Palash's Law of Fluid Dynamics

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Abstract: ***This paper presents a new sub branch of mechanical physics:- The Palash's law of fluid dynamics, shortly known as PLFD, that is valid for liquids as well as gases and that deals with the understanding of motion of fluids of different properties through an enclosed medium. This research document incorporates a range of theory and mathematical equations, calculations and proofs that will give the world an authority to make accurate calculations of values for different properties of the fluids in motion like:- velocity, acceleration, direction, density etc.***

Keywords: Fluid current, Fluid velocity, Fluid Circuits, Force, Resistance, Laminar flow, Turbulent flow, Physics, Fluid mechanics, Motion, Liquids, Gases.

1. Purpose

The purpose of this research paper is to give the world a better sense of motion of liquids and gases, and also provide the world with the methods to do mathematical calculations related to the motion of fluid through an enclosed medium.

2. Significance

This research contributes all the mathematical equations with proofs that will open the gates to calculate the properties of motion for the fluids having different properties and provide betterment in the fields including civil engineering, mechanical physics, advance physics and even rocket science.

3. Introduction of the law

To understand the basic definition of palash's law, we first have to understand the meaning of the term "fluid current". Fluid current is the rate of flow of fluid volume with respect to the time and can be written in a form of an equation as shown below:-

$$f_c = \frac{dv}{dt}$$

$$dV = f_c dt$$

$$= \int_{V_1}^{V_2} dV = f_c \int_{t_1}^{t_2} dt$$

$$\Delta V = f_c \Delta t$$

this is the simplest form of equation for the fluid current, where fluid current is denoted by the "f_C". "V" is the volume and "t" is the time. This equation can be simplified in the form:-

$$f_c = \frac{1}{\rho}.\left(\frac{dm}{dt}\right)$$

Now this equation gives a clear sense about what fluid current is. Here "ρ" is the density of the fluid and "m" is the mass of the fluid. It can be understood from the above equation that how is fluid current is affected by the density of the fluid flowing through the medium. Fluid current is inversely proportional to density, that states that the fluid flowing through the medium having lesser density will have a greater value of fluid current and vice versa, which can be also understood logically. But the above equation is not the final formulae that is to be used, it is just a basic definition for it, and the final formulae is yet to come ahead in this research paper.

Now after getting the key idea about the fluid current, we shall look forward to learn about the Palash's law of fluid dynamics.

<u>The law</u>

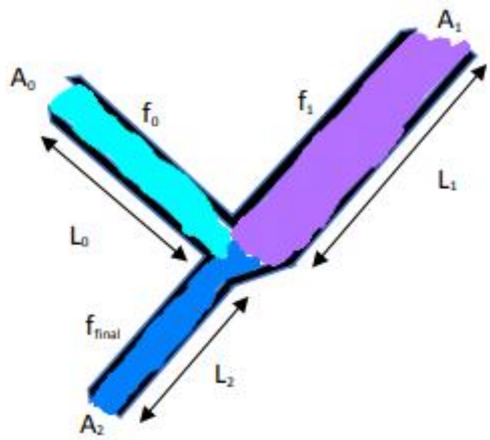
The Palash's law states that:-

"the vector addition of all the fluid currents in a closed system always equates to zero".

To understand this law mathematically, we will have to take a case of fluid circuit:-

Suppose there are 2 different fluids having unique density in a system flowing through 2 different tubes of unique cross section as shown in the figure below

Here 2 fluids of different densities are flowing through the tubes 1 and 2. By assuming that the variation in their density is large, we know that both the liquids will enter the third tube. Both the fluids before meeting at the junction will have their own unique value of fluid current as "f_0" and "f_1" and similarly after passing the junction, the final value of fluid current will be unique from the initial value of fluid current. Now by using PLFD, This is to be expressed mathematically as:-



$$(f_c)_0 + (f_c)_1 - (f_c)_{final} = 0$$

$$(f_c)_0 + (f_c)_1 = (f_c)_{final}$$

$$(\frac{1}{\rho}\frac{dm}{dt})_0 + (\frac{1}{\rho}\frac{dm}{dt})_1 = (\frac{1}{\rho}\frac{dm}{dt})_{final}$$

or

$$(\frac{1}{\rho}\frac{dm}{dt})_0 + (\frac{1}{\rho}\frac{dm}{dt})_1 = (f_c)_{final} \qquad (1)$$

This is the method to find the value of fluid current in the unknown tube.

