Dynamic Modeling of Blood Flow and Pressure in the Cardiovascular System

Ashutosh Kumar Upadhyay

School of Computational and Integrative Sciences, Jawaharlal Nehru University, New Delhi 110067, India
Email: ashuscis7[at]gmail.com

Abstract: The cardiovascular system is a complex network of blood vessels responsible for transporting vital nutrients, oxygen, and hormones throughout the body. In this paper, we presented a mathematical model for simulating blood flow and pressure in the cardiovascular system, integrating principles from fluid dynamics and vessel mechanics. The model incorporates the continuum equation and Navier - Stokes equations to describe fluid behavior, while considering the viscoelastic properties of vessel walls. Through computational simulations, we investigated the effects of varying blood density, vessel wall thickness, and elasticity on blood flow distribution and pressure gradients. Our findings provided insights into the physiological implications of fluid - structure interactions within the cardiovascular system, highlighting the importance of dynamic modeling in cardiovascular research and clinical practice.

Keywords: Cardiovascular system, Blood flow dynamics, Pressure distribution, Fluid - structure interactions, Dynamic modeling

1. Introduction

The cardiovascular system plays a fundamental role in maintaining homeostasis and ensuring proper organ function by facilitating the circulation of blood throughout the body. Central to this process is the intricate interplay between blood flow dynamics and vessel mechanics within the cardiovascular network [25, 34, 53]. Traditional models have offered valuable insights into cardiovascular physiology; however, they often oversimplify the complexities of blood flow behavior and vessel wall mechanics. Dynamic modeling approaches have emerged as promising tools for capturing the dynamic interplay between fluid dynamics and vessel mechanics, providing a more comprehensive understanding of cardiovascular function [13, 32, 36, 55, 74]. Many models have been shown for simulating blood flow and pressure in the cardiovascular system, leveraging principles from fluid dynamics and vessel mechanics. Blood pressure measurement assesses the force exerted on blood vessel walls and helps determine the rate of blood circulation. Blood vessels form a robust network of tubes that continually transport blood throughout the body, playing a vital role in the circulatory system's function [4, 16, 46, 65, 73]. There are three main types of blood vessels: arteries, which carry blood away from the heart; capillaries, facilitating the exchange of substances between blood and tissues; and veins, returning blood to heart from capillaries. Aorta is largest artery, transports oxygen - rich blood from heart to the body. Its larger diameter enables a higher blood flow rate compared to other arteries [6, 23, 38, 43, 66]. Constriction of aorta requires heart to exert some force. Research predominantly focuses on pressure exerted on blood vessel walls. Studies have suggested that the narrowing of blood vessels is a contributing factor to high blood pressure development, leading to disruptions in blood flow. Many papers specifically examines high diastolic blood pressure, considering the varying density of blood flow between veins and arteries [14, 42, 50, 63, 70]. It investigates, impact of pressure on vessel walls' thickness and blood flow speed. The study on cardiovascular fluid dynamics and vessel mechanics encompasses a wide range of experimental and computational studies. Early models often relied on simplified assumptions, neglecting the complex interactions between blood flow and vessel walls [18, 44, 67, 72]. Recent advancements have led to the development of dynamic models that account for fluid - structure interactions within the cardiovascular system. These models integrate principles from continuum mechanics, fluid dynamics, and solid mechanics to simulate the behavior of blood flow and vessel deformation under physiological conditions. Key contributions include the incorporation of viscoelastic properties in vessel wall modeling, the investigation of hemodynamic factors influencing cardiovascular health, and the development of patient - specific models for clinical applications [1, 11, 24, 54, 62].
Theoretical Framework and Formulation of the Problem:
Our dynamic model is based on the continuum equation and Navier - Stokes equations, which describe the conservation of mass and momentum for fluid flow, respectively. The model accounts for incompressible Newtonian flow through a network of cylindrical vessels, considering the viscoelastic deformation of vessel walls [8, 9, 35, 56]. The governing equations are solved numerically using finite element methods, allowing for the simulation of blood flow and pressure distribution within the cardiovascular system [Figure 1].

