

Some Inverse Multiplicative Topological Indices of a Graph

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Abstract: Different versions of topological indices are studied in many papers in chemical graph theory. We introduce some inverse multiplicative topological indices such as inverse second multiplicative Zagreb index, inverse multiplicative modified first Zagreb index, inverse multiplicative first hyper Zagreb index, inverse multiplicative second hyper Zagreb index, inverse multiplicative atom bond connectivity index, inverse multiplicative geometric arithmetic index and inverse fifth multiplicative sum connectivity index.

Keywords: Degree, inverse multiplicative topological indices, molecular graph, multiplicative topological indices

1. Introduction

Let $G = (V, E)$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex $u \in V(G)$ is denoted by d_u and is the number of vertices adjacent to u . The edge connecting the vertices u and v is denoted by uv [1]. The basic properties of Zagreb coindices by taking example of a chemical compound methyl cyclopentane was investigated by Kiruthika [2]. Many degree based multiplicative topological indices such as first, second multiplicative Zagreb indices, Narumi-Katayama index, new multiplicative version of first Zagreb index, first and second hyper-Zagreb indices, multiplicative generalized Zagreb indices, multiplicative sum connectivity indices, multiplicative atomic bond connectivity index and multiplicative geometric arithmetic index of Jahangir graph were computed by W.Gao et al.[3]. For notation and concepts not defined in this paper we refer to [4]. A topological index is a numeric quantity associated with graph which characterize the topology of a graph and is invariant under graph automorphism [5]. The multiplicative first and second inverse Nirmala indices of a graph were defined in [6]. Some degree-based multiplicative topological indices of nanostar dendrimers were studied by S. Mondal et al.[7]. By using the definition of the multiple Zagreb indices for the extremal graphs, topological indices were computed by Ghorbani and N. Azim in [8]. Several multiplicative topological indices of linear $[n]$ -anthracene, V-anthracene nanotube and nanotori were studied [9]. Multiplicative degree based topological indices of some chemical structures in drugs were investigated in [10]. Multiplicative versions of Zagreb indices of $TUC_4C_8(S)$ nanotubes was studied by M. R. Farahani [11].

The multiplicative first Zagreb index $PM_1(G)$ and multiplicative second Zagreb index $PM_2(G)$ are defined as [12-14]

$$PM_1(G) = \prod_{uv \in E(G)} (d_u + d_v) \quad (1)$$

$$PM_2(G) = \prod_{uv \in E(G)} (d_u \times d_v) \quad (2)$$

Fifth multiplicative sum connectivity index of a graph is defined as [15]

$$S_5 \Pi(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{S_u + S_v}} \quad (3)$$

We introduce some inverse multiplicative topological indices as

$$IZ_2 \Pi(G) = \prod_{uv \in E(G)} \frac{1}{d_u \times d_v} \quad (4)$$

$$IZ_1^* \Pi(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v} \right) \quad (5)$$

$$IH \Pi_1(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v} \right)^2 \quad (6)$$

$$IH \Pi_2(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u \times d_v} \right)^2 \quad (7)$$

$$IABC \Pi(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_u \times d_v}{d_u + d_v - 2}} \quad (8)$$

$$IGA \Pi(G) = \prod_{uv \in E(G)} \frac{d_u + d_v}{2\sqrt{d_u \times d_v}} \quad (9)$$

$$IS_5 \Pi(G) = \prod_{uv \in E(G)} \sqrt{S_u + S_v} \quad (10)$$

In this paper inverse multiplicative second Zagreb index ($IZ_2 \Pi(G)$), inverse multiplicative modified first Zagreb index ($IZ_1^* \Pi(G)$), inverse multiplicative first hyper Zagreb index ($IH \Pi_1(G)$), inverse multiplicative second hyper Zagreb index ($IH \Pi_2(G)$), inverse multiplicative atom bond connectivity index ($IABC \Pi(G)$), inverse multiplicative geometric arithmetic index ($IGA \Pi(G)$) and inverse fifth multiplicative sum connectivity index ($IS_5 \Pi(G)$) and corresponding multiplicative topological indices ($Z_2 \Pi(G)$, $Z_1^* \Pi(G)$, $H \Pi_1(G)$, $H \Pi_2(G)$, $ABC \Pi(G)$, $GAP \Pi(G)$ and $S_5 \Pi(G)$) are computed for methyl cyclopentane.

