Modelling of a Micro-Hydro Turbine System for Electricity Generation - A Case Study of Hhaynu Micro Hydropower Plant - Tanzania

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Abstract: Mini and Micro-hydropower plants are used to supply electricity to the rural and off-grid areas of many developing countries like Tanzania. Their power capacity ranges from lower capacity of 5 kW to a higher capacity of 100 kW which is equivalent of supplying electricity from a few households to several villages in the region. The main challenge that have influenced to undertake this research study is centred on the possibility of modelling and performance optimization of a designed micro-hydro turbine system that can meet the dynamic load demand from the growing rural and off-grid users of many developing countries like Tanzania and also at the same time achieve high energy utilization efficiency with minimum energy losses. The methods used in this research study are based on the field work and site data measurements that lead to determination of hydraulic and turbine transfer functions as inputs to system design and modelling that will optimise the system performance characteristics. The results from data analysis and modelling shows that the feasible water flow discharge for the designed micro-hydropower plant is 0.45 m³/s with the gross head of 25m that gives a turbine output (mechanical)power of 79.5 kW.

Keywords: Micro-hydro turbine, electricity, penstock pipe, transfer function, Hhaynu, Tanzania

1. Introduction

Micro-hydropower plant modelling can be done by taking into consideration the physical arrangement of its hydraulic turbine system. The general arrangement of the micro hydro turbine system consists of a weir, intake, canal, forebay, penstock and turbine/generator system. The weir is used to divert the water from a river and direct it to the intake and canal. At the end of the canal, there is a forebay which is a temporary water storage tank and is usually connected to an elevated head penstock (pipe) that carry water flow discharge with high pressure and velocity to the water turbine unit which is connected to the generator that rotated and produces electricity as shown on Figure 1.1 below.

In micro-hydro-turbine modelling, the tangential characteristics of the micro-hydropower plant are determined by the penstock dynamics, turbine dynamics, turbine control dynamics, generator dynamics and load dynamics as shown in Figure 1.2.
The turbine conduit dynamics is related to the dynamics of the water flow in the penstock which is determined by the water flow rate value $Q$, pressure head $H$, flow velocity $U$ and the length of the penstock $L$. In this relationship, the turbine and penstock dynamics are determined by the four basic relations between the turbine power $P_m$, velocity of the water in the penstock $U$ and the acceleration of water column $H$ as shown in Figure 1.3 below.

On the other hand, the turbine/generator dynamics are determined by the power/torque and the rotational speed of the prime mover which has to be maintained and controlled by turbine control dynamics through gate position from the servo motor. With linear blocks, the micro hydropower plant model can be presented as shown in Figure 1.4 below as a closed system with a feedback loop. The plant dynamics consists of gate positioner, hydraulic system and turbine/generator system. On the other hand, load disturbances are related to the unpredicted load that may be introduced into the system and may affect the system performance and stability by reducing/increasing the generator rotational speed and hence the frequency.
2. Methods and Materials

2.1 Hydraulic Turbine dynamics

Electricity generated from micro hydropower plants is an important renewable energy source in many rural areas of the developing countries which provide significant flexibility during its operation phase (off-grid operation). When most of the micro hydro turbine is in operation, their dynamic operating behaviour is mostly determined by the transient characteristics of the water flow in the penstock, although from the literature it is noted that the turbine energy conversion process has been involved non-dynamic characteristics [3]. In broad categories, micro-hydro power plants system models can be classified into two major segments based on the complexity of their transfer functions/equations involved in the modelling process i.e. the linear models which sometimes called non-elastic model and non-linear models which sometimes called elastic model.

In the modelling of a micro-hydro turbine system, the use of transfer functions/equations with linear and non-linear behaviour has to be used. The transfer functions are mathematical equations which are essential tools for the simulation of micro hydro turbine systems which will be explained in details in the following sub-sections.

