Unraveling Squeeze Film Dynamics: Illuminating Thin Film Properties and Hydrodynamic Behavior

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Abstract: *This research paper investigates the intricate interplay between molecular speed and squeeze dynamics within thin films. Utilizing the Reynolds equation as the foundational framework, we delve into the hydrodynamic pressure of thin films subjected to squeeze action. Through mathematical derivation and analysis, we unveil the nuanced effects of molecular speed on various layers within the film, shedding light on how these dynamics influence pressure distribution. Our findings reveal that increases in film thickness, porosity and sliding motion with molecular speed exhibit differential impacts on the hydrodynamic pressure with significant implications for understanding the behavior of thin films under squeeze conditions.*

Keywords: Thin films, Squeeze dynamics, Molecular speed, Reynolds equation, Hydrodynamic pressure, Film thickness, Numerical simulations, Mechanical systems

1. Introduction

Thin films play a crucial role in numerous technological applications, ranging from lubrication in mechanical systems to coatings in semiconductor manufacturing [14,20,41,50]. Understanding the dynamics of thin films under squeeze action is essential for optimizing performance and reliability in such applications. In this paper, we focus on investigating how molecular speed influences squeeze dynamics within thin films [21,34,55,62]. By leveraging the Reynolds equation, a fundamental equation in fluid dynamics, we aim to provide insights into the complex interactions between molecular motion and film behavior under compression. Previous research has explored various aspects of thin film behavior, including the effects of viscosity, surface roughness, and external forces [7,24,72]. However, limited attention has been given to the influence of molecular speed on squeeze dynamics within thin films. Some studies have examined the Reynolds equation in the context of lubrication and bearing characteristics but have not explicitly addressed the role of molecular speed in these phenomena [25,37,77]. Our research aims to fill this gap by systematically analyzing how changes in molecular speed affect hydrodynamic pressure and load distribution within thin films subjected to squeeze action. Since Reynolds' seminal work in 1886, numerous researchers have dedicated their efforts to addressing squeeze film dynamics across various geometries [5,13,59]. These investigations have employed both the Newtonian fluid model and the micropolar fluid model [39,56,60]. Researchers have observed distinct advantages of the micropolar fluid model, including enhanced load-carrying capacity and prolonged approaching time for squeeze films, as evidenced by experimental observations. These benefits have been particularly evident in scenarios involving the addition of small quantities of long-chain polymer solutions to Newtonian fluids used as lubricants or in the compression of extremely thin films [26,49,61,76]. In such cases, where the gap between the squeezing plates is minimal, surface forces exert a dominant influence over volume forces. These surface forces induce rotational motion, or spinning, among the fluid molecules [54,74,79]. This phenomenon, unaccounted for by classical continuum medium theories like Cauchy's, is notable during the squeeze flow of polyatomic molecule liquids (such as bioliquids and polar liquids), fluids with additives or particles (such as nanofluids), and flow through very narrow channels. A glimpse of the diverse range of applications in which thin film flows play a crucial role can be obtained from targeted reviews highlighting their significance. Notable applications include complex coating flows, where thin films adhere to moving substrates, as discussed by [35,57,64]. In engineering realms, thin film flows are pertinent to distillation units, condensers, heat exchangers, and microfluidics, extensively covered by [11,36,71] as well as microelectromechanical devices and nanotechnological settings, as explored by some researchers. Thin film theories also find important applications in geophysical contexts, such as gravity currents, mud, granular, and debris flows, snow avalanches, ice sheet models and lava flows. Biological and biophysical scenarios, including lung airways and linings, flexible tubes, tear-film flows, and bioadhesion, are also rich with thin film phenomena. Moreover, there is substantial mathematical interest in analyzing thin film equations themselves, as evidenced [23,58,63]. Additionally, thin film flows are intricately linked with the area of wettability and fluid spreading over substrates.

