Vague Generalized $\beta^*$ - Closed Sets in Vague Topological Spaces

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Abstract: The aim of this paper to explore the notation of vague sets and define a new class of vague generalized $\beta^*$-closed sets in topological spaces. We are characterizations several examples and produced some theorems are given to explained the concepts to in this paper.

Keywords: Vague set, Vague topology, Vague generalized $\beta^*$- closed set

1. Introduction

Topology is a mathematical study of the properties that are preserved through deformations, twisting and stretchings of objects. Topology began with the study of curves, surface band other objects in the plane. The objects of topology are often formally defined as topological spaces.

Topology was introduced by Johann Benedict Listing in the 19th century. Although it was not until the first decades of 20th century that the idea of Topological space was developed. By the middle of the 20th century, topology had become a major branch of Mathematics.

In 1970, Levine [3] initiated the study of generalized closed set. The concept of fuzzy sets was introduced by Zadeh [2] who introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set as a mathematical tool to solve problems and vagueness in everyday life. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. In 1970, At Anassov [1] generalized this idea to intuitionistic fuzzy sets. The theory of fuzzy topology was introduced by C. L. Chang [4] in 1968 several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. AtAnassov [1] introduced the degree of nonmembership / falsehood (F) in 1986 and defined the intuitionistic fuzzy set as a generalization of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehre [5] as an extension of fuzzy set theory in 1993.

In this paper we introduce the concept of Vague generalized $\beta^*$ -closed sets and their properties are obtained. Also its relationship with other existing sets is compared and discussed with example.

2. Preliminaries

Definition 2.1[7].

A vague set A in the universe of discourse U is characterized by two membership functions given by:
(i) A true membership function $t_A : U \to [0,1]$ and
(ii) A false membership function $f_A : U \to [0,1]$

Where $t_A(x)$ is a lower bound on the grade of membership of x derived from the “evidence for x”, and $f_A(x)$ is a lower bound on the negation of x derived from the “evidence for x”, and $t_A(x)+ f_A(x)\leq 1$. Thus, the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x),1-f_A(x)]$ of $[0,1]$. This indicates that if the actual grade of membership of x is $\mu(x)$, then, $t_A(x)\leq \mu(x) \leq 1-f_A(x)$. The vague set A is written as $A = \{x, t_A(x), 1-f_A(x)\}/\mu(U)$ where the interval $[t_A(x),1-f_A(x)]$ is called the vague value of A in X, Denoted by VA(x).

Definition 2.2[7]:

Let A and B be vague sets of the form $A = \{x, t_A(x), 1-f_A(x)\}/\mu(X)$ and $B = \{y, t_B(y), 1-f_B(y)\}/\mu(X)$ then
a) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1-f_A(x) \leq 1-f_B(x)$ for all $x \in X$

b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

c) $A = \{x, t_A(x), 1-f_A(x)\}/\mu(X)$

d) $A \cap B = \{x, \min(t_A(x), t_B(x)), \min(1-f_A(x), 1-f_B(x))\}/\mu(X)$

e) $A \cup B = \{x, \max(t_A(x), t_B(x)), \max(1-f_A(x), 1-f_B(x))\}/\mu(X)$

For the sake of simplicity, we shall use the notation $A = \{x, [t_A, 1-f_A]\}$ instead of $A = \{x, [t_A(x), 1-f_A(x)]\}/\mu(X)$.

Definition 2.3:[5]

A subset A of a topological space $(X, \tau)$ is called
a) A preclosed set if $cl(int(A)) \subseteq A$

b) A semi-closed set if $int(cl(A)) \subseteq A$

c) A regular closed set if $A = cl(int(A))$

d) A $\alpha$-closed set if $cl(int(cl(A))) \subseteq A$

e) (V) A closed set if $cl(A) = A$

Definition 2.4:

A subset A of a topological space $(X, \tau)$ is called
1) A generalized closed set (briefly g- closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X

2) A generalized closed set (briefly sg- closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in X

3) A generalized semi-closed set (briefly gs-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X

4) A generalized semi pre closed set (briefly gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X

5) A generalized pre closed set (briefly gp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X

6) A generalized $\alpha$-closed set (briefly $\alpha$g-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$