2.2 Hydraulic Turbine transfer function

Most linear hydropower models are calculated around an operating point and they are extracted from basic equations of turbine and penstock hydraulic characteristics. The model equations are based on the transfer function which relates to the turbine mechanical output power to the gate opening signal. Considering the studies based on system stability, the formulation of the representation of the hydraulic turbine is based on the following assumptions:
(a) The penstock pipe is in-elastic and water is incompressible
(b) The hydraulic resistance in the system is negligible
(c) The water flow velocity in the pipe varies directly with the gate opening and with the root square of the net head
(d) The hydraulic turbine output power is proportional to the product of the head and volume of flow

2.3 Penstock Pipe and Turbine system

In a micro hydro turbine system, the penstock is the pipe which carries water discharge from the head tank (forebay)

to the turbine unit. The velocity of the water in the penstock will be determined by the pressure head/elevation between the intake and the powerhouse. The appropriate water velocity and pressure head inside the penstock will determine the turbine speed and hence the generator speed but due to the fluctuation of water volume in the penstock pipe, the speed of the turbine/generator varies as well which affect generator rotation speed and frequency. In a normal application, the speed and frequency of the micro-hydro generator system must be maintained at a constant nominal value of 1500 RPM and frequency of 50 Hz but when connected to a local grid there is usually a tolerance of ±5%. So, in order to maintain this constant generator speed and frequency value, most micro-hydro turbine system regulates the rate of water discharge to the turbine which in turn change the turbine-generator speed by the use of speed governing system.

During the turbine operation when water discharge flow through a penstock pipe, the micro-hydro turbine system responses consists of two scenarios:
(a) The transient response: which is the initial state of the turbine-generator speed response caused by the initial pressure change due to the introduction of water flow to the turbine system? In this case, the system is unstable due to changes in turbine power and speed.
(b) The steady state response: when the turbine-generator system stabilizes and gives the steady-state values of speed and mechanical power

2.3.1 Penstock pipe modelling

A: Discharge \( Q(s) \) and Pressure head \( H(s) \)

Considering the water flow in the penstock pipe from the inlet as point 1 (water entrance) and the outlet as point 2 (water exit) in Figure 2.1 below, the volumetric flow rate in the penstock \( Q \) is given by:

\[
Q = A_1 U_1 = A_2 U_2
\]

(1.1)

where; \( Q \) = water discharge \( (\text{m}^3/\text{s}) \), \( A \) = penstock cross-sectional area \( (\text{m}^2) \) and \( U \) = water flow velocity in the penstock \( (\text{m/s}) \), \( H \) or \( H_s \) = Water height \( (\text{m}) \) (In steady state operating condition, \( H = H_s \) while during transient’s condition \( H\neq H_s \))
Water flowing in the penstock from the intake at point 1 and exit at point 2 and when considering the volume flow rate in the penstock, the amount of water flow at point 1 and point 2 will remains the same [4] and thus the water flow discharge \( Q \) can be written as:

\[
Q = AU
\]

From the above equation, the value of \( Q \) can be interpreted as the rate of change volume discharge \( \Delta v \) (m\(^3\)) with a change in time \( \Delta t \) (s) and depends on the water flow velocity \( U \) and penstock cross section area \( A \).

So applying \( \Delta v \) to the above equation, the change in volume flow of the water is given by:

\[
\frac{\Delta v}{\Delta t} = AU
\]

Where: \( \Delta v \) = change in volume flow and \( \Delta t \) = change in time

Re-arranging the above equation gives, \( \Delta v = AU \Delta \) and when differentiating with respect to time it gives, \( dv = AU(t)dt \)

Simplifying the above equation based on the water flow velocity in the penstock as a function of time, the change in volume flow \( \Delta v \) becomes:

\[
\Delta v = AU(t)\Delta t
\]

On the other hand, the water flow in the penstock from the inlet and the exit can be related to the Bernoulli’s theorem that:

\[
\frac{U_1^2}{2g} + \frac{p_1}{\rho g} + H_1 = \frac{U_2^2}{2g} + \frac{p_2}{\rho g} + H_2
\]

Since both the penstock inlet and exit are exposed to the atmospheric pressure, then \( p_1 = p_2 \) and also the velocity at the penstock inlet \( U_1 \) is usually very small compared to the velocity at the penstock exit \( U_2 \) due to large volume of water from the forebay to the penstock inlet, so in this case, \( U_1 \) is set to be zero \( (U_1 = 0) \)

Thus, the above equation can be simplified and becomes:

\[
\frac{-U_2^2}{2g} + 0 + (H_1 - H_2) = 0
\]

Let \( H \) represent the difference between \( H_1 \) and \( H_2 \) then the water flow exit velocity \( U_2 \) in the penstock can be expressed as:

\[
U_2 = \sqrt{2gH}
\]

This is the exit velocity of the water in the penstock which is the function of pressure head \( H \) and gravity. This is valid for most micro-hydro turbine systems as the velocity of the water flow in the penstock depend on the pressure head \( H \).