2. Materials and Methods

A molecular graph or a chemical graph is a graph such that its vertices correspond to the atoms and edges to the bonds. The molecular graph of a chemical compound methyl cyclopentane is shown in figure (1) with six vertices having degrees 1, 2 and 3. The edge partition used to compute inverse multiplicative topological indices and multiplicative topological indices are given in table (1) and (2). In table (3) computed values of some multiplicative topological indices are presented. Graphs of multiplicative and inverse

multiplicative topological indices are shown in figure 2 for comparison.

3. Results and Discussion

Theorem 1: Inverse multiplicative second Zagreb index of methyl cyclopentane is 0.8333.

Proof. This theorem is proved by using figure (1) and table (1). Inverse multiplicative second Zagreb index

$$IZ_2\Pi(G) = \prod_{uv \in E(G)} \frac{1}{d_u \times d_v}$$

$$= |E_{1,3}| \left(\frac{1}{1 \times 3}\right) \times |E_{2,3}| \left(\frac{1}{2 \times 3}\right) \times |E_{2,2}| \left(\frac{1}{2 \times 2}\right)$$

$$= 0.8333.$$

Theorem 2: Inverse multiplicative modified first Zagreb index of methyl cyclopentane is 6.666.

Proof. This theorem is proved by using figure (1) and table (1). Inverse multiplicative modified first Zagreb index

$$IZ_1^*\Pi(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v}\right)$$

$$= |E_{1,3}| \left(\frac{1}{1} + \frac{1}{3}\right) \times |E_{2,3}| \left(\frac{1}{2} + \frac{1}{3}\right) \times |E_{2,2}| \left(\frac{1}{2} + \frac{1}{2}\right)$$

$$= 6.666.$$

Theorem 3: Inverse multiplicative first hyper Zagreb index of methyl cyclopentane is 7.407.

Proof. This theorem is proved by using figure (1) and table (1). Inverse multiplicative first hyper Zagreb index

$$IH\Pi_1(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u} + \frac{1}{d_v}\right)^2$$

$$= |E_{1,3}| \left(\frac{1}{1} + \frac{1}{3}\right)^2 \times |E_{2,3}| \left(\frac{1}{2} + \frac{1}{3}\right)^2 \times |E_{2,2}| \left(\frac{1}{2} + \frac{1}{2}\right)^2$$

$$= 7.407.$$

Theorem 4: Inverse multiplicative second hyper Zagreb index of methyl cyclopentane is 0.0012.

Proof. This theorem is proved by using figure (1) and table (1). Inverse multiplicative second hyper Zagreb index

$$IH\Pi_2(G) = \prod_{uv \in E(G)} \left(\frac{1}{d_u \times d_v}\right)^2$$

$$= |E_{1,3}| \left(\frac{1}{1 \times 3}\right)^2 \times |E_{2,3}| \left(\frac{1}{2 \times 3}\right)^2 \times |E_{2,2}| \left(\frac{1}{2 \times 2}\right)^2$$

$$= 0.0012.$$

Theorem 5: Inverse multiplicative atom bond connectivity index of methyl cyclopentane is 14.69.

Proof. This theorem is proved by using figure (1) and table (1). Inverse multiplicative atomic bond connectivity index

$$IABC\Pi(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_u \times d_v}{d_u + d_v - 2}}$$

$$= |E_{1,3}| \sqrt{\frac{1 \times 3}{1+3-2}} \times |E_{2,3}| \sqrt{\frac{2 \times 3}{2+3-2}} \times |E_{2,2}| \sqrt{\frac{2 \times 2}{2+2-2}}$$

$$= 14.69.$$

Theorem 6: Inverse multiplicative geometric arithmetic index of methyl cyclopentane is 4.743.