7) and U is $\alpha$-open set in X
Let A be VS of a VTS \((X,\tau)\) then
\[\text{Vscl}(A) = \bigcup \{K/ K \text{ is a VSCS in } X \text{ and } A \subseteq K\}\]
\[\text{Vsint}(A) = A \cap \text{Vcl}(\text{Vint}(A))\]

### Definition 2.11:[9]
Let A be VS of a VTS \((X,\iota)\), then the Vague alpha interior of A \((\text{Vint}(A))\) and Vague alpha closure of A \((\text{Vcl}(A))\) are defined by
1) \(\text{Vint}(A) = U \{G/ G \text{ is a VOS in } X \text{ and } G \subseteq A\}\)
2) \(\text{Vcl}(A) = A \cap \text{Vcl}(\text{Vint}(A))\)

### Definition 12
Let A be VS of a VTS \((X,\tau)\), then
1) A VS of a VTS \((X,\tau)\), then the Vague alpha interior of A \((\text{Vint}(A))\) and Vague alpha closure of A \((\text{Vcl}(A))\) are defined by
2) \(\text{Vcl}(A) = A \cap \text{Vcl}(\text{Vint}(A))\)

### Definition 2.13:[9]
A VS of a VTS, is of \((X,\tau),\text{ said to be Vague generalized closed set(VGCS)}\)
If \(\text{Vcl}(A) \subseteq U\) whenever \(A \subseteq U\) and U is VOS in \(X\).

### Definition 2.14:[9]
A VS of a VTS \((X,\tau),\text{ is said to be Vague generalized semi closed set(VGSCS)}\)
If \(\text{Vscl}(A) \subseteq U\) whenever \(A \subseteq U\) and U is VOS in \(X\).

### Definition 2.15:[9]
A VS of a VTS \((X,\tau),\text{ is said to be Vague alpha generalized closed set(VAACS)}\)
If \(\text{Vcl}(A) \subseteq U\) whenever \(A \subseteq U\) and U is VOS in \(X\).

### Definition 2.16:[6]
Let \((X,\tau)\) be a VTS and A = \(\{<x, [t_1,1-f_i] >\}\) be vague set in \(X\). Then the vague interior and a vague closure are defined by
1) \(\text{Vint}(A) = U \{G/ G \text{ is an VOS in } X \text{ and } G \subseteq A\}\)
2) \(\text{Vcl}(A) = \cap \{K/ K \text{ is a VCS in } X \text{ and } A \subseteq K\}\)

### 4. Vague Generalized \(\beta^*\) – Closed Sets

### Definition 3.1:
A VS A is said to be vague generalized \(\beta^*\)-closed set \((\text{VG}\beta^*\text{CS})\) in \((X,\tau)\) if \(\text{Vcl}(A) \subseteq U\) whenever \(A \subseteq U\) and U is \(\text{VOS in } X\). The family of all \text{VG}\beta^*\text{CSs} of a VTS\((X,\tau)\) is denoted by \(\text{VG}\beta^*\text{CS}(X)\).

#### Example 3.2
Let \(X = \{a, b\}\) and \(\tau = (0, 1)\) is a vague topology on \(X\), where \(U = \{x < [0.4, 0.7] [0.4, 0.6] >\}\)
\(A = \{x < [0.3, 0.7] [0.4, 0.5] >\}\)
\(U^c = \{x < [0.3, 0.6] [0.4, 0.6] >\}\)
is \text{VG}\beta^*\text{CS} of \(X\).

#### Theorem 3.3:
Every VGCS is \text{VG}\beta^*\text{CS} but not conversely.

#### Proof:
Let A be VGCS in \(X\) and let \(A \subseteq U\) and U be VOS in \((X,\tau)\).
Since \(\text{Vcl}(A) \subseteq \text{Vcl}(A)\) and by hypothesis, \(\text{Vcl}(A) \subseteq U\).
Therefore, A is \text{VG}\beta^*\text{CS} in \(X\).
Example 3.4
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.4, 0.7 \ [0.4, 0.6] >\}$, $A = \{x < 0.3, 0.7 \ [0.4, 0.6] >\}$, $U^c = \{x < 0.3, 0.6 \ [0.4, 0.6] >\}$ is VG$\beta$*CS in $X$ but not VGCS in $X$.