Consider the water volume flow between the inlet and exit of the penstock pipe, the total change in volume of water entering the penstock \( (\Delta v_{in}) \) during the change in time \( \Delta t \) should be equal to the change in volume of water exit from the penstock \( (\Delta v_{exit}) \). The above relation is based on the law of conservation of energy.

The equation \( \Delta v = AU(t)\Delta t \) can be expanded to determine the volume of water entering and exit the penstock as follows:

\[
\Delta v_{exit} = A_2U_2(t)\Delta t
\]

But \( A_2 = \frac{\pi D_2^2}{4} \) where \( D_2 \) = diameter of the penstock pipe at the exit point

Then:

\[
\Delta v_{exit} = \frac{\pi D_2^2}{4} \times \sqrt{2gH} \times \Delta t
\]

When considering the \( \Delta H(t) \) as the amount of water pressure head in the penstock at the incremental time \( \Delta t \), then the rate of change water volume flow at the inlet \( \Delta v_{inlet} \) is given by:

\[
\Delta v_{inlet} = -A_1 \times \Delta H(t) = \left(-\frac{\pi D_1^2}{4}\right) \times \Delta H(t)
\]
The rate of water flow discharge per unit time on the penstock must be equal to the volumetric flow rate on the penstock, which gives:

\[
\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 \times \sqrt{\frac{g}{t}} H(t)
\]

But, \(\sqrt{\frac{g}{t}} H(t) = U(t)\)

Which gives;

\[
\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 \times U(t)
\]

The above water flow velocity with time \(U(t)\) is the velocity at the inlet of the penstock;

\[U(t) = U_1(t)\]

In this case when considering the two water flow locations on the penstock inlet and exit, the inlet and exit water flow velocities may be represented as follows:

\[A_1 U_1(t) = A_2 U_2(t)\]

Substituting the value of \(U_1\) gives;

\[U_1(t) = \frac{A_2}{A_1} U_2(t) = \left(\frac{D_2}{D_1}\right)^2 \times U_2(t)\]

Which can also be written as;

\[
\frac{U_1(t)}{U_2(t)} = \left(\frac{D_2}{D_1}\right)^2
\]

Substituting the above values to the equation \(\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 \times U(t)\) will give the following;

\[
\frac{dQ(t)}{dt} = \frac{dH(t)}{dt} + \left(\frac{D_2}{D_1}\right)^2 \times U_2(t)
\]

The velocity \(U_1(t)\) represents the water flow velocity at the penstock inlet and the velocity \(U_2(t)\) is the flow velocity at the penstock exit. Since the speed of the water flows in the penstock pipe will be determined by the approach velocity from the intake which is given by;

\[U_1(t) = \frac{D_1}{\sqrt{t}} \times \sqrt{\frac{2gH(t)}{t}}\]

Substituting the above gives;

\[U_1 = \frac{D_1}{\sqrt{t}} \times \sqrt{\frac{2gH(t)}{t}}\]

The above water flow velocity with time \(U(t)\) gives the following relation;

\[\frac{dH(t)}{dt} = \frac{A_1}{D_1^2} \times \frac{1}{2} \times \frac{1}{\sqrt{t}} \times \sqrt{\frac{2gH(t)}{t}}\]

To maintain the constant speed of the turbine at a given time, the water flow velocity \(U_2(t)\) at the exit of the penstock must be constant \(U_2\).

