices and m edges. Let u∈ V(G) and e ∈ E(G).Then $d_2$-distance degree of point and line vertices in $\overline{G^{xy}}$ are

(i) $d_{2\overline{(G^{++})}}(u)$ = 2$d_G(u)$ and $d_{2\overline{(G^{++})}}(e)$ = 2.

(ii) $d_{2\overline{(G^{+-})}}(u)$ = 2$d_G(u)$ and $d_{2\overline{(G^{+-})}}(e)$ = n-2.

(iii) $d_{2\overline{(G^{-+})}}(u)$ = n-1 and $d_{2\overline{(G^{-+})}}(e)$ = 2.

(iv) $d_{2\overline{(G^{--})}}(u)$ = (n+m-1)-2$d_G(u)$ and $d_{2\overline{(G^{--})}}(e)$ = n- 2.

**Theorem 1.1:** The leap first Zagreb polynomial of $G^{++}$ transformation graph is $4x^{2((n+m-1)-2d_G(u))}+8x^{(2n+2m-4)-2d_G(u)} + 4x^{2(n+m-3)}$.

**Proof:** Partition the edge set $E(G^{++})$ in three sets $E_1$,$E_2$ and $E_3$, where $E_1$ = {uv | u,v∈E(G)},$E_2$ = {ue |the vertex u is incident to the edge e in G}and $E_3$ = {ef | e,f∈ E(G)} and so $|E_1|$=4,$|E_2|$=8 and $|E_3|$=4.By using proposition (1.1) we have if u∈V(G) then $d_{2(G^{++})}(u)$ = (n+m-1)-2$d_G(u)$ and if e∈ E(G) then $d_{2(G^{++})}(e)$ = n+m-3.

$$LM_1(G,x) = \sum_{uv\in E(G^{++})} x^{d_{2(G^{++})}(u)+d_{2(G^{++})}(v)}$$
$$=\sum_{uv\in E_1(G^{++})} x^{((n+m-1)-2d_G(u))+((n+m-1)-2d_G(u))} + \sum_{uv\in E_2(G^{++})} x^{((n+m-1)-2d_G(u))+(n+m-3)} + \sum_{uv\in E_3(G^{++})} x^{(n+m-3)+(n+m-3)}$$
$$= 4x^{2((n+m-1)-2d_G(u))}+8x^{(2n+2m-4)-2d_G(u)} + 4x^{2(n+m-3)}.$$

**Theorem 1.2:** The leap second Zagreb polynomial of $\overline{G^{++}}$ transformation graph is $2x^{(2d_G(u))^2}+8x^{4d_G(u)} + 2x^4$.

**Proof:** Partition the edge set $E(\overline{G^{++}})$ in three sets $E_1$,$E_2$ and $E_3$, where $E_1$ = {uv | u,v∈E(G)},$E_2$ = {ue| the vertex u is not incident to the edge e in G}and $E_3$ = {ef|e,f∈ E(G)} and so $|E_1|$=2,$|E_2|$=8 and $|E_3|$=2.By using proposition (1.2) we have if u∈V(G) then $d_{2\overline{(G^{++})}}(u)$= 2$d_G(u)$ and if e∈E(G) then $d_{2\overline{(G^{++})}}(e)$ = 2.

$$LM_2(G,x) = \sum_{uv\in E\overline{(G^{++})}} x^{d_{2(G^{++})}(u)\times d_{2(G^{++})}(v)}$$
$$=\sum_{uv\in E_1\overline{(G^{++})}} x^{(2d_G(u))\times(2d_G(u))} + \sum_{uv\in E_2\overline{(G^{++})}} x^{2d_G(u)\times(2)} + \sum_{uv\in E_3\overline{(G^{++})}} x^{2\times 2}$$
$$= 2x^{(2d_G(u))^2}+8x^{4d_G(u)} + 2x^4.$$

**Path graph $P_4$**

Proposition 2.1.Let G be a graph with n vertices and m edges. Let u∈ V(G) and e∈ E(G).Then the $d_2$-distance degree of point and line vertices in $G^{xy}$ are

(i) $d_{2(G^{++})}(u)$ = 6-2$d_G(u)$ and $d_{2(G^{++})}(e)$ = m+1.