We clearly see that $(f_c)_{final}$ doesn't depend on the angles at which the system of tubes are arranged. So it doesn't matter the way the fluid circuit is arranged.

From the equation 1 we see that the fluid that is approaching towards the junction, is given a positive value for its fluid current, and the one moving away from the junction is been given a negative value of its fluid current. So from here we conclude one more important theorem for our law as:-

"the fluid that approaches towards the junction will have a positive value for its fluid current, and the fluid moving away from the junction will have a negative value for its fluid current, and thus the summation of all this fluid current equates to zero".

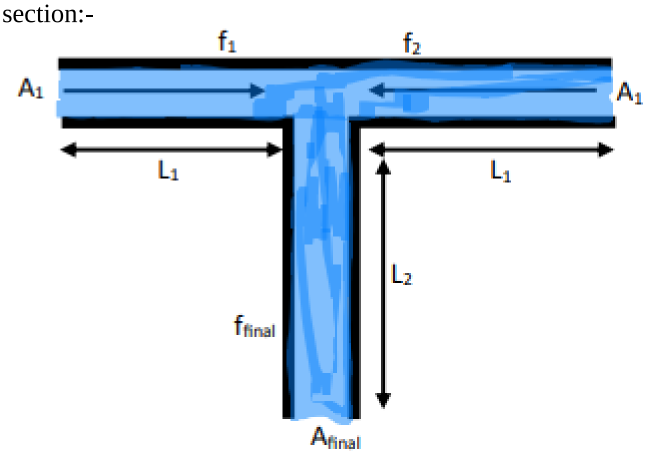
This law seems very similar to the Kirchoff's law for electrical circuits in current and electricity, but the above equation is just a simpler version to demonstrate Palash's law, and a more detailed idea of fluid current is provided in this research paper ahead.

To show that if this law is valid or not, in the next part of this research paper

I will present the complete proof for PLFD:-

Proof of PLFD

To proof this law, lets again take a case of fluid circuit with 2 same fluids of flowing in 2 different tubes of same cross section:-



$A_{final} = 2A_1$

Now using PLFD:

$$(f_c)_1 + (f_c)_2 - (f_c)_{final} = 0$$

$$(f_c)_1 + (f_c)_2 = (f_c)_{final}$$

$$\frac{1}{\rho}(\frac{dm}{dt})_1 + \frac{1}{\rho}(\frac{dm}{dt})_2 = \frac{1}{\rho}(\frac{dm}{dt})_{final}$$

Now,

$$M = \rho V = \rho AL$$

$$\frac{1}{\rho}(\frac{d\,[A.l]}{dt})_1 + \frac{1}{\rho}(\frac{d\,[\rho.A.l]}{dt})_2 = \frac{1}{\rho}(\frac{d\,[\rho.A.l]}{dt})_{final}$$

$$(\frac{d\,[A.l]}{dt})_1 + (\frac{d\,[A.l]}{dt})_2 = (\frac{d\,[A.l]}{dt})_{final}$$

Area of the tubes is uniform in time, so:-

$$A_1.(\frac{dl}{dt})_1 + A_2.(\frac{dl}{dt})_2 = A_{final}.(\frac{dl}{dt})_{final}$$

Here $\frac{dl}{dt}$ term is simply derivative of length with respect to time, that equals velocity of the fluid. Therefore:-

$$(A_1.v_1 + A_2.v_2) = A_{final}.v_{final} \;\ldots\ldots(2)$$

From the above we can clearly see that this is the equation of continuity that is an existing law and is proved mathematically.

Therefore from equation 2 we proof that Palash's law is valid and true.

Also we get:-

$$A_{final} = 2A_1$$

Therefore, the value of final velocity will be:-

$$v_{final} = \frac{(v1 + v2)}{2}$$

Now next I will present a more functional formulae and definition for the fluid current, in the further part of this research paper.