Thus, the above equation can be written as;

\[\frac{dH(t)}{dt} = \frac{dU_2(t)}{dt} + \frac{1}{2} \times \frac{1}{U_2^2} \times \frac{1}{\sqrt{t}} \times \sqrt{\frac{2gH(t)}{t}}\]

Taking Laplace transform of the above equation, assuming initial condition to be zero gives the transfer function as follows:

\[sQ(s) = sH(s) + \frac{1}{2} \times \frac{1}{U_2^2} \times \frac{1}{\sqrt{t}} \times \sqrt{\frac{2gH(s)}{s}}\]

Also, from the design formula, water flow discharge at the exit \(Q_2\) is given by the following relation;

\[Q_2 = A_2 U_2 = \frac{\pi D_2^2}{4} U_2\]

In this case, considering the turbine design parameter values of \(Q_2 = 0.45\text{m}^3/\text{s}\) and \(D_2 = 460\text{mm}\)

Putting these values to the above equation gives the following results for the water flow velocity in the penstock pipe:

\[U_2 = \frac{4Q}{\pi D_2^2} = 2.7\text{ m/s}\]

Then, substituting the values of \(U_2\) and using \(g = 9.81\text{ m/s}^2\) on the equation;

\[sQ(s) = sH(s) + \frac{1}{2} \times \frac{1}{U_2^2} \times \frac{1}{\sqrt{t}} \times \sqrt{\frac{2gH(s)}{s}}\]

\[sQ(s) = sH(s) + \frac{2gH(s)}{2.7}\]

Re-arranging the above equation gives the following relation;

\[\frac{H(s)}{Q(s)} = \frac{1}{s+7.27}\]

This is the transfer function of the pressure head \(H(s)\) to flow discharge \(Q(s)\) in the penstock pipe which is represented in the following Figure 2.2 block diagram.

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**Figure 2.2:** Step signal input \(Q(s)\) to the signal response \(H(s)\)
On the other hand, when considering the water flow velocity $U$ inside the penstock pipe and in relation to gate opening $G$ and pressure head $H$, the following equation applies with per unit values [5].

$$\bar{U} = \bar{G} \sqrt{\bar{H}} \quad (1.28)$$

For the non-linear model, the water flow velocity in the penstock can be very difficult to determine, but when substituting the value of velocity $U$ with discharge $Q$ value as $\bar{Q} = A\bar{U}$. In this case, the following condition applies for gate valve section: $0 \leq \bar{G} \leq A$ and in per unit values this can be written as $0 \leq \bar{G} \leq 1$ [6]

$$\bar{Q} = A \bar{G} \sqrt{\bar{H}} \quad (1.29)$$

$$\sqrt{\bar{H}} = \frac{\bar{G}}{\bar{A}}$$

Thus;

$$H = \frac{Q^2}{G^2} A^2 \quad (1.30)$$

The above equation can be represented in the following block diagram with the input gate valve position $G$ and flow discharge $Q$ and output pressure head $H$ as shown in Figure 2.3 below.

![Figure 2.3: Formulation of hydraulic head H equation based on Q and G during modelling](image)

B: Discharge $Q(s)$ and Mechanical Power $P(s)$

The Power output per second ($P(t)$) from the hydropower plant depends on the water discharge, weight per second, pressure head and system efficiency. This relation is given by the following equation.

$$P(t) = mQ\eta H(t) \quad (1.31)$$

where; $P(t) = \text{power per second}$, $m = \text{mass of water flow per second}$, $Q = \text{discharge}$ and $H = \text{pressure head}$ and $\eta$ is the hydraulic efficient.

From the above formula the value of flow discharge $Q$ and pressure head $H$ are usually maintained at a constant value at the turbine entry so, in this case the weight of water flowing per second can be expressed in terms of water density $\rho$ and water volume $v$ as follows:

$$P(t) = \rho v Q\eta H(t) \quad (1.32)$$

From the above equation the volume $v$ per second is referred to as flow discharge, so the above equation can be re-written as:

$$P(t) = \rho Q^2 \eta H(t) \quad (W) \quad (1.32)$$

or in terms of kW capacity

$$P(t) = Q^2 \eta H(t) \quad (kW)$$

Re-arranging the above equation and taking the Laplace transform we get;

