International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

Dynamics of Inventory Management with Time-Varying Demand and Deterioration: A Comprehensive Cost and Profit Analysis

Vivek Vijay

Swami Keshvanand Institute of Technology Management &Gramothan, Jaipur
Email: *vivekvijayvargia1984[at]gmail.com*

Abstract: ***In this inventory model, we study the system for items with time varying demand. The demand is power pattern. The deterioration of items is proportional to time. The production rate is given by the deterioration and demand. Shortages are not allowed in this model. Holding cost, ordering cost, replenishment costs are considered in this model. Our objective in this model is to minimize the total cost and maximize the profit. For validation of the model, the numerical example is also available which show the sensitivity of the cost and profit with respect to variations in parameters.***

Keywords: Inventory, deterioration, power pattern demand, holding cost

1. Introduction

We know that in any business, the management of inventory plays the most important role to begin and establish the business. Due to this management a business can pursue properly and develop in remarkable situation. The inventory level directly affect to economy of business as well as market value of the particular industry. Everyone knew that the storage of goods is very necessary for the business but it is not easy to handle the goods for best sale and profit. The demand of any item in the market is very effective from usability, necessity. But some time we can increase the demand of any item to show their advantage in multidisciplinary cases. Display and approach of any product can increase the demand in the market. It is not possible that everyone have the product in his hand. And it is not possible that a product can survive for a long time. Any item have a fixed lifetime for best use. After this lifetime the item start deteriorate and not suitable for use. For some items like medicine, electric items etc have long lifetime. But fruits, vegetable, sweets and some other food items have very short lifetime.

Many researchers have been developed the different inventory model including or without including deterioration. Deterioration is very important factory to manage the inventory. Demand of any item is not constant remain but it varies time to time or it may be dependent on inventory level. The optimal policy of inventory management has developed by Donalson [16] in his research on Inventory replenishment policy for a linear trend in demand: an analytical solution. Datta and Pal[14] has developed an order level inventory system with power demand pattern for items with variable rate of deterioration. Goswami and Chaudhuri[1] have published an EOQ model for an inventory with a linear trend in demand and finite rate of replenishment considering shortages. Bose, Goswami and Chauduri[9] have been published an EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. Chakrabarti and Chaudhuri[4] have given an EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles. Dye[3] has published a note on "an EOQ model for items with weibull distributed deterioration, shortages and power demand pattern". Singh, Singh and Dutt [13] have been developed an EOQ model for perishable items with power demand and partial backordering. Ghasemi and Damghani [6] presented A robust simulation-optimization approach for pre-disaster multi-period location–allocation–inventory planning Sakaguchi [11] has presented an inventory model for an inventory system with time varying demand rate. Mishra and Singh [15] have developed a deteriorating inventory model with time dependent demand and partial backlogging. Hsueh [2] has developed an inventory control model with consideration of remanufacturing and product life cycle. Sharma and Preeti[12] have developed an inventory model for optimum ordering interval for random deterioration with selling price and stock dependent demand rate and shortages. Rajeswari and Vanjikkodi [7] have developed a deteriorating inventory model with power demand and partial backlogging. Rajeswari and Vanjikkodi [8] have been published an inventory model for items with two parameter weibull distribution deterioration and backlogging. Mishra and Singh [10] have given a Partial backlogging EOQ model for queued customers with power demand and quadratic deterioration: computational approach. Tripathy, Sharma, and Sharma [5] has given an EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility.

Many of inventory models have power pattern demand and no deterioration. The length of cycle time is fixed. Now here, we developed an economic production quantity inventory model. In which deterioration of inventory is allowed form initially up to end of cycle time. The demand of items is taken power pattern demand. Replenishment policy of items is also considered in this model. The inventory level is proportional to power pattern demand with effect of deterioration. In this model we calculate the total cost in cycle time and sales revenue generated from the production. Now we calculate the profit according to this inventory system. These are the functions of many parameters. We establish a numerical example to check optimization of this model.