Proof. This theorem is proved by using figure (1) and table (1).

Inverse geometric arithmetic index

$$IGA\Pi(G) = \prod_{uv \in E(G)} \frac{d_u + d_v}{2\sqrt{d_u \times d_v}}$$

$$= |E_{1,3}| \frac{1+3}{2\sqrt{1 \times 3}} \times |E_{2,3}| \frac{2+3}{2\sqrt{2 \times 3}} \times |E_{2,2}| \frac{2+2}{2\sqrt{2 \times 2}}$$

$$= 4.743.$$

Theorem 7: Inverse fifth multiplicative sum connectivity index of methyl cyclopentane is 303.5.

Proof. This theorem is proved by using figure (1) and table (2).

Inverse fifth multiplicative sum connectivity index

$$IS_5\Pi(G) = \prod_{uv \in E(G)} \sqrt{S_u + S_v}$$

$$= |E_{3,5}| \sqrt{3+5} \times |E_{5,5}| \sqrt{5+5} \times |E_{4,5}| \sqrt{4+5} \times |E_{4,4}| \sqrt{4+4}$$

$$= 303.5.$$

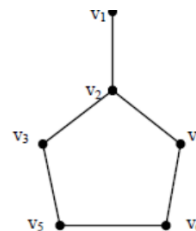


Figure1: Molecular graph of methyl cyclopentane.

Table 1: (d_u, d_v) partition

(d _u , d _v)	(1,3)	(2,3)	(2,2)
Number of edges	1	2	3

Table 2: (S_u, S_v) partition

(S _u , S _v)	(3,5)	(5,5)	(4,5)	(4,4)
Number of edges	1	2	2	1

Table 3: Multiplicative topological indices of methyl cyclopentane

Topological indices	Z ₂ Π(G)	Z ₁ [*] Π(G)	HΠ ₁ (G)	HΠ ₂ (G)	ABCΠ(G)	GAPΠ(G)	S ₅ Π(G)
Values	432	480	38400	31104	2.449	5.091	0.0529

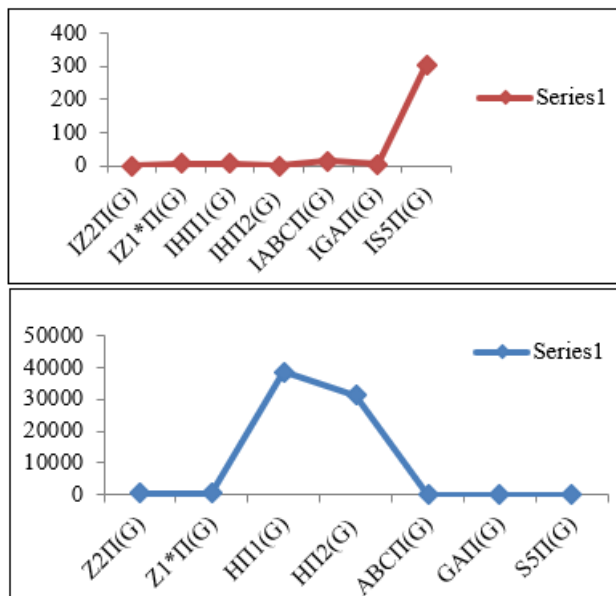


Figure 2: Graphs of inverse multiplicative and multiplicative topological indices of methyl cyclopentane

4. Conclusion

Inverse multiplicative second Zagreb index, inverse multiplicative modified first Zagreb index, inverse multiplicative first hyper Zagreb index, inverse multiplicative second hyper Zagreb index, inverse multiplicative atom bond connectivity index, inverse multiplicative geometric arithmetic index and inverse fifth multiplicative sum connectivity index of methyl cyclopentane are obtained. Also these multiplicative and inverse multiplicative topological indices are graphically studied.

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