Theorem 3.5
Every $VaCS$ is VG$\beta$*CS but not conversely.

Proof:
Let $A$ be $VaCS$ in $X$ and let $A \subseteq U$ and $U$ be VOS in $X$. By hypothesis, $Vcl(Vint(Vcl(A))) \subseteq A$. Since $A \subseteq Vcl(A)$, $Vint(Vcl(Vint(A))) \subseteq A$. Hence $Vcl(A) \subseteq A \subseteq U$. Therefore $A$ is VG$\beta$*CS in $X$.

Example 3.6:
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.6, 0.4 \ [0.4, 0.6] >\}$, $A = \{x < 0.3, 0.6 \ [0.4, 0.6] >\}$, $U^c = \{x < 0.6, 0.4 \ [0.4, 0.6] >\}$ is VG$\beta$*CS in $X$ but not $VaCS$ in $X$.

Theorem 3.7:
Every $VRCS$ is VG$\beta$*CS but not conversely.

Proof:
Let $A$ be a VRCS in $X$. By definition, $A = Vcl(Vint(A))$. This implies $Vcl(A) = Vcl(Vint(A))$. Therefore $Vcl(A) = A$. That is $A$ is VCS in $X$. By Therefore $A$ is VG$\beta$*CS in $X$.

Example 3.8:
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.3, 0.7 \ [0.7, 0.8] >\}$, $A = \{x < 0.2, 0.3 \ [0.7, 0.8] >\}$, $U^c = \{x < 0.3, 0.7 \ [0.7, 0.8] >\}$ is VG$\beta$*CS in $X$ but not VRCS in $X$.

Theorem 3.9:
Every $VaGCS$ is VG$\beta$*CS but not conversely.

Proof:
Let $A$ be $VaGCS$ in $X$ and let $A \subseteq U$ and $U$ be VOS in $(X, \tau)$. $A \cup Vcl(Vint(Vcl(A))) \subseteq U$.

This implies $Vcl(Vint(Vcl(A))) \subseteq U$ and $Vint(Vcl(Vcl(A))) \subseteq U$.

Thus $Vcl(A) = A \cup Vint(Vcl(Vint(A))) \subseteq U$. Therefore $A$ is VG$\beta$*CS in $X$.

Example 3.10:
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.7, 0.8 \ [0.7, 0.8] >\}$, $A = \{x < 0.4, 0.5 \ [0.4, 0.6] >\}$, $U^c = \{x < 0.2, 0.3 \ [0.2, 0.3] >\}$ is VG$\beta$*CS in $X$ but not $VaGCS$ in $X$.

Theorem 3.11:
Every $V\beta$*CS is VSCS but not conversely.

Proof:
Let $A$ be $V\beta$*CS in $X$. Then $Vint(Vcl(Vint(A))) \subseteq A$. Since $A \subseteq Vcl(A)$, $Vint(Vcl(A)) \subseteq A$. Hence $A$ is VSCS in $X$.

Example 3.12:
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.4, 0.7 \ [0.4, 0.6] >\}$, $A = \{x < 0.3, 0.7 \ [0.4, 0.6] >\}$, $U^c = \{x < 0.3, 0.6 \ [0.4, 0.6] >\}$ is VSCS in $X$ but not $V\beta$*CS in $X$.

Theorem 3.13:
Every $V\beta$*CS is VGCS but not conversely.

Proof:
Let $A$ be $V\beta$*CS in $X$. Then $Vint(Vcl(Vint(A))) \subseteq A$. Since $A \subseteq Vcl(A)$, $Vint(Vcl(A)) \subseteq A$. Here $A$ is vague semi closed sets in $X$.

Example 3.14:
Let $X = \{a, b\}$ and $\tau = (0, G, 1)$ is a vague topology on $X$, where $U = \{x < 0.4, 0.7 \ [0.4, 0.6] >\}$, $A = \{x < 0.3, 0.7 \ [0.4, 0.5] >\}$, $U^c = \{x < 0.3, 0.6 \ [0.4, 0.6] >\}$ is VGCS but not $V\beta$*CS in $X$.

5. Diagrammatic Representation
References


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