(ii) $d_{2(G^{+-})}(u)$ = 6-2$d_G(u)$ and $d_{2(G^{+-})}(e)$ = m+1.

(iii) $d_{2(G^{-+})}(u)$ = 3 and $d_{2(G^{-+})}(e)$ = m+1.

(iv) $d_{2(G^{--})}(u)$ = 2$d_G(u)$ and $d_{2(G^{--})}(e)$ = m+1.

Proposition 2.2. Let G be a graph with n vertices and m edges. Let u∈ V(G) and e∈ E(G).Then $d_2$-distance degree of point and line vertices in $\overline{G^{xy}}$ are

(i) $d_{2\overline{(G^{++})}}(u)$ = n-2 and $d_{2\overline{(G^{++})}}(e)$ = n-2.

(ii) $d_{2\overline{(G^{+-})}}(u)$ = n-2 and $d_{2\overline{(G^{+-})}}(e)$ = n-2.

(iii) $d_{2\overline{(G^{-+})}}(u)$ = n-1 and $d_{2\overline{(G^{-+})}}(e)$ = n-2.

(iv) $d_{2\overline{(G^{--})}}(u)$ = 6-2$d_G(u)$ and $d_{2\overline{(G^{--})}}(e)$ = n-2.

**Theorem 2.1:** The hyper leap first Zagreb polynomial of $G^{++}$ transformation graph is $3x^{(2(6-2d_G(u))^2}+6x^{(6-2d_G(u)+n)^2} + 2x^{(2n)^2}$.

**Proof.** Partition the edge set $E(G^{++})$ in three sets $E_1$,$E_2$ and $E_3$, where $E_1$ = {uv | u,v∈E(G)},$E_2$ = {ue |the vertex u is incident to the edge e in G}and $E_3$ = {ef|e,f∈ E(G)} and so $|E_1|=3, |E_2|=6$ and $|E_3|=2$.By using proposition (2.1) we have if u∈V(G) then $d_{2(G^{++})}(u) = 6\text{-}2d_G(u)$ and if e∈ E(G) then $d_{2(G^{++})}(e) = n$.

$HLM_1(G,x) = \sum_{uv\in E(G^{++})} x^{(d_{2(G^{++})}(u)+d_{2(G^{++})}(v))^2}$
$=\sum_{uv\in E_1(G^{++})} x^{(6-2d_G(u))+(6-2d_G(u))^2} +$
$\sum_{uv\in E_2(G^{++})} x^{(6-2d_G(u)+n)^2} + \sum_{uv\in E_3(G^{++})} x^{(n+n)^2}$
$=3x^{(2(6-2d_G(u))^2}+6x^{(6-2d_G(u)+n)^2} + 2x^{(2n)^2}$.

**Theorem 2.2:** The hyper leap second Zagreb polynomial of $\overline{G^{++}}$ transformation graph is $10x^{16}$.

**Proof:** Partition the edge set $E(\overline{G^{++}})$ in three sets $E_1$,$E_2$ and $E_3$, where $E_1$ = {uv|u,v∈E(G)},$E_2$ = {ue |the vertex u is not incident to the edge e in G}and $E_3$ = {ef|e,f∈ E(G)} and so $|E_1|=3, |E_2|=6$ and $|E_3|=1$.By using proposition (2.2) we have if u∈V(G) then $d_{2\overline{(G^{++})}}(u)$= n-2 and if e∈E(G) then $d_{2\overline{(G^{++})}}(e) = 2$.

$HLM_2(G,x) = \sum_{uv\in E\overline{(G^{++})}} x^{(d_{2\overline{(G^{++})}}(u) \times d_{2\overline{(G^{++})}}(v))^2}$
$=\sum_{uv\in E_1\overline{(G^{++})}} x^{((n-2)\times(n-2))^2} + \sum_{uv\in E_2\overline{(G^{++})}} x^{((n-2)\times 2)^2} +$
$\sum_{uv\in E_3\overline{(G^{++})}} x^{(2\times 2)^2}$
$= 10x^{16}$.