Fluid Current

In the beginning of this paper we defined fluid current as:

$$f_c = \frac{1}{\rho}.(\frac{dm}{dt})$$

but as we will look forward for the problems in fluid circuit, we notice that this equation doesn't give detailed information of the properties of the fluid, so therefore we will rewrite the above equation to derive a more functional formulae to calculate fluid current.

Using Newton's third law:

$$\sum_{i=1}^{n} F_i = ma$$

$$\sum_{i=1}^{n} F_i = m\frac{dv}{dt}$$

$$\sum_{i=1}^{n} F_i \int_{t_1}^{t_2} dt = m\int_{v1}^{v2} dv$$

$$\sum_{i=1}^{n} F_i.\triangle t = m.\triangle v$$

Therefore, we get equation for mass in terms of force as:

$$m = \sum_{i=1}^{n} F_i.\left(\frac{\triangle t}{\triangle v}\right) \ldots\ldots\ldots (3)$$

If the change in velocity and time is small:

$$m = \sum_{i=1}^{n} F_i.\left(\frac{dt}{dv}\right) \ldots\ldots\ldots (3)$$

The reason why the summation term is given is that when the fluid moving in its own medium would face different forces throughout its trajectory we would need to address it properly in the given equation. As in the latter of this paper we would add different entities in the fluid circuit problems that would create a huge impact on the forces acting on individual fluids, that would indirectly affect the properties of motion of fluids.

Therefore we substitute this equation for mass in our required equation:-

$$f_c = \frac{1}{\rho}.\left(\frac{d\,[\sum_{i=1}^{n} Fi]}{dt}.\left\{\frac{dt}{dv}\right\}\right)$$

$$f_c = \frac{1}{\rho}.\left(\frac{\sum_{i=1}^{n} Fi}{v}\right) \ldots..\textit{Fuid Current Formula}$$

This is a more functional formula to calculate the fluid current.

Unit of fluid current

Basic unit for fluid current is:- $N\text{-}m^{2}\text{-}s\text{-}kg^{-1}$

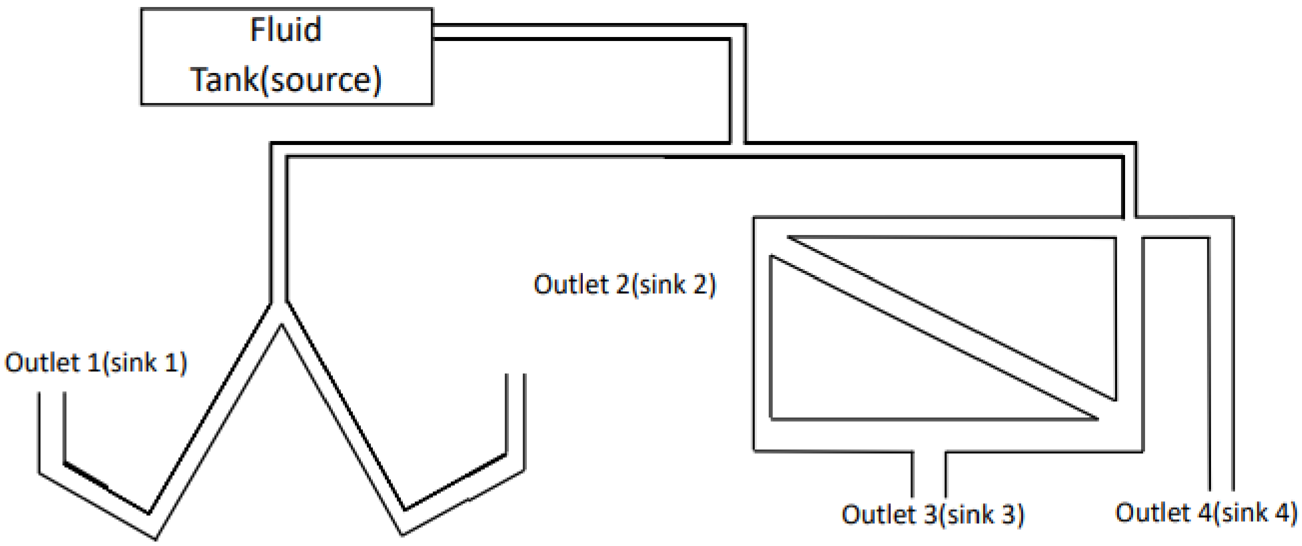
The SI unit of fluid current is:- Shimpi (S)

Fluid Circuits

"A fluid circuit is a network of tubes interlinked with each other at junctions to form a pathway for a fluids to travel through in the desired trajectory or direction."