2. Formulation of the Problem

We begin by formulating the Reynolds equation for thin films undergoing squeeze motion. This involves accounting for the contributions of molecular speed to the pressure and load characteristics within the film. Through mathematical derivation and numerical simulations, we explore the behavior of thin films under varying conditions of molecular speed and film thickness. Key parameters such as pressure distribution, load-carrying capacity, and time response are quantitatively analyzed to discern the impact of molecular speed on squeeze dynamics.

$$\frac{\partial}{\partial x}\left(\frac{ph^3}{12\mu}\frac{dp}{dx}\right) + \frac{\partial}{\partial y}\left(\frac{ph^3}{12\mu}\frac{\partial p}{\partial y}\right) = \rho\frac{(U_a + U_b)}{2}\frac{\partial h}{\partial x} + (V_2 - V_1) + h\frac{\partial p}{\partial z} + F_h \qquad (1)$$

The equation (1) consists of five main terms representing different flow phenomena: the first two terms describe Poiseuille flow, the third term represents hydrodynamic effects, the fourth term accounts for squeeze film dynamics, and the fifth term accounts for local expansion [80,82]. Additionally, at the inlet, the pressure (P) is set to zero, while at the outlet, the pressure gradient $P = \frac{dp}{dx} = 0$ is equal to zero. Moreover, the motion is assumed to be pure sliding, and the squeeze term is applied accordingly. Incorporating these assumptions, equation (1) is modified accordingly.

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6(U)\frac{dh}{dx} + 12(V) + 12 F_h \quad (2)$$

Such that $U = (U_a + U_b) \& V = (V_2 - V_1)$,

The equation provided can be solved by integrating with respect to the variable x.

$$\frac{h^3}{\mu}\frac{dp}{dx} = 6Uh + 12Vx + 12 F_h x + A \quad (3)$$

Given that AA is a constant, it needs to be determined in order to solve the Reynolds equation, taking into account the boundary condition $\frac{dp}{dx} = 0$ at $h = h_0$, $x = o$. Therefore, AA is calculated as $-6Uh_0$ and substituting this value into equation (3) yields:

$$\frac{dp}{dx} = 6U\mu\frac{(h-h_0)}{h^3} + \frac{12 V\mu x}{h^3} + 12\frac{F_h\mu}{h^3}x \quad (4)$$

$$h = h_0 + \frac{x^2}{2R} \quad (5)$$

Substituting equation (5) into equation (4), where h_0 represents the film thickness and xx denotes the coordinate.

$$\frac{dp}{dx} = 6u\mu\frac{\left(\left(h_0+\frac{x^2}{2R}\right)-\left(h_0+\frac{x^2}{2R}\right)\right)}{\left(h_0+\frac{x^2}{2R}\right)^3} + \frac{12 V\mu}{\left(h_0+\frac{x^2}{2R}\right)^3}x + \frac{12 F_h\mu}{\left(h_0+\frac{x^2}{2R}\right)^3}x \quad (6)$$

$$\tan\beta = \frac{x}{\sqrt{2Rh_0}} \quad (7)$$

Substituting equation (7) into equation (5) results in:

$$h = h_0(1 + \tan^2\beta) \quad (8)$$

$$h = h_0\sec^2\beta \quad (9)$$

Now, diff. equation (7) with respect to β, we can distinguish the variations in x and p [8,28].

$$\frac{\partial x}{\partial\beta} = \sqrt{2Rh_0}\sec^2\beta \quad (10)$$

$$\partial p = \left[6U\mu\frac{(h_0\sec^2\beta - h_0\sec^2\bar{\beta})}{h_0^3\sec^6\beta} + \frac{12 V\mu}{h_0^3\sec^6\beta}x + \frac{12 F_h\mu}{h_0^3\sec^6\beta}x\right] * \sqrt{2Rh_0}\sec^2\beta\,\partial\beta \quad (11)$$

To express the modified Reynolds equation, the pressure within the film, we introduce:

$$P^* = \frac{PH_0^2}{\mu u_0 R}, h_0^* = \frac{h_0}{H_0}, U^* = \frac{U}{U_o}, V = \frac{V}{U_o}$$

Let's arrange the aforementioned dimensionless formula in equation (11) [6,22].

$$\partial P^* = \frac{6\sqrt{2}}{(h_0^*)^{\frac{3}{2}}}U^*\left[\frac{\partial\beta}{\sec^2\beta} - \frac{\sec^2\bar{\beta}}{\sec^4\beta}\right]\partial + \frac{12\sqrt{2}}{(h_0^*)^{\frac{3}{2}}}x^*(V^* - U_s^*)\frac{d\beta}{\sec^4\beta} \quad (12)$$