$$H(s) = \frac{P(s)}{\eta Q^2} \quad (1.34)$$

Substituting the value of $H(s)$ to the above equation from the transfer function of Equation 5.28 we get;

$$\bar{P}(s) \approx \frac{\bar{Q}(s)^2}{\bar{G}(s)^2} \quad (1.35)$$

Applying the following system design parameters, flow discharge $Q = 0.45 \text{ m}^3/\text{s}$ and hydraulic turbine efficiency $= 72\%$ to the above equation gives the following Laplace equation;

$$\frac{\bar{P}(s)}{\bar{Q}(s)} = 0.146 \frac{s}{s+7.27} \quad (1.36)$$

The following Figure 2.4 is the transfer function block diagram of turbine power $P(s)$ to flow discharge $Q(s)$ as shown in details below.

![Figure 2.4: Block diagram of Q(s) signal input to P(s) signal output](image)

It is noted that in this case there is a gain value of 0.146 on the transfer function for the flow discharge, $Q(s)$ to the power output $P(s)$ compared to a previous transfer function for $Q(s)$ and $H(s)$.

When relating the input signals for pressure head $H(s)$ and flow discharge $Q(s)$, the power output signal can be derived with the following relation based on mechanical power, head and flow in the turbine as follows:

For the transfer function that relates turbine velocity to the turbine head is given as [7];

$$TF(s) = \frac{\Delta P}{\Delta H} \quad (1.37)$$
But for a non-linear turbine model for this design where the pressure wave in the penstock and water compressibility is not considered, then the above equation becomes:

$$TF(s) = \frac{1}{\tau_w s}$$  \hspace{1cm} (1.38)

where; \(\tau_w\) = Water starting time (1.79 seconds)

From the equation 5.38, substituting the \(\tilde{U}\) with \(\frac{\bar{Q}}{A_p}\) we get the following:

$$TF(s) = \frac{\delta \tilde{U}}{\Delta H} = \frac{1}{\tau_w s}$$  \hspace{1cm} (1.39)

where; \(A_p\) = Area of the penstock pipe

Simplifying the above equation, we get:

$$\Delta \tilde{Q} \Delta H = \frac{A_p}{\tau_w s}$$  \hspace{1cm} (1.40)

In relation to modelling, the value of \(A_p\) is taken as a gain (constant), so in this case, the mechanical power \(P_m\) developed in a micro-hydro turbine depends on the net pressure head \(\Delta H\) and water flow discharge \(\Delta Q\).

Then the equation 1.41 below relates the turbine mechanical power \(P_m\) in with the net head \(\Delta H\) and net flow discharge \(\Delta Q\) as follows:

$$\bar{P}_m = \Delta \tilde{Q} \Delta H$$  \hspace{1cm} (1.41)

But theoretically, power from the micro-hydro turbine due to water flow discharge is given by equation 1.12. So, in this case, the parameters \(\rho g\) are the power gains for the system model and are represented as a constant value.

When considering per unit values of the mechanical power output, the nominal operating point of the hydro turbine need to be considered.

Thus, at the nominal operating point;

$$P_N = \rho g H(Q_N - Q_v)$$ and also \(Q_N = AG\sqrt{\Delta H}\)  \hspace{1cm} (1.42)

where; \(P_N\) = Nominal power of turbine, \(H_s\) = Water height, \(Q_v\) = Nominal water flow, \(Q\) = Volumetric flow losses.

When normalizing the above turbine power equation, we get:

$$P_m (p.u.) = K P_u (Q_{pu} - Q_v)$$  \hspace{1cm} (1.43)

Simplifying the above equation, we get:

$$P_m (p.u.) = K_p H_{pu} (Q_{pu} - Q_v)$$  \hspace{1cm} (1.44)

In this case, the value \(K_p\) = power gain given by \(\rho g\) in nominal values and \(K_p = \frac{1}{1 - \phi(p.u)\text{ in p.u. values.}}\)

The above Equation 1.44 with p.u. values can be simulated based on the set parameters as shown on the block diagram of Figure 2.5 below.