These circuits are present in the civil structures like huge buildings or skyscrapers, in rockets, in labs, and in many different places.

The fluid circuit consists of three different parts, that are fluid source, fluid sink and a network. A typical example is shown below.



Here in the diagram the fluid tank is the fluid source through which the fluid is introduced in the network, and there are 4 outlets through which the fluid is drained out at different rates. Here the outlets need not have to necessarily be drain pipes, instead it can be some kind of applications, like taps, flush, showers or any water application entity.

Now I will introduce different entities in the fluid circuit ahead, to make more detailed research on how their presence affects the properties of motion of the fluids in closed mediums.

Electrical Motors in Circuits

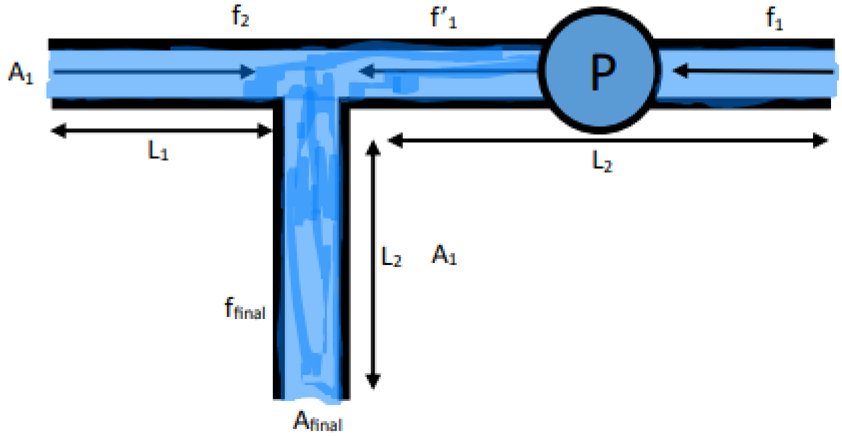
Now we will discuss about the impact of presence of motor in the fluid circuit and how it affects the motion of the fluid.

The presence of motor in the circuit will be represented by the symbol:

Here the "P" denotes the power that the motor has. The motor is such an entity that largely affects the velocity of fluid incoming and outgoing through it.

Now to look at how to deal with this situation to solve Mathematical problems for fluid circuit, I will present the method to calculate the fluid current for this case.

We will again take a case of fluids flowing through the junctions:-



Here one of the tube consists of a electrical motor. The power of the motor is "P". We would assume the velocity of fluid on the RHS of the motor to be υ_i and υ_f for the fluid on LHS of the motor.

So mathematically:-

The power of the motor is:-

$$P = \frac{V^2}{R}$$

"V" is the potential difference supplies to motor and r is internal resistance.

The power is also represented by the followingformula:-

$$P = F\upsilon \rightarrow F = \frac{P}{\upsilon}$$

Therefore:

$$F = \frac{V^2}{R \triangle \upsilon}$$

Here the velocity υ is the velocity of the fluid approaching out of the motor.

Therefore using PLFD for above case:-

$$(fc^{|})_1 + (f_c)_2 = (f_c)_{final}$$

$$\frac{1}{\rho 1}.(\frac{\sum_{i=1}^{n} Fi}{\upsilon})_1 + \frac{1}{\rho 2}.(\frac{\sum_{i=1}^{n} Fi}{\upsilon})_2 = (f_c)_{final}$$

$$\frac{1}{\rho 1}.(\frac{\frac{V^2}{R \triangle \upsilon} - 6\pi\eta r\upsilon}{\upsilon})_1 - \frac{1}{\rho 2}.(\frac{6\pi\eta r\upsilon}{\upsilon})_2 = (f_c)_{final}$$

Here the term $6\pi\eta r\upsilon$ is the viscous force acting on the moving fluid, and η is the viscosity coefficient of the specific fluid.