Proposition 3.1. Let G be a graph with n vertices and m edges. Let u∈ V(G) and e∈ E(G).Then the $d_2$-distance degree of point and line vertices in $G^{xyz}$ are
(i) $d_{2(T^{001})(G)}(u) = n$ and $d_{2(T^{001})(G)}(e)$ =m.
(ii)$d_{2(T^{0+1})(G)}(u)$ = 5,4 and $d_{2(T^{0+1})(G)}(e)$ =m+1.
(iii) $d_{2(T^{011})(G)}(u)$ = n-1 and $d_{2(T^{011})(G)}(e)$ =0.
(iv) $d_{2(T^{++1})(G)}(u)$ = 0 and $d_{2(T^{++1})(G)}(e)$ = 4.

Proposition 3.2. Let G be a graph with n vertices and m edges. Let u∈ V(G) and e∈ E(G).Then the degree of point and line vertices in $\overline{G^{xyz}}$ are
(i) $d_{2\overline{(T^{001})}(G)}(u) = 0$ and $d_{2\overline{(T^{001})}(G)}(e)$ =m-1.
(ii)$d_{2\overline{(T^{0+1})}(G)}(u)$ = 2,3 and $d_{2\overline{(T^{0+1})}(G)}(e) = m + 1$.
(iii) $d_{2\overline{(T^{011})}(G)}(u)$ = 1,2 and $d_{2\overline{(T^{011})}(G)}(e) = m - 1$.
(iv) $d_{2\overline{(T^{++1})}(G)}(u)$ = 2,3 and $d_{2\overline{(T^{++1})}(G)}(u) = m + 1$.

**Theorem 3.1:** The leap first Zagreb polynomial of $T^{001}(G)$ transformation graph is $3x^{2(n-1)}$.

**Proof:** Partition the edge set $E(T^{001}(G))$ in three sets $E_1$,$E_2$ and $E_3$,so $|E_1|=0, |E_2|=0$ and $|E_3|=3$.By using proposition (3.1) we have if u∈V(G) then $d_{2(T^{001}(G))}(u) = n$ and if e∈ E(G) then $d_{2(T^{001}(G))}(e)$ = n-1.
$LM_1(T^{001}(G),x) = \sum_{uv\in E(T^{001}(G))} x^{d_{2(T^{001}(G))}(u) + d_{2(T^{001}(G))}(v)}$
$=\sum_{uv\in E_1(T^{001}(G))} x^{(n)+(n)} + \sum_{uv\in E_2(T^{001}(G))} x^{(n)+(n-1)} +$
$\sum_{uv\in E_3(T^{001}(G))} x^{(n-1)+(n-1)}$
$=3x^{2(n-1)}$.

**Theorem 3.2:** The leap first Zagreb polynomial of $(\overline{T^{0+1}(G)})$ transformation graph is $3x^{2(n-2)}+6x^{(2n-2)}+x^{2n}$.

**Proof:** Partition the edge set $E(\overline{T^{0+1}(G)})$ in three sets $E_1$,$E_2$ and $E_3$,so $|E_1|=3, |E_2|=6$ and $|E_3|=1$.By using proposition (3.2) for figure refer we have, if u∈V(G) then $d_{2\overline{(T^{0+1}(G))}}(u)$= 2,3 and if e∈E(G) then $d_{2\overline{(T^{0+1}(G))}}(e) = n$. We compute leap first Zagreb polynomial for $d_{2\overline{(T^{0+1}(G))}}(u)$= 2,
$LM_1((\overline{T^{0+1}}(G),x) = \sum_{uv\in E\overline{(T^{+01}(G))}} x^{d_{2\overline{(T^{0+1}(G))}}(u)+ d_{2\overline{(T^{+01}(G))}}(v)}$
$=\sum_{uv\in E_1\overline{(T^{0+1}(G))}} x^{(n-2)+(n-2)} +$
$\sum_{uv\in E_2\overline{(T^{0+1}(G))}} x^{(n-2)+(n)} + \sum_{uv\in E_3(\overline{(T^{0+1}(G))}} x^{n+n}$
$=3x^{2(n-2)}+6x^{(2n-2)}+x^{2n}$.