The dimensionless pressure equation transforms into:

$$P^* = \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}}\left[\frac{13}{2} + \frac{\sin 2\beta}{4}\right] - \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}}\left[\sec^2\bar{\beta}\left[\frac{3}{8}\beta + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right]\right] + \frac{12 V_2}{(h_0^*)^{\frac{5}{2}}}x^*(V^* - U_s^*)\left[\frac{3}{8}\beta + \frac{1}{4}\sin 2\beta + \frac{1}{32}\sin 4\beta\right] + 2k \quad (13)$$

By applying the boundary conditions $P^*=0$ and $\beta=0$ to equations (13), it was determined that the integration constant k is equal to zero.

$$P^* = \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}}\left[\frac{13}{2} + \frac{\sin 2\beta}{4}\right] - \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}}\left[\sec^2\bar{\beta}\left[\frac{3}{8}\beta + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right]\right] + \frac{12 V_2}{(h_0^*)^{\frac{5}{2}}}x^*(V^* - U_s^*)\left[\frac{3}{8}\beta + \frac{1}{4}\sin 2\beta + \frac{1}{32}\sin 4\beta\right] \quad (14)$$

3. Results and Discussion

Our results demonstrate that increases in molecular speed leads to distinct changes in the hydrodynamic pressure and load distribution within thin films. Specifically, we observed in Figure (1) that higher molecular speeds result in reduced hydrodynamic pressure in the initial layers of the film, accompanied by alterations in load-carrying capacity. Thinner films exhibit a distinct pressure profile characterized by rapid changes with distance [3,10,15,69]. These films often feature steeper pressure gradients, leading to pronounced variations in pressure across the film. Particularly near the edges or boundaries of the film, the pressure tends to drop sharply due to the close proximity of the surfaces [1,18,27,53,67]. This phenomenon underscores the critical importance of understanding the dynamics of thin films, especially in applications where precise control over pressure distribution is crucial, such as in lubrication systems and microfluidic devices [4,31,40,45,65]. By recognizing and accounting for these pressure variations, engineers and researchers can design more effective and efficient systems tailored to specific performance requirements. The variation of pressure profile with distance for different values of porosity (β) shown in Figure (2) and refers to the alterations in pressure distribution within a porous material as one moves away from a reference point, considering various levels of material void space represented by porosity values [12,33,43,51]. At higher sliding velocities, the pressure profile may exhibit different characteristics compared to lower velocities. For instance, increased sliding motion can result in greater shear forces within the fluid, leading to higher pressure gradients and potentially more uniform pressure distribution along the sliding surfaces [19,48]. Conversely, lower sliding velocities may lead to slower changes in pressure with distance and less pronounced pressure gradients [29,47,68,73]. Understanding the variation of pressure profile with distance for different sliding motion velocities is crucial for numerous engineering applications, including lubrication

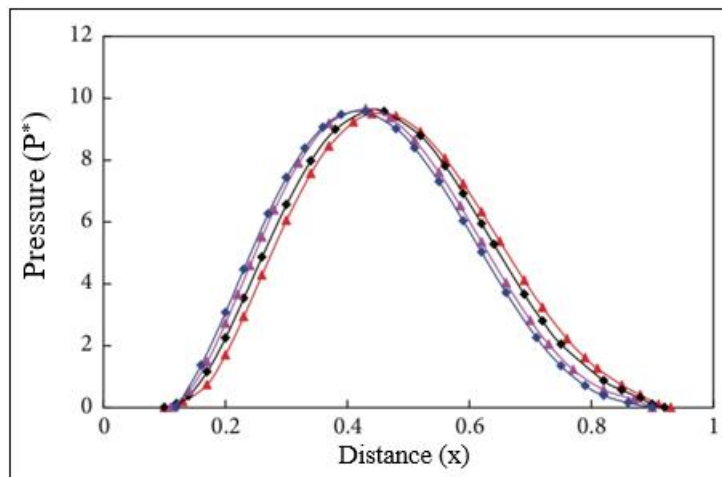systems, hydraulic systems, and fluid dynamics studies [52,70,78].