2. Assumption and Notation

- C0 = Setup cost per unit item.
- Ch = Holding cost per unit item per unit time.
- CR = Replenishment cost per unit time per unit item.
- t1 = Replenishment period.
- r = Average demand = d/T.
- T = Length of cycle time.
- θ = θ(α, t) = θ0αt = Deterioration rate of items. Where θ0 is constant.
- β, n are demand parameter.
- The power pattern demand is D(t) = $\begin{cases} (\beta-1)r\dfrac{t^{\frac{1-n}{n}}}{T^{\frac{1-n}{n}}}, & 0\le t\le t_1 \\ r\dfrac{t^{\frac{1-n}{n}}}{T^{\frac{1-n}{n}}} & t_1\le t\le T \end{cases}$
- Q = Initially inventory quantity.
- α = deterioration parameter.
- I(t) = Inventory level at time t.
- C(t1, T) = Total cost per cycle time.
- A = replenishment cost money per order.
- s = selling price per unit item.
- S(t1, T) = total sales revenue
- P(t1, T) = total profit.

3. Mathematical Model

We consider a continuous inventory system over an infinite time horizon with power pattern demand. Inventory deterioration is also allowed in the system. Let I(t) be stock level at time t. Replenishment period t_1 is the time duration in which replenishment size is being added to inventory. The production of inventory starts from initially at time t = 0. Also sale, managing, deterioration and procurement starts form this time t = 0. In a cycle time, when the production stop then the inventory level decrease very fast and become zero at time T. The mathematical model for such system is follows:

$$\frac{d}{dt}I(t)+\theta(\alpha,t)I(t)=(\beta-1)\frac{rt^{\frac{1-n}{n}}}{nT^{\frac{1-n}{n}}},\quad 0\le t\le t_1 \qquad ...(1)$$

$$\frac{d}{dt}I(t)+\theta(\alpha,t)I(t)=\frac{rt^{\frac{1-n}{n}}}{nT^{\frac{1-n}{n}}},\quad t_1\le t\le T \qquad ...(2)$$

The boundary conditions are $I(0)=Q,\ I(T)=0$.

Solution of differential equation (1) is

$$I(t)e^{\frac{\theta_0\alpha t^2}{2}}=\frac{(\beta-1)r}{T^{\frac{1-n}{n}}}\left(t^{\frac{1}{n}}+\frac{\theta_0\alpha t^{\frac{1}{n}+2}}{2(2n+1)}\right)+X$$

After using the boundary condition $I(0)=Q$

$$I(t)=\frac{(\beta-1)r}{T^{\frac{1-n}{n}}}\left[t^{\frac{1}{n}}-\frac{\theta_0\alpha n}{1+2n}t^{\frac{1}{n}+2}+\frac{\theta_0^2\alpha^2(2n-1)}{8(1+2n)}t^{\frac{1}{n}+4}\right]+Q\left[1-\frac{\theta_0\alpha t^2}{2}+\frac{\theta_0^2\alpha^2t^4}{8}\right] \qquad ...(3)$$

Solution of differential equation (2) is

$$I(t)e^{\frac{\theta_0\alpha t^2}{2}}-\frac{r}{T^{\frac{1-n}{n}}}\left(t^{\frac{1}{n}}+\frac{\theta_0\alpha}{2(2n+1)}t^{\frac{1}{n}+2}\right)+X$$

After using the boundary condition $I(T)=0$

$$I(t)=--\frac{r}{T^{\frac{1-n}{n}}}\left[t^{\frac{1}{n}}+T^{\frac{1}{n}}-\frac{\theta_0\alpha}{2(1+2n)}\left\{2nt^{\frac{1}{n}+2}+(2n+1)t^2T^{\frac{1}{n}}-T^{\frac{1}{n}+2}\right\}+\frac{\theta_0^2\alpha^2}{8(1+2n)}\left\{(2n-1)t^{\frac{1}{n}+4}+(2n+1)t^4T^{\frac{1}{n}}-2t^2T^{\frac{1}{n}+2}\right\}\right]$$