The computed values leap and hyper leap Zagreb polynomials are given in the tables (1-3).

**Table 1:** $LM_1(G,x)$, $LM_2(G,x)$, $HLM_1(G,x)$ and $HLM_2(G,x)$ of $G^{xy}$, $\overline{G^{xy}}$ in triangle with pendant edge graph

| **Polynomial→** | $LM_1(G,x)$ | $LM_2(G,x)$ | $HLM_1(G,x)$ | $HLM_2(G,x)$ |
|---|---|---|---|---|
| G++ | $4x^{2((n+m-1)-2d_G(u))}+8x^{(2n+2m-4)-2d_G(u)}+4x^{2(n+m-3)}$ | $4x^{(n+m-1-2d_G(u))^2}+8x^{(n+m-1-2d_G(u))(n+m-3)}+4x^{(n+m-3)^2}$ | $4x^{4((n+m-1)-2d_G(u))^2}+8x^{((2n+2m-4)-2d_G(u))^2}+4x^{(2(n+m-3))^2}$ | $4x^{(n+m-1-2d_G(u))^4}+8x^{[(n+m-1-2d_G(u))(n+m-3)]^2}+4x^{(n+m-3)^4}$ |
| G+- | $4x^{2(n+m-1-2d_G(u))}+8x^{(n+2m-2d_G(u))}+2x^{2(m+1)}$ | $4x^{(n+m-1-2d_G(u))^2}+8x^{(n+m-1-2d_G(u))(m+1)}+2x^{(m+1)^2}$ | $4x^{(2(n+m-1-2d_G(u))^2}+8x^{((n+2m-2d_G(u))^2}+2x^{4(m+1)^2}$ | $4x^{(n+m-1-2d_G(u))^4}+8x^{[(n+m-1-2d_G(u))(m+1)]^2}+2x^{(m+1)^4}$ |
| G-+ | $2x^{2m}+8x^{(n+2m-3)}+4x^{2(n+m-3)}$ | $2x^{m^2}+8x^{m(n+m-3)}+4x^{(n+m-3)^2}$ | $2x^{4m^2}+8x^{(n+2m-3)^2}+4x^{4(n+m-3)^2}$ | $2x^{m^4}+8x^{(m(n+m-3))^2}+4x^{(n+m-3)^4}$ |
| G-- | $2x^{4d_G(u)}+8x^{2d_G(u)+m+1}+2x^{2(m+1)}$ | $2x^{(2d_G(u))^2}+8x^{(2d_G(u)(m+1))}+2x^{(m+1)^2}$ | $2x^{(4d_G(u))^2}+8x^{(2d_G(u)+(m+1))^2}+2x^{(2(m+1))^2}$ | $2x^{(2d_G(u))^4}+8x^{(2d_G(u)(m+1))^2}+2x^{(m+1)^4}$ |
| $\overline{G^{++}}$ | $2x^{4d_G(u)}+8x^{2+d_G(u)}+2x^4$ | $2x^{(2d_G(u))^2}+8x^{4d_G(u)}+2x^4$ | $2x^{(2d_G(u))^2}+8x^{(2+d_G(u))^2}+2x^{16}$ | $2x^{(2d_G(u))^4}+8x^{(4d_G(u))^2}+2x^{16}$ |
| $\overline{G^{+-}}$ | $2x^{4d_G(u)}+8x^{2d_G(u)+n-2}+4x^{2(n-2)}$ | $x^{(2d_G(u))^2}+8x^{(2d_G(u)(n-2))}+4x^{(n-2)^2}$ | $2x^{(4d_G(u))^2}+8x^{(2d_G(u)+(n-2))^2}+4x^{4(n-2)^2}$ | $x^{(2d_G(u))^4}+8x^{(2d_G(u)(n-2))^2}+4x^{(n-2)^4}$ |
| $\overline{G^{-+}}$ | $4x^{2(n-1)}+8x^{n+1}+2x^4$ | $4x^{(n-1)^2}+8x^{2(n-1)}+2x^4$ | $4x^{4(n-1)^2}+8x^{(n+1)^2}+2x^{16}$ | $4x^{(n-1)^4}+8x^{(2(n-1))^2}+2x^{16}$ |