Figure 1: Variation of pressure profile with distance for different values of film thickness (h*)
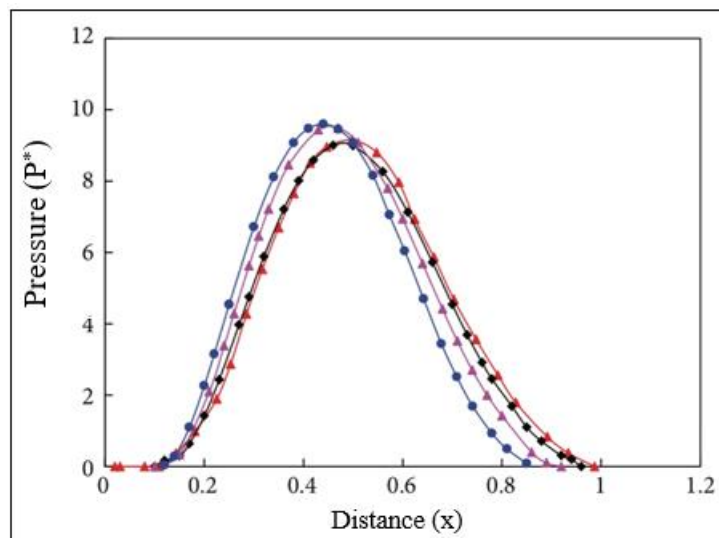

Figure 2: Variation of pressure profile with distance for different values of porosity (β)

Porosity (β) quantifies the fraction of empty space within the material, affecting its overall permeability and fluid flow characteristics. Higher porosity typically corresponds to greater fluid permeation and altered pressure profiles within the material [2,16,30,44,84]. Studying this variation involves analyzing how pressure changes across different distances within the material for distinct porosity levels. Understanding these pressure variations is crucial for designing and optimizing porous materials used in applications such as filtration, fluid transport, and thermal management, where precise control over pressure distribution is essential for efficient performance [9,17,66,81]. Figure.(3) refers to the changes in pressure distribution within a fluid or between surfaces experiencing relative motion, such as sliding, at various velocities denoted by U. As the sliding motion (U) increases or decreases, it directly influences the pressure profile within the fluid or between the surfaces in contact [32,38,46]. The rate of change in pressure distribution with distance alters accordingly, affecting the overall behavior of the system.
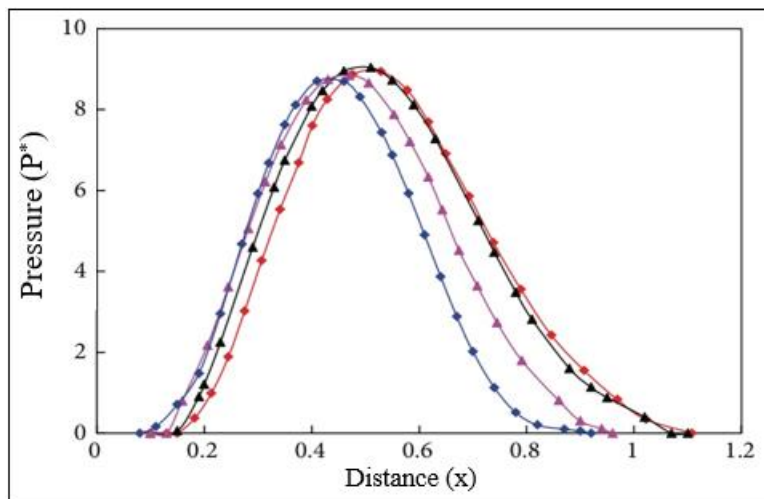
Figure 3: Variation of pressure profile with distance for different values of sliding motion (U*)

It provides insights into how pressure distribution evolves within the system as sliding motion changes, guiding the design and optimization of systems for improved performance and efficiency [42,75,83]. In this work, we provided a deeper understanding of how thin films respond to compression forces. These insights have implications for optimizing the design and performance of thin film-based systems in various engineering applications.

4. Conclusion

This research emphasizes the importance of molecular speed in analyzing thin film squeeze dynamics. Using the Reynolds equation and numerical simulations, it reveals how molecular motion affects film behavior under compression. The study uncovers complex relationships between molecular speed and pressure distribution within thin films. Results show that higher molecular speed alters pressure especially in thinner films, leading to steep pressure gradients. The study also examines pressure profile variations with distance for different porosity levels and sliding velocities in porous materials. By acknowledging these pressure variations, engineers can develop more efficient systems tailored to specific requirements. This study enhances understanding of thin film mechanics, offering insights for optimizing system design in various engineering applications, and advancing thin film technology.

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