We introduced a mathematical model to characterize the pressure generated by blood flow within the layers of elastic, varying thickness in arteries, capillaries, and veins [33, 48, 68].

\[
\mu \frac{\partial^2 u}{\partial r^2} = \frac{1}{\rho} \frac{\partial p}{\partial z} \tag{1}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0 \tag{2}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (rv) = 0 \tag{3}
\]

We have introduced a novel parameter termed "elastic fiber." This variable encapsulates the elastic properties within the structure under consideration [59, 78]. For instance, in the context of blood vessels, elastic fibers would denote the presence of proteins like elastin, which confer elasticity to the vessel walls [27, 51, 76].

\[
a = \frac{u}{R (Lt)} \tag{4}
\]

Inner layers of blood vessels refer to the innermost portions or surfaces of the blood vessel walls. These layers are crucial in determining the dynamics of blood flow within the vessels [2, 15, 26, 45, 47].

\[
u = 0, \text{ at } r = 0 \tag{5}
\]

\[
u = 0, \text{ at } r = d. \tag{6}
\]

When blood flows through blood vessels, it moves smoothly along the vessel walls, adjusting to changes in vessel diameter along its path. This sliding phenomenon ensures that blood flow remains continuous and unimpeded despite variations in vessel size [41, 69, 84].

\[
u(r, z) = \frac{u}{2} \frac{\partial p}{\partial z} (-r^2 + a^2) \tag{7}
\]

When we replaced the concept of blood sliding smoothly through blood vessels while accounting for differences in vessel diameter, as described in equation (7), into the Navier - Stokes equations outlined in equation (2), we obtained a set of equations [22, 49, 75].

\[
\frac{\partial^2 w}{\partial r^2} = \frac{1}{\rho \mu} \frac{\partial p}{\partial z} \left(2 - \frac{r^2}{a^2} + \frac{4}{a^4} \right) \tag{8}
\]

This condition implies that the blood adheres to the vessel walls and does not slip or move relative to them [5, 39, 61].

\[
w = 0, \text{ at } (r = 0) \tag{9}
\]

\[
w = 0, \text{ at } (r = d) \tag{10}
\]

Now process involves incorporating the constraints imposed by the condition into the mathematical framework, resulting in an expression that describes how the velocity of blood flow varies across radial dimension of vessel [28, 60, 82].

\[
w(r, z) = \frac{1}{2 \rho \mu} \frac{\partial p}{\partial z} \left(r^2 - \frac{r^4}{12a^2} + \frac{2r^2}{3} - \frac{r^4}{12a^2} + \frac{r^2}{3} + \frac{4}{a^4} \right) \tag{11}
\]

When we inserted equation (10) into equation (4), we derived what is known as the dynamic continuity equation. This process involves integrating the expression derived from equation (10) into the broader framework outlined in equation (4) [3, 12, 30, 79].

\[
\frac{1}{r} \frac{\partial}{\partial r} \left(rv^2 + \frac{1}{2 \rho \mu} \frac{\partial p}{\partial z} \left(r^2 - \frac{r^4}{12a^2} + \frac{2r^2}{3} - \frac{r^4}{12a^2} + \frac{4}{a^4} \right) \right) \tag{12}
\]

This condition describes how the velocity of blood flow, represented by v (r, z), is influenced or constrained at the boundaries or surfaces of the vessel layers. In simpler terms, it outlines how the flow of blood is affected by factors such as vessel geometry and the surrounding environment at these specific points within the blood vessel structure [7, 19, 31, 37].

\[
\frac{\partial v}{\partial t} = - \frac{1}{2 \rho \mu} \frac{\partial p}{\partial z} \left(\frac{1}{3} - \frac{\mu v^2}{6a} + \frac{\mu v^2}{6a} - \frac{hL^3}{2} + \frac{hL^3}{24a} - \frac{\mu L^5}{4a} \right) \tag{13}
\]
By incorporating a non-dimensional variable and associated parameters, we are introducing a standardized way of representing certain aspects of the system under study [21, 54].

\[
\bar{L} = \frac{1}{d} \bar{h} = \frac{\bar{h}}{d} \quad \bar{p} = \frac{\bar{p}}{\mu \bar{R}^2} \quad \bar{Z} = \frac{Z}{L}
\]

(14)

This non-dimensional allows for easier comparison and analysis across different scales [64, 83].