| | | | | |
|---|---|---|---|---|
| $\overline{G^{--}}$ | $4x^{2(n+m-1-2d_G(u))}+8x^{(2n+m-3-2d_G(u))}$ | $4x^{(n+m-1-2d_G(u))^2}+8x^{(n+m-1-2d_G(u))(n-2)}$ | $4x^{4(n+m-1-2d_G(u))^2}+8x^{(2n+m-3-2d_G(u))^2}$ | $4x^{(n+m-1-2d_G(u))^4}+8x^{[(n+m-1-2d_G(u))(n-2)]^2}$ |

**Table 2:** $LM_1(G,x)$, $LM_2(G,x)$, $HLM_1(G,x)$ and $HLM_2(G,x)$ of $G^{xy}, \overline{G^{xy}}$ in path graph $P_4$

| | | | | |
|---|---|---|---|---|
| G++ | $3x^{2(6-2d_G(u))}+6x^{(10-2d_G(u))}+2x^{2n}$ | $3x^{(6-2d_G(u))^2}+6x^{n(6-2d_G(u))}+2x^{16}$ | $3x^{4(6-2d_G(u))^2}+6x^{(10-2d_G(u))^2}+2x^{64}$ | $3x^{(6-2d_G(u))^4}+6x^{[n(6-2d_G(u))]^2}+2x^{256}$ |
| G+- | $3x^{2(6-2d_G(u))}+6x^{(10-2d_G(u))}+x^{2n}$ | $3x^{(6-2d_G(u))^2}+6x^{(6-2d_G(u))n}+x^{n^2}$ | $3x^{4(6-2d_G(u))^2}+6x^{((10-2d_G(u))^2}+x^{4n^2}$ | $3x^{(6-2d_G(u))^4}+6x^{[(6-2d_G(u))n]^2}+x^{n^4}$ |
| G-+ | $3x^{2(n-1)}+6x^{n+m}+2x^{2(m+1)}$ | $3x^{(n-1)^2}+6x^{(n-1)(m+1)}+2x^{(m+1)^2}$ | $3x^{(2(n-1))^2}+6x^{(n+m)^2}+2x^{64}$ | $3x^{(n-1)^4}+6x^{((n-1)(m+1))^2}+2x^{(m+1)^4}$ |
| G-- | $3x^{2(2d_G(u))}+6x^{(4+2d_G(u))}$ | $3x^{(2d_G(u))^2}+6x^{8d_G(u)}$ | $3x^{(4d_G(u))^2}+6x^{(4+2d_G(u))^2}$ | $3x^{(2d_G(u))^4}+6x^{(8d_G(u))^2}$ |
| $\overline{G^{++}}$ | $1ox^4$ | $1ox^4$ | $1ox^{16}$ | $1ox^{16}$ |
| $\overline{G^{+-}}$ | $11x^4$ | $11x^4$ | $11x^{16}$ | $11x^{16}$ |
| $\overline{G^{-+}}$ | $3x^{2(n-1)}+6x^{(n+m-2)}+x^{2(m-1)}$ | $3x^{(n-1)^2}+6x^{(n-1)(m-1)}+x^{(m-1)^2}$ | $3x^{(2(n-1))^2}+6x^{(n+m-2)^2}+x^{(2(m-1))^2}$ | $3x^{(n-1)^4}+6x^{((n-1)(m-1))^2}+x^{(m-1)^4}$ |
| $\overline{G^{--}}$ | $3x^{2(6-2d_G(u))}+6x^{(n+m+1-2d_G(u))}+2x^{2(m-1)}$ | $3x^{(6-2d_G(u))^2}+6x^{(6-2d_G(u))(m-1)}+2x^{(m-1)^2}$ | $3x^{[2(6-2d_G(u))]^2}+6x^{(n+m+1-2d_G(u))^2}+2x^{(2(m-1))^2}$ | $3x^{(6-2d_G(u))^4}+6x^{[(6-2d_G(u))(m-1)]^2}+2x^{(m-1)^4}$ |