Now if we assume the density to be constant in all tubes:-

$$(\frac{\frac{V^2}{R \triangle \upsilon} - 6\pi\eta r\upsilon}{\upsilon})_1 - (\frac{6\pi\eta r\upsilon}{\upsilon})_2 = (\frac{6\pi\eta r\upsilon}{\upsilon})_{final}$$

The initial velocities can be simply found out using the principle of conservation of mechanical energy, if it is coming from a fluid source above at height "h"

$$Mgh = \frac{1}{2} M \,(\upsilon_f^2)_{final}$$

Tesla Valve or valvular conduit in Circuits

In this part we would discuss what is the tesla valve and its impact of its existence in the fluid circuits.

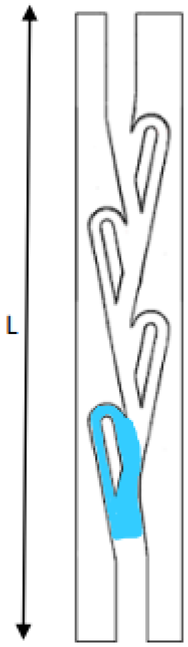
"Tesla valve is a one way valve and rigid piece of instrument with no moving parts that was invented by the great scientist Nikola Tesla. It is a series of interconnected, and asymmetric, droplet-shaped loops that are called slits, to move fluid in a single direction."

Tests showed the valve to be quite capable of directing forward flows, but less adept at reversing the flow.

The way a tesla valve works is that, when a liquid or gas is moving in the low flow direction, it creates backflows that increase the amount of energy and pressure necessary to force the liquid or gas in that direction. In the opposite direction, the liquid or gas can run through the main channel with very little opposition.

A more detailed explanation on working of this valve is not given into this paper.

The Tesla valve is explained using the image below:-



here the region highlighted in blue color is **one slit** of the valve. In this case for the image on left, there are in total four slits. L is the length of the valve, A is the area of the tube in the Tesla valve.

Now we know that the valve creates an objective resistance on the fluid moving through it, that is to be termed as ***"demurance"*** that is denoted by **"D_T"**.

It can be understood logically that the demurance of the valve will increase with the increase in the number of slits **"N"** in the valve, and also increase in the length **"L"** of the valve, and the fluid with higher value of viscosity coefficient **"η"** would face greater demurance by the valve. The fluid moving with higher velocity **"υ"** will correspond to higher value of demurance due to greater interference with the fluid molecules and energy losses through the slits. The demurance of the Tesla valve is also somehow related to the area **"A"** of the valve through which the fluid flows. Experimentally it can be seen that the value of demurance comes to be inversely proportional to the area of the valve.

Considering the above statements, lets see how the terms in relation to the demurance are in proportion to each other:-

$$D_T \infty \upsilon^{\alpha} \infty \eta^{B} \infty\, L^{\delta} \infty\, N \,\infty \left(\frac{1}{A}\right)\Psi$$

$$D_T \infty \left(\frac{N\upsilon\eta L}{A}\right)$$

Therefore the terms α, β, δ, ψ are the powers of the terms in proportionality with demurance. To find the values of these terms, we would need to do a dimension analysis.

Now as we know that demurance is a resistive force, D_T will have the dimension of the force.

Therefore:-

$$D_T = [MLT^{-2}]$$

As we already know that:-

$$[\eta] = [ML^{-1}T^{-1}];\ [\upsilon] = [LT^{-1}]$$

$$[A_T] = [L^2]$$

And N is a numerical and φ is the proportionality constant, therefore the are dimensionless quantity.

So now writing the dimension equation:- $[MLT^{-2}] = [LT^{-1}]^{\alpha}.[ML^{-1}T^{-1}]^{\beta}.[L]^{\delta}.[L^2]^{-\Psi}$.