\[
\frac{\bar{p} \bar{h}^2}{\mu} = \frac{\partial \bar{p}}{\partial \bar{z}} \delta (\mu, L)
\]

(15)

\[
\delta (\mu, L) = \frac{L^3}{2} - \frac{\mu \bar{R}^5}{62} + \frac{\mu \bar{R}^3}{64} - \frac{\bar{h} \bar{L}^2}{2} + \frac{\mu \bar{R}^3 \bar{L}^5}{2} - \frac{\mu \bar{h} \bar{b}^2}{6}
\]

(16)

This condition outlines how pressure varies within the vessel walls, considering factors such as vessel geometry and the surrounding environment.

\[
\bar{p} = 0, \text{at } z = 1
\]

(17)

\[
\frac{\partial \bar{p}}{\partial z} = 0, \text{at } z = 0
\]

(18)

\[
\bar{p} = \frac{2 \rho g \mu \bar{R}^2}{L^3} \int_0^1 V(t, z) \quad dt
\]

(19)

Hemodynamics refers to the study of how blood flows through the blood vessels in the body. The speed or rate at which blood moves through these vessels is affected by some factors, like blood pressure, vascular resistance and viscosity.

\[
Q_x = \frac{1}{T} \int_0^L V(t, z) \quad dz
\]

(20)

\[
V(t, z) = \frac{1}{2 \mu \bar{R}^2} \left( Z^2 - ZL \right) + \frac{dL}{Lh}
\]

(21)

When we insert equation (20) into equation (21), we derive a new expression or equation. This process involves replacing the variables or terms in equation (21) with the corresponding terms from equation (20). By doing so, we obtain a modified equation that incorporates the information or relationships described in both equations (21) and (20).

\[
Q_x = \frac{1}{L} \int_0^L \frac{1}{2 \mu \bar{R}^2} \left( Z^2 - ZL \right) + \frac{dL}{Lh}
\]

(22)

By substituting equation (22) into equation (21), we arrive at a dimensionless flow rate,

\[
\tilde{Q}_x = \frac{Q_x}{L^3}
\]

(23)

\[
\tilde{Q}_x = \int_0^z \frac{1}{L^3} \left( \frac{p \bar{L}^3}{2 \mu \bar{R}^2} \right) (Z^2 - ZL) \quad dz + \frac{dL}{Lh}
\]

(24)

\[
\tilde{Q}_x = \frac{1}{L} \int_0^z \frac{\partial \bar{p}}{\partial z} (Z^2 - ZL) \quad dz + \frac{dL}{Lh}
\]

(25)

To derive the dimensionless pressure (p) from equation (26), we need to take the derivative of the equation with respect to the appropriate variables involved.

\[
\frac{\partial \bar{p}}{\partial z} = \left[ \left( \frac{L^3}{2} - \frac{\mu \bar{R}^5}{62} + \frac{\mu \bar{R}^3}{64} - \frac{\bar{h} \bar{L}^2}{2} + \frac{\mu \bar{R}^3 \bar{L}^5}{2} - \frac{\mu \bar{h} \bar{b}^2}{6} \right) \right] \quad dz
\]

(26)

When we substitute equation (26) into equation (25), we obtain a new expression that incorporates the information provided by both equations.

\[
\tilde{Q}_x = \frac{1}{L} \int_0^z \left( \frac{p \bar{L}^3}{2 \mu \bar{R}^2} \right) (Z^2 - ZL) \quad dz + \frac{dL}{Lh}
\]

(27)

\[
\tilde{Q}_x = \left( \frac{L^3}{2} - \frac{\mu \bar{R}^5}{62} + \frac{\mu \bar{R}^3}{64} - \frac{\bar{h} \bar{L}^2}{2} + \frac{\mu \bar{R}^3 \bar{L}^5}{2} - \frac{\mu \bar{h} \bar{b}^2}{6} \right) \frac{dL}{Lh}
\]

(28)