**Table 3:** $LM_1(G,x)$, $LM_2(G,x)$, $HLM_1(G,x)$ and $HLM_2(G,x)$ of $G^{xyz}, \overline{G^{xyz}}$ in path graph $P_4$

| | | | | |
|---|---|---|---|---|
| $T^{001}(G)$ | $3x^{n+2}$ | $3x^{2n+1}$ | $3x^{(n+2)^2}$ | $3x^{(2n+1)^2}$ |
| $T^{0+1}(G)$ | $3x^{10}+6x^9+x^{2n}$ | $3x^{(n+1)^2}+6x^{20}+x^{16}$ | $3x^{4(n+1)^2}+6x^{(2n+1)^2}+x^{4n^2}$ | $3x^{225}+6x^{200}+x^{256}$ |
| $T^{011}(G)$ | $12x^3$ | 00 | $12x^9$ | 00 |
| $T^{++1}(G)$ | $6x^4+3x^8$ | $3x^{(n)^2}$ | $6x^{(n)^2}+3x^{(2(n))^2}$ | $3x^{(n)^4}$ |
| $\overline{T^{001}(G)}$ | $12x^2$ | 00 | $12x^n$ | 00 |
| $\overline{T^{0+1}(G)}$ | $3x^{2(n-2)}+6x^{(n+2)}+x^{2(m+1)}$ | $3x^n+6x^{2n}+x^{2(m+1)}$ | $3x^{(n)^2}+6x^{(2(n-1))^2}+x^{4(m+1)^2}$ | $3x^{(n)^2}+6x^{(2n)^2}+x^{(4(m+1))^2}$ |
| $\overline{T^{011}(G)}$ | $6x^{n-2}$ | 6x | $6x^{(n-2)^2}$ | 6x |
| $\overline{T^{++1}(G)}$ | $3x^4+6x^6$ | $3x^4+6x^8$ | $3x^{16}+6x^{36}$ | $3x^{16}+6x^{64}$ |

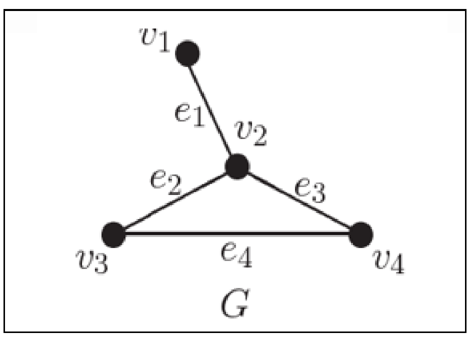


**Figure 1:** Triangle with pendant edge graph pen


**Figure 2:** Path graph $P_4$

**Figure 1:** Triangle with pendant edge graph and figure 2. Path graph $P_4$.

## 4. Conclusion

Leap first, second, hyper leap first, second Zagreb polynomials of $G^{xy}, \overline{G^{xy}}$ for triangle with pendant edge graph and also $G^{xy}, \overline{G^{xy}}$, $G^{xyz}$ and $\overline{G^{xyz}}$ transformation graphs in path graph $P_4$ are obtained. The leap first Zagreb polynomial is equal to leap second Zagreb polynomial and hyper leap first Zagreb polynomial is equal to hyper leap second Zagreb polynomial in $\overline{G^{++}}$ and $\overline{G^{+-}}$ transformation for path graph $P_4$. Leap second and hyper leap second Zagreb polynomial are zero for $T^{011}(G)$ and $\overline{T^{001}(G)}$.

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