Therefore deriving equations to find the value of terms in power:-

$$M^{\beta}.L^{(\alpha-\beta+\delta-2\Psi)}.T^{-(\alpha+\beta)} = MLT^{-2}$$

Therefore we get the following equations:-

$$\beta = 1;\ (\alpha - \beta + \delta - 2\psi) = 1\ ;\ -(\alpha + \beta) = -2$$

Therefore we get the values as follows:

$$\beta = 1\ ;\ \alpha = 1$$

Now there are 2 unknown values for the above equation as:

$$(\delta - 2\psi) = 1$$

It is clear that the demurance is proportional to L and area.

Therefore:

$$\delta \neq 0\ \ ;\ \ \psi \neq 0$$

and it can be logically or experimentally understood that:

$$\delta \in +ve\ R$$

as the increase in length results in greater value of demurance.

Therefore: possible value of terms can be:

$$\delta = 2\ \ ;\ \ \psi = -\frac{1}{2}$$

Now summing the above proportionality relation into a formulae as:-

$$D_T = \phi\,\left(\frac{L^2\upsilon\eta N}{\sqrt{A}}\right)$$

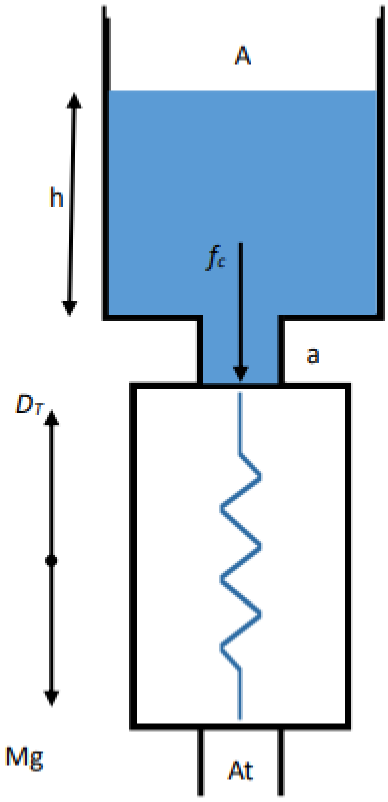
This is an important formula to calculate demurance for the Tesla valve.

Where **"φ"** is the proportionality coefficient, also called as the Palash's coefficient of demurance.

The unit for demurance is:- Newton

Palash's Apparatus to calculate demurance coefficient:

The values of the palash's coefficient depends on the dimensions of the tesla valve, as well as the properties of the fluid (density and viscosity). To determine the value of coefficient of specific fluid, I represent the formula for φ:-



In the equillbrium when the fluid just starts to move through the valve:-

$$D_T = (\rho gh).\ A_c$$

$$\frac{\varphi\ \{N\upsilon L^2\eta\}}{\sqrt{At}} = (\rho gh).A_c$$

Now here, using the continuity equation:-

$$A\left[-\frac{dh}{dt}\right] = av$$

$$-A\int_{h1}^{h2} dh = av\int_{t1}^{t2} dt$$

$$-A\ (\triangle h) = av\ (\triangle t)$$

Therefore we get equation for the velocity from here:-

$$v = -\frac{A}{a}\ (\frac{\triangle h}{\triangle t})$$

Now, substituting this value of the velocity in the equation 1:-

$$\frac{\varphi\{N\upsilon L^2\eta\}}{\sqrt{At}} = (\rho gh).A_c$$

$$\frac{\varphi\{NL^2\eta\}}{\sqrt{At}}(-\frac{Ac}{a})\frac{\Delta h}{\Delta t} = (\rho gh).A_c$$

$$\frac{\varphi\{NL^2\eta\}}{\sqrt{At}}(-\frac{1}{a})\frac{\Delta h}{\Delta t} = \rho gh$$

$$\boxed{\varphi = \frac{\rho gha\sqrt{At}}{NL^2\eta\{-\frac{\Delta h}{\Delta t}\}}}$$