When turbine efficiency is considered during modelling then the output turbine power obtained is the mechanical power given by the following equation:

$$P_m = \rho g H \Delta Q \eta_t$$  \hspace{1cm} (1.45)

The above equation 1.45 is the general equation used to calculate the potential power output from a hydro turbine system with known site parameters of head \(H\) and water flow discharge \(Q\). The required amount of water flow is allowed to pass through the gate valve and discharged to the turbine and cause the turbine to rotate which produce hydraulic power. In the conversion process, the output hydraulic power value must be multiplied by a power gain which is \(\rho g\) factor from the formula and also turbine efficiency \(\eta_t\) in order to obtain mechanical power as shown in Figure 2.5 below.

2.3.2 Hydraulic Turbine modelling

In micro-hydro turbine modelling with a small variation around an operating point, consideration should be made on steady-state operating conditions and also on their linearized condition which can be related by the Taylor series linear equation as follows for the change in flow discharge and mechanical power respectively [8]:

$$\Delta Q = a_{11} \Delta H + a_{12} \Delta G + a_{13} \Delta \omega$$  \hspace{1cm} (1.46)

$$\Delta P_m = a_{21} \Delta H + a_{22} \Delta G + a_{23} \Delta \omega$$  \hspace{1cm} (1.47)

Where the constants:
a_{11} and a_{21} = partial derivative of discharge and mechanical power with per unit deviation in head
a_{12} and a_{22} = partial derivative of discharge and mechanical power with per unit deviation in gate position
a_{13} and a_{23} = partial derivative of discharge and mechanical power with per unit deviation turbine speed.

The above two equations are related to the small change of water flow discharge, ΔQ and change in mechanical power, ΔP_m with respect to change in water head ΔH, change in gate position ΔG and change in turbine speed Δω. Applying partial derivative to the above equations in relation to the penstock water flow dynamics, the transfer functions of the micro hydro turbine can be developed as follows:

Equation 1.46: \( \Delta Q = a_{11}\Delta H + a_{12}\Delta G + a_{13}\Delta \omega \) with partial derivative gives as follows:
\[ a_{11} = \frac{\partial Q}{\partial H} \text{ and } a_{12} = \frac{\partial Q}{\partial G} \text{ and } a_{13} = \frac{\partial Q}{\partial \omega} \]

Equation 7.47: \( \Delta P_m = a_{21}\Delta H + a_{22}\Delta G + a_{23}\Delta \omega \)
\[ a_{21} = \frac{\partial P_m}{\partial H} \text{ and } a_{22} = \frac{\partial P_m}{\partial G} \text{ and } a_{23} = \frac{\partial P_m}{\partial \omega} \]

In this case, the penstock-turbine Laplace transform transfer function related to the change of mechanical power \( \Delta P_m \), to the change in gate position, \( \Delta G \), is given by [9]:
\[ \frac{\Delta P_m(s)}{\Delta G(s)} = \frac{a_{23} - (a_{12}xG + a_{13}xG^2)xT_w}{1 + a_{11}xT_w} \] (1.48)

When considering the hydraulic turbine change in mechanical power \( \Delta P_m \) to the change in gate valve position \( \Delta G \), the developed transfer function in the block diagram is given in Figure 2.6 below.

\[ \Delta G(s) \rightarrow \frac{a_{23} - (a_{12}xG + a_{13}xG^2)xT_w}{1 + a_{11}xT_w} \rightarrow \Delta P_m(s) \]

Figure 2.6: Block diagram transfer function of the linearized turbine model

To develop the system, transfer function we have to consider for an Ideal hydraulic turbine (lossless turbine) at rated speed and head with the initial values of partial derivatives as follows [6]:
\[ a_{11} = 0.5, a_{12} = 0, a_{13} = 1 \]
and
\[ a_{21} = 1.5, a_{22} = -1, a_{23} = 1 \]

Substituting the above partial derivative values for the ideal turbine we obtain the following short form linearized first-order transfer function:
\[ \frac{\Delta P_m(s)}{\Delta G(s)} = \frac{1-xT_w}{1+0.5xT_w} \] (1.49)

The above Equation 1.49 has been represented in a block diagram as the deviation in gate position and mechanical power as shown in Figure 2.7 below.

\[ \Delta G(s) \rightarrow \frac{1-xT_w}{1+0.5xT_w} \rightarrow \Delta P_m(s) \]

Figure 2.7: Stead state turbine-penstock transfer function block diagram

From the above Equation 1.49, it noted that the main characteristic of the transfer function is that behaves as a non-minimum phase system on the Laplace transform s-plane.

