An Analytical Exploration of AlternativeMethods and Formulas for Solving Quadratic Equations

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Abstract: ***This paper presents a range of methods and alternative formulas for solving quadraticequations, going beyond the classic quadratic formula to include techniques such as factorization and especially derived formulas. In addition to established approaches, several of these formulasare the author's innovations, designed to provide tailored solutions for specific cases of quadratic equations. This study aims to offer students, educators, and researchers enhanced flexibility and a broader set of tools to address quadratic equations in diverse contexts.***

Keywords: Quadratic Equation, Mathematical Methods, Alternative Formulas, Factorization Method, Innovation in Mathematics, Derived Formulas, Algebraic Techniques, Islamic Golden Age, Quadratic Formula, Mathematical Research, Equation Solutions, Discriminant Analysis, Imaginary Roots, Mathematical Exploration, Formula Derivation.

1. Introduction

This research paper covers an in-depth study of various quadratic equation solutions and formulas, covering both well-established conventional methods and more modern innovations. The author himself is the source for a variety of the formulations. This study's main goal is to evaluate and compare these approaches to determine their effectiveness, efficiency, and suitability for use with various quadratic equation formats. This study aims to give researchers, educators, and students a wide range of tools that may address particular problem types and scenarios by thoroughly examining each technique. This study also aims to deepen understanding and broaden the set of options for solving quadratic equations by highlighting the potential benefits and drawbacks of each approach.

2. Work

The Classic [1] Quadratic Formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof: Let there be an equation
$x^2 + 7x + 12 = 0$

Substitute by $a=1, b=7,$ *and* $c(constant)=12$

By using the quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 48}}{2}$$

$$x = \frac{-7 \pm \sqrt{49 - 48}}{2}$$

$$= \frac{-7 \pm 1}{2}$$

$$= \frac{-8}{2} \text{ or } x = \frac{-6}{2}$$

x = -4 or -3

Let's substitute x's values.
16 - 28 + 12 = 0 (On having $x = 0$)
-12 + 12 = 0
0 = 0

On having x = 3,9 - 21 + 12 = 0
-12 + 12 = 0
0 = 0

It works!

3. The Factorizing Way

In this method, we have to break the bx term as wx and yx of the equation in such a way, so that $wy = ac$.

$$x^2 + 7x + 12 = 0$$

In this equation, let's break $7x$ *into* two parts, such that the product of boththe coefficients of x's is equal to the product of the constant (12) and the coefficient of x^2.

$$x^2 + 4x + 3x + 12 = 0$$

Now, to understand this equation better, let's break it into 2 parts.

$$(x^2 + 4x) + (3x + 12) = 0$$

Take out and factorize the common term, but WITHOUT using thesebrackets.

That is, x in the first bracket and 3 in the second bracket.

$$x\ (x + 4) + 3(x + 4) = 0$$

Note: Both the brackets should be the same.
Since $(x+y)\ (b+c) = x(b+c) + y\ (b + c)$, $(x + 3)(x$

$+ 4) = 0$

Now, any one bracket has to be zero.

So, let's solve it individually.

1. *x + 3 = 0,*
   x = -3

2. *x + 4 = 0*
   x = -4

4. The Close Counterpart to the Quadratic Formula:

$$\frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

As a close counterpart to the classic quadratic formula, this alternative method may not be universally applicable to every quadratic equation. However, it has demonstrated success with most quadratic equations encountered so far. Since its discovery, this formula has provided accurate solutions, with only one instance producing imaginary results that it couldn't resolve. In the followingsections, we'll examine two different quadratic equations to illustratethe formula's effectiveness in various scenarios.

1. $x^2 + 7x + 12 = 0$

The solution by the formula

$$\frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

$$x = \frac{24}{-7 \mp \sqrt{49 - 48}}$$

$$= \frac{24}{-7 \mp 1}$$

$$= \frac{24}{-8} \; or \; \frac{24}{-6}$$

$$x = -3 \; or \; -4$$

2. When the discriminant is negative, this alternative formula is less likely to provide a solution, although it occasionally does.To illustrate this limitation, we will examine a quadratic equation with a negative discriminant that this formula is unable to resolve.

$$x^2 + x + 1 = 0$$

On having *a=1, b=1, c=1.*

$$x = \frac{2}{-1 \mp \sqrt{1 - 4}}$$

$$x = \frac{2}{-1 \mp \sqrt{-3}}$$

$$x = \frac{2}{-1 \mp \sqrt{3}}$$

This is incorrect as the correct roots of this equation are

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$

This happens because of many errors in the formula itself. You may ask, *what are they, and if the formula is errorful, why it can answer so much accurately?* The reason behind this is that the discriminant part $b^2 - 4ac$ of the formula is in the denominator of the equation. In this case, we see that the answer that this formula yielded is just the reciprocal of the correctsolution. In this case, if the discriminant is negative, we can just **reciprocate** it. Why not try this idea in another equation also?