Or

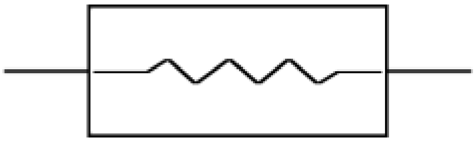
$$\boxed{\varphi = \frac{\rho gha\sqrt{At}}{NL^2\eta\{-H(t)\}}}$$

where -H(t) is the declination of the level of the height of the water as a function of time, and At or A_t is the area of the telsa valve

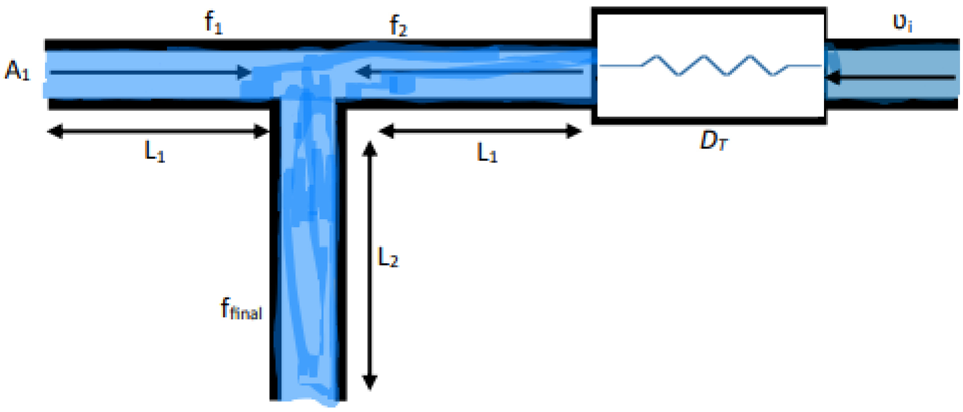
Therefore the above equation gives the value of Palash's coefficient of demurance.

So, we can now include the tesla valve in our fluid circuit problems by giving it a unique significance, and solve problems related to it.

For fluid circuit problems for Tesla valve, we would denote the Tesla valve by the symbol:-



We will now look at a similar fluid problem as in above pages for the Tesla valve:-



using PLFD, and assuming same density:-

$$-(\frac{D+6\pi\eta r \upsilon}{\upsilon})_1 - (\frac{6\pi\eta r \upsilon}{\upsilon})_2 = -(\frac{6\pi\eta r \upsilon}{\upsilon})_{final}$$

There are negative signs to every fluid current as, only resistive forces are acting in system.

$$(\frac{D+6\pi\eta r \upsilon}{\upsilon})_1 + (\frac{6\pi\eta r \upsilon}{\upsilon})_2 = (\frac{6\pi\eta r \upsilon}{\upsilon})_{final}$$

$$(\frac{n\{\upsilon i\}\eta L+6\pi\eta r \upsilon}{\upsilon})_1 + (\frac{6\pi\eta r \upsilon}{\upsilon})_2 = (\frac{6\pi\eta r \upsilon}{\upsilon})_{final}$$

Therefore:-

$$\boxed{(\frac{n\{\upsilon i\}\eta L+6\pi\eta r \upsilon}{\upsilon})_1 + (\frac{6\pi\eta r \upsilon}{\upsilon})_2 = (f_c)_{final}}$$

One important thing to note is that the tesla valve has a finite value of demurance if the fluid flows in one specific direction of valve where it experiences interference and energy losses. The fluid will have negligible demurance if the tesla valve is inverted 180^0 in horizontal plane, where the fluid flows with ease without any significant interference with the molecules, and have negligible energy losses. In that case the value of demurance is

$$D_T \approx 0$$

Therefore the direction of the fluid through the valve should be carefully observed before mentioning it in the PLFD equation.

Conclusion

Palash's law of fluid dynamics provides the methods to understand properly the concept of fluid motion in an enclosed system, and provides all the necessary formulae to calculate the properties of motion of fluids. Palash's law of fluid dynamics will help all engineers and scientists to accurately calculate the terms in fluid problems.

References

[1] Newton's third law of motion
[2] Tesla valve/ valvular conduit

Images by:-

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[4] pfnicholls.com
[5] TU Delft Repositories