The term \( T_w \) is related to the micro-hydropower turbine starting time which depends on the speed of water flow in the penstock. It can be defined as the time required to accelerate water flow in the penstock pipe from the minimum speed at the water entering the penstock to a maximum speed of the water exit from the penstock.

Mathematically:
\[ T_w = \frac{L \times Q}{H \times A_p \times g} = 1.79 \text{ seconds} \] (1.50)
where;
\( L \) = length of the penstock (162 m), \( Q \) = design discharge (0.45 m³/s), \( H \) = design head (25 m), \( A_p \) = area of the penstock pipe (0.166 m²) and \( g \) = acceleration due to gravity (9.81 m²/s).

The value of \( T_w \) is an important parameter used in micro-hydropower modelling and this value usually varies because it depends on the speed (\( U \)) of the water flow in the penstock. Typical values of \( T_w \) for micro-hydropower systems range between 0.5 seconds to 4.0 seconds [6].

In this case, using the \( T_w \) value of 1.79 seconds for the turbine design and by referring to the equation 1.50 and Figure 2.8, the stead-state turbine-penstock transfer function for the change in gate valve position \( \Delta G \) in relation to the change in mechanical power \( \Delta P_m \) can be represented in the following block diagram. Also, in this case, an integrator and feedback control loop has been introduced on the model together with the signal filter as shown in Figure 2.8 below.

\[ \Delta G(s) \rightarrow -1.79s+1 \rightarrow \frac{0.895s+1}{s} \rightarrow 1/s \rightarrow \Delta P(S) \]

Figure 2.8: Block diagram for stead state turbine-penstock of a linearized micro hydro turbine model

On the other hand, the micro-hydropower systems with short and medium penstock length like that of Hhaynu river in Mbulu, the design consideration has been based on a non-linear system model, where the transfer function does not consider the travelling pressure wave and water compressibility, thus gives the simplified transfer function as follows [7]:
\[ \frac{\Delta U(s)}{\Delta H(s)} = \frac{1}{xT_w} \] (1.51)

Substituting the \( T_w \) value on the above equation gives:
\[ \frac{\Delta U(s)}{\Delta H(s)} = \frac{1}{1.79s} \]
In hydraulic modelling, the relationship between power output signal \( P(s) \) to the water flow input signal \( Q(s) \) is obtained by substituting the values to the above equation 1.51 with \( H(s) \) values from equation 1.34 on which we obtain the following relation:

\[
\frac{\Delta U(s)}{P(s)} = \frac{1}{\eta Q^2} = \frac{1}{s T_w} \quad (1.52)
\]

\[
\frac{\Delta P(s)}{\Delta Q(s)} = \frac{1}{\eta Q^2 s T_w} = 0.26 \quad (1.53)
\]

where; \( Q = 0.45 \text{ m}^3/\text{s}, \eta = 0.72 \) and \( s T_w = 1.79 \) seconds.

This is the transfer function between the water flow velocity in the penstock to the power output from the turbine.

### 3. Conclusion

In the hydraulic turbine system model, the following parameters were used as modelling parameters to develop system transfer functions, water flow discharge, \( Q \) through the penstock pipe and pressure water head, \( H \) which have been obtained from the site measurements. The input parameter of hydraulic turbine system model is the water flow discharge/velocity through the penstock pipe with gating system to the turbine unit on which when modelled produce turbine power (Mechanical Power), \( P_m \) as output parameter. The modelling results shows that, the mechanical power produced is related to the amount of water flow discharge/velocity in the penstock pipe to the turbine system. Using the design parameters as modelling values for \( Q \) and \( H \), the output turbine (mechanical) power, \( P_m \) obtained during modelling is 79.5 kW with the hydraulic turbine system efficiency, \( \eta_m \) of 72% with water starting time, \( T_w \) of 1.79 seconds.

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### References