2. Results and Discussion

Computational simulations are conducted to investigate the effects of varying blood density, vessel wall thickness, and elasticity on blood flow dynamics and pressure gradients. The relationship between blood vessel pressure and the velocity of blood pumped by the heart is fundamental to understanding the dynamics of circulation within the human body. This relationship is intricately influenced by the cross-sectional area of different types of blood vessels, including the aorta, arteries, veins, and capillaries. The aorta is a largest artery responsible for carrying oxygen with blood from heart and exhibits high blood flow velocity, resulting in elevated pressure within artery. However, as blood traverses through arteries into veins, its velocity gradually decreases, leading to a reduction in pressure. This decrease in velocity and pressure is attributed to the increased total cross-sectional area of the vascular system as blood moves away from the heart. Notably, during the systole, or contraction phase of the heart, blood velocity in arteries is higher compared to diastole, the relaxation phase, reflecting the dynamic nature of blood flow throughout the cardiac cycle [54, 81]. Moreover, Figure 3 emphasizes significant pressure differences between veins and capillaries, particularly evident when comparing blood velocity across their cross-sectional areas.
In capillaries, where crucial oxygen and nutrient exchange with tissues occur, pressure dramatically decreases due to their smaller diameter, facilitating efficient gas and nutrient exchange. Overall, these observations underscore the complex interplay between blood vessel pressure, flow velocity, and cross-sectional area, highlighting the intricate mechanisms that govern circulatory dynamics within the human body [80, 84]. Figure (2) and (3) illustrate the relationship between blood vessel pressure and the velocity of blood pumped by the heart. This relationship is influenced by the cross-sectional area of each type of blood vessel. Notably, during systole (contraction phase of the heart), blood velocity in arteries is higher compared to diastole (relaxation phase). Figure 3 highlights significant pressure differences between veins and capillaries, particularly evident when comparing blood velocity across their cross-sectional areas. Pressure decreases dramatically in capillaries due to their smaller diameter [17, 58, 77]. The dynamics of blood flow
within the circulatory system is crucial for comprehending various physiological processes and their implications for health. One important observation in this regard is the relationship between the thickness of blood vessel walls and the rate of blood flow. It has been observed that blood vessels with thicker walls, such as the aorta, tend to have higher rates of blood flow [10, 29, 40]. This suggests that the structural integrity provided by thicker walls may contribute to more efficient propulsion of blood throughout the body. Conversely, blood vessels with thinner walls, like veins and capillaries, typically exhibit lower rates of blood flow. These observations highlight the intricate interplay between vessel morphology and hemodynamic parameters, underscoring the importance of vessel thickness in regulating blood flow dynamics. This understanding serves as a foundation for elucidating circulatory physiology and its role in maintaining homeostasis [20, 57, 71]. In Figure (4) and (5), we observed the relationship for rate of flow and the thickness of vessels. Volume of blood flowing vessels varies according to vessel thickness. With thicker walls, as seen in the aorta, blood flow rate is higher. Conversely, veins and capillaries, characterized by thinner walls, have lower flow rates.

![Diagram showing blood flow rates in veins and capillaries](image)

**Figure 5:** Relationship between blood flow rate and thickness of wall in vein and capillary

3. Conclusion

In this paper, we have explored the intricate relationship between blood vessel pressure, blood flow velocity, and vessel morphology within the context of circulatory dynamics. By developing a mathematical model, we have elucidated how the pressure exerted by the heart varies across different types of blood vessels, with the aorta exhibiting particularly high pressures due to its role in carrying oxygenated blood from the heart. Our analysis has underscored the crucial role of vessel diameter in regulating blood flow rates, with arterial blood flow generally being faster compared to venous and capillary flow. Additionally, we have highlighted the impact of vessel wall thickness on blood flow, observing higher flow rates in vessels with thicker walls, such as the aorta, and lower rates in vessels with thinner walls, like veins and capillaries. These findings deepen our understanding of circulatory physiology and provide insights into the mechanisms underlying cardiovascular health and disease. Through visual representations, we have visually depicted the relationship between blood flow rate and vessel thickness, further enhancing our comprehension of circulatory dynamics. This study contributed to the body of knowledge surrounding blood flow dynamics, paving the way for future research aimed at improving our understanding of circulatory function and its clinical implications.

References


Technology and Exploring Engineering, 10 (2), 176 - 180, (2020).


Volume 13 Issue 5, May 2024
Fullly Refereed | Open Access | Double Blind Peer Reviewed Journal
www.ijsr.net

Paper ID: SR24518032944
DOI: https://dx.doi.org/10.21275/SR24518032944
1198