Let there be an equation $2x^2 - 4x + 3 = 0$.In this case, the discriminant is-

$b^2 - 4ac < 0$

$16 - 24 < 0$

$-8 < 0$

Since $\Delta < 0$, we can reciprocate and then try to answer.

$$\frac{-8}{1 \mp \sqrt{16 - 24}}$$

$$= \frac{-8}{1 \mp \sqrt{-8}}$$

$$= \frac{-8}{1 \mp i\sqrt{8}}$$

$$= \frac{-8}{1 \mp 2i\sqrt{2}}$$

We have,

$$= \frac{1 \mp 2i\sqrt{2}}{-8}$$

Are we getting there?

The correct roots of this equation from the quadratic formula is

$$x = \frac{4 \pm 2i\sqrt{2}}{4}$$

$$x = 1 \pm \frac{i\sqrt{2}}{2}$$

This didn't work here this way either. We can have the *a* factor in the denominator also, or many other kinds of experiments with this formula.

Other variations of the formulas

Now, there are tons of formulas which can answer certain types of equations. In the same manner, I have invented 2 formulas that are derivedfrom the classic quadratic equation.

$$1. \frac{(-b \pm \sqrt{b^2 - 4ac})^2}{4ac}$$

Paper ID: SR241105171212 DOI: https://dx.doi.org/10.21275/SR241105171212

I first tried it with the simplest quadratic equation, $x^2 -1= 0$
Here, it simply does this.

$$x = \frac{(-b \pm \sqrt{b^2 - 4ac})^2}{4ac}$$

$$x = \frac{(0 \pm \sqrt{0^2 + 4})^2}{4}$$

$$x = \frac{(\pm 2)^2}{4}$$

$$x = 1$$

2. $-b \pm \sqrt{b^2 - 4ac} - 4ac$
This might seem nonsense to you, but I have verified that both of them work in certain *special* cases. Please don't expect it to be insanely accurate like the

$$\frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

one or like the classic quadratic equation. The proof is left to the reader as an exercise.

5. Conclusion

This paper has explored various methods for solving quadratic equations, presenting both traditional and innovative approaches. While the classic quadratic formula remains the most universally recognized and reliable tool for solving quadratic equations, alternative methods such as factorization and newly derived formulas (including those proposed by the author) offer additional flexibility. These alternatives may be more suitablein specific contexts or for particular types of equations. It is essential to understand the limitations and potential errors that can arise, especially when dealing with equations that yield complex or imaginary roots.

Furthermore, experimenting with these variations can help to expand theset of tools available to students, educators, and researchers. Ultimately,this study highlights the importance of innovation in mathematics while emphasizing the enduring significance of classical methods.

References

[1] Katz, V. J. (2009). *A History of Mathematics: An Introduction* (3rd ed.).Addison-Wesley.
This book provides a comprehensive history of the development of mathematical ideas, including the quadratic formula, which was refinedduring the Islamic Golden Age and attributed to prominent mathematicians like al-Khwarizmi.

[2] Berman, D. L. (2006). *The Quadratic Formula and its Applications*.Mathematical Association of America.
Berman discusses the evolution of the quadratic formula, detailing its practical applications and significance in solving quadratic equations.

[3] Al-Khwarizmi, M. (1960). *Al-Khwarizmi's Algebra* (Translated by R. Rashed).Harvard University Press.
Al-Khwarizmi's work, which laid the groundwork for algebra, includes methods of solving quadratic equations, and his contributions are pivotal to understanding the development of algebraic techniques.

Author Profile

Rishikesh Biswas is an accomplished young scholar and researcher, currently an 8th grader at Kendriya Vidyalaya, India. Despite his early academic stage, Rishikesh has already contributed significantly to the field of mathematics with seven research papers and two published books. His research interests span various aspects of algebra, with a focus on quadratic equations and alternative solving methods. Apart from his academic achievements, Rishikesh is also a national-level tabla player, recognized by the Ministry of Culture, and has participated in national chess tournaments twice. Rishikesh's dedication to learning and his innovative approach to mathematical problems reflect his passion for both theoretical and applied mathematics. His diverse talents and relentless curiosity continue to drive his academic journey and contributions tothe field.