Correcting Common Misunderstandings of Lorentz-Fitzgerald Length Contraction: A Guide to Accurate Measurements

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Abstract: ***This article points out the mistake in a Lorentz Fitzgerald length contraction by explaining the restriction in the use of Lorentz Fitzgerald length contraction. It has been explained that for the two different points on the positive X axis (i.e. x_1 and x_2), there can not be the same time value. Means one cannot measure two different distances by using Lorentz transformation simultaneously. So, the value vt_1 and vt_2 cannot be the same. Means we can't take $vt_1 = vt_2 = vt$ by saying that length x_1 and x_2 are measured at same time "t" so measured simultaneously. This yields different results when we calculate for Lorentz Fitzgerald length contraction. So, correction of the calculations is advised.***

Keywords: Lorentz Fitzgerald length contraction, Length contraction, Corrected Length contraction, Lorentz transformation, Special theory of Relativity, Special Relativity

1. Introduction

In the special theory of Relativity proposed by albert einstein, consequences of the postulates of the special theory of relativity leads to relative length contraction when two inertial reference frames move with uniform rectilinear velocity with reference to each other. That relative length contraction is known as Lorentz Fitzgerald contraction because originally that was described by Hendric Lorentz and G.F. Fitzgerald. The erroneous logic used while calculating Lorentz Fitzgerald length contraction is shown in this article and correction is suggested.

Article

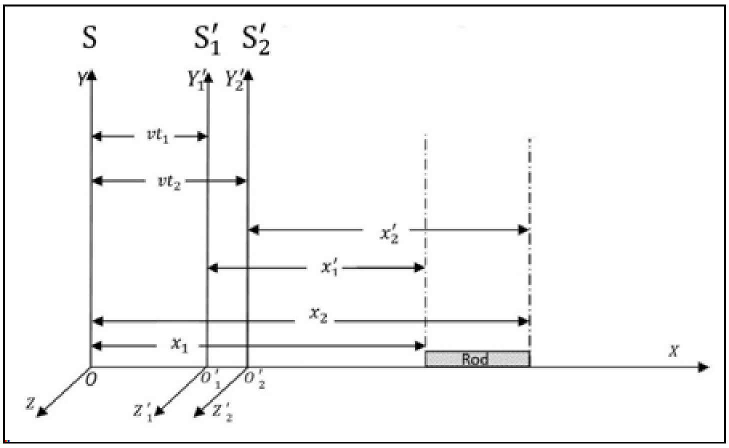

Figure 1

S′ inertial reference frame is moving with uniform rectilinear velocity "v" along with positive X axis.

Rod is placed along with the positive X axis in a stationary reference frame S, Co-ordinates of the proximal end of the Rod is (x_1, 0, 0, t_1) and co-ordinates of the distal end is (x_2, 0, 0, t_2)

S'_1 is a place where S′ reach when it travels a distance vt_1 in a time t_1 while transforming the point x_1.

S'_2 is a place where S′ reach when it travels a distance vt_2 in a time t_2 while transforming the point x_2.

Let's derive corrected length contraction first.

Now,

Deriving Corrected length contraction using Lorentz transformation equations.

Lorentz transformation equations for distance or x co-ordinates of the object are,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In figure 1 above the Rod is put along the X axis. Length of the rod in relation to stationary inertial reference frame or non-primed inertial reference frame S is proper length l_0. Length of the rod in relation to moving inertial reference frame or primed inertial reference frame S′ is contracted length l

Proper length $\quad l_0 = x_2 - x_1 \quad$ ... (3)

Contracted length $\quad l = x_2{}' - x_1{}' \quad$ ... (4)

Now let's put value of (1) in equation no (4).

$$l = x_2{}' - x_1{}'$$

$$l = \frac{x_2 - vt_2}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$l = \frac{(x_2 - x_1) - \frac{v}{c}(ct_2 - ct_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(x_2 - x_1) - \frac{v}{c}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{l_0 - \frac{v}{c} l_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$Contracted\ length\ \ l = l_0 \frac{(1 - \frac{v}{c})}{\sqrt{1 - \frac{v^2}{c^2}}} \quad where\ v < c$$

Above equation shows the value of contracted length l.

Length contraction is Δl

$$\Delta l = l_0 - l$$

$$\Delta l = l_0 - l_0 \frac{(1 - \frac{v}{c})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta l = l_0 \left(1 - \frac{(1 - \frac{v}{c})}{\sqrt{1 - \frac{v^2}{c^2}}}\right) \qquad \text{(where, v<c)}$$

In a theory of special relativity, in a described calculation, vt_2 and $vt_1$are considered same with the argument that $x_2{}'$and $x_1{}'$ are measured simultaneously, so t_2 and $t_1$remains same and vt_2becomes equal to vt_1. So, that gets cancelled during calculations.

But The phrase "$x_2{}'$and $x_1{}'$ are measured simultaneously" is an impossible task for $x_2 \neq x_1$.

Let's understand this.

See Figure 2 for the Lorentz transformation Shown here.

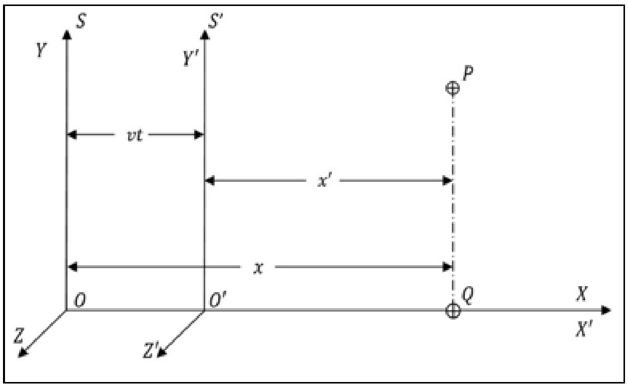

Figure 2

Here, vt is the distance of S′ frame from the point O.

This is the distance that Reference frame S′ travels along the X axis in a time t.

Now what is this time t!

This is the time taken by photon traveling with the speed c from O to co-ordinate x of the point P in reference frame S. The point along the X axis with the same x co-ordinate value of the point P is denoted as Q in the figure 2, So, $x = OQ$.

So,

$ct = x$. Here c is the constant, so, value of t is depending on the value of x.

As the value of x changes, value of the t will also change.

If we consider the value of v constant,

when we calculate x', then value of vt will depend on the value of x.

$$vt = xv/c$$

So, for $x_1 \neq x_2 \Leftrightarrow vt_1 \neq vt_2$

Lorentz transformation equations are equations, those are made to transform co-ordinates of the point, only at one time, which time $t = x/c$. If the value of x changes, then time t will also get changed according to the value of x, you can't choose that time t according to your free will for particular x and can't choose same t for two different x values.

So, we can't measure both x_2 and x_1 simultaneously. Because when we apply Lorentz transformation Equation for point $x_2{}'$, the value of x_2 will be equal to ct_2 where t_2 will be specific for x_2. In that time t_2 the primed or moving reference frame S′ will move a distance vt_2 . which is also a specific distance depending on the value of t_2 and so, also depending on the value of x_2.

when we apply Lorentz transformation Equation for point $x_1{}'$, the value of x_1 will be equal to ct_1 where t_1 will be specific for x_1. In that time t_1 the moving reference frame S′ will move a distance vt_1. which is also a specific distance depending on the value of t_1 and so, also depending on the value of x_1.

So, the value of vt_2 can't be equal to vt_1, means a position of reference frame S′ can't be at same place when we are applying the Lorentz transformation equations for two different points having two different value of x co-ordinates.

So, measuring distance for proximal and distal end of the rod simultaneously in a moving reference frame is not possible, because when we apply Lorentz transformation equation value of t′ will be different for different x values when velocity v of the moving reference frame will remain constant. So, we can calculate value of $x_1{}'$ and $x_2{}'$ for respective value x_1 and x_2 at same velocity v, but we can't calculate them at the same time t.

Let's look at the length contraction described in the book "Relativity, The special and The General theory by Albert einstein, 1916"?

There, the rod is put in a moving reference frame with proximal end on the origin (0,0). There, time t_1 becomes 0, and time t_2 becomes t and, contracted length is $l = x_2 - 0 = x_2$

proper length is $l_0 = x_2{}' - x_1{}' = \Upsilon(x_2 - vt - x_1 + vt)$

And so,

$$l_0 = \Upsilon(x_2 - x_1)$$
$$l_0 = \Upsilon(x_2 - 0)$$
$$l_0 = \Upsilon x_2$$
$$l_0 = \Upsilon l$$
$$l = \frac{l_0}{\Upsilon}$$
$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

So, in that calculation we can find contracted length as above. You may find the mistake in the calculation there because there t_1 is taken as 0. And vt is canceled anyhow by taking $t_2 = t = 0$.

The line is there, "Lorentz transformation value of both end of the rod at time t=0 is as below"

Means in that case, in the book of the theory of relativity it is mentioned that the distance of the distal end of the rod placed in a moving reference frame can be calculated by Lorentz transformation equation in relation to stationary reference frame S by taking t=0 value in the vt

This is the mistake, if proper distance x_2 is not zero, then ct_2 cannot be zero.

If ct_2 cannot be zero, then vt_2 cannot be zero. So, by using Lorentz transformation equation one cannot take the value of t=0 for the point having value of x coordinate more than zero. So, mistake is there in the Book of Relativity by albert Einstein that should be corrected as shown above.

At the time t=0, origin of S' frame coincides with the origin of the frame S, and for that situation vt must be 0. origin of S' frame moves a distance vt, when we try to transform x co-ordinate of any point along the X axis having value more than 0 if we consider a velocity v of S'more than 0. So, when we try to transform the x co-ordinate of the distal end of the Rod having non 0 length than, x_2 value must be more than 0, that leads to value of ct_2 must be more than 0, that further leads to value of vt_2 must be more than 0 and that will not allow the origin of the moving or primmed reference frame S' to be coincided with origin of the non-primed or stationary reference frame S.

2. Conclusion

There is a mistake in Lorentz Fitzgerald length contraction, by assuming $vt_1 = vt_2$ when $x_1 \neq x_2$. Corrected calculations must show contracted length l by equation A and not by equation C.

$$l = l_0 \frac{(1 - \frac{v}{c})}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ...\ (A) \quad (where\ v < c.)$$

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} \quad ....(C)$$

So, correction is suggested in relevant theory.

Similar mistakes should also be corrected in the calculation of the time dilation.

References

[1] A book, "Relativity, The special and The General theory by Albert einstein, 1916"

[2] A book "RELATIVITY REFUTED" by Dr. Hasmukh Devidas Rathod "2024". isbn 9789334028751 or isbn 97988991701007.