The total demand in cycle time [0, T] is given as

$$D=\int_0^T D(t)dt=\int_0^{t_1}\frac{(\beta-1)rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}}dt+\int_{t_1}^{T}\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}}dt$$

$$=rT+r(\beta-2)\frac{t_1^{\frac{1}{n}}}{T^{\frac{1}{n}-1}} \qquad ...(5)$$

Now we calculate the different cost related to this inventory model

Ordering cost = C_0 ...(6)

Holding cost = HC = $C_h\int_0^T I(t)dt$

$$=C_h\left[\frac{(\beta-1)r}{T^{\frac{1-n}{n}}}\left\{\frac{n}{n+1}t_1^{\frac{1}{n}+1}-\frac{\theta_0\alpha n^2}{(1+2n)(1+3n)}t_1^{\frac{1}{n}+3}+\frac{\theta_0^2\alpha^2n(2n-1)}{8(1+2n)(1+5n)}t_1^{\frac{1}{n}+5}\right\}+Q\left\{t_1-\frac{\theta_0\alpha}{6}t_1^3+\frac{\theta_0^2\alpha^2}{40}t_1^5\right\}\right]$$

$$-\frac{C_hr}{T^{\frac{1}{n-1}}}\left[\frac{2n+1}{n+1}\left(T^{\frac{1}{n}+1}-t_1^{\frac{1}{n}+1}\right)-\frac{\theta_0\alpha}{2(1+2n)}\left\{\frac{12n^2-n-2}{3(1+3n)}T^{\frac{1}{n}+3}-\frac{2n^2}{1+3n}t_1^{\frac{1}{n}+3}-\frac{1+2n}{3}t_1^3T^{\frac{1}{n}}-t_1T^{\frac{1}{n}+2}\right\}\right] \qquad ...(7)$$

Replenishment cost = C_R = AR(T) = $\frac{A}{T}$ ...(8)

The total cost per unit time per unit item is given as

$C(t_1,T)=\frac{1}{T}$(Ordering cost + Holding cost) + Replenishment cost

$$=\frac{C_0}{T}+C_h\frac{(\beta-1)r}{T^{\frac{1}{n}}}\left\{\frac{n}{n+1}t_1^{\frac{1}{n}+1}-\frac{\theta_0\alpha n^2}{(1+2n)(1+3n)}t_1^{\frac{1}{n}+3}\right.$$

$$+\frac{\theta_0^2\alpha^2 n(2n-1)}{8(1+2n)(1+5n)}t_1^{\frac{1}{n}+5}\Bigg\} + \frac{C_hQ}{T}\left\{t_1 - \frac{\theta_0\alpha}{6}t_1^3 + \frac{\theta_0^2\alpha^2}{40}t_1^5\right\}$$

$$-\frac{C_h(2n+1)r}{(n+1)T^{\frac{1}{n}}}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) + \frac{C_h\theta_0\alpha r}{2(1+2n)T^{\frac{1}{n}}}\Bigg\{\frac{12n^2-n-2}{3(1+3n)}T^{\frac{1}{n}+3}$$

$$-\frac{2n^2}{1+3n}t_1^{\frac{1}{n}+3} - \frac{1+2n}{3}t_1^3T^{\frac{1}{n}} - t_1T^{\frac{1}{n}+2}\Bigg\} + \frac{A}{T} \qquad ...(9)$$

The sales revenue is

$$S(t_1,T) = s\int_0^T D(t)dt = srT + \frac{sr(\beta-2)t_1^{\frac{1}{n}}}{T^{\frac{1}{n}-1}} \qquad ...(10)$$

Now the profit per unit time per unit item is given as

$$P(t_1,T) = \frac{1}{T}S(t_1,T) - C(t_1,T)$$

$$= sr + \frac{sr(\beta-2)t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} - \frac{C_0}{T} - \frac{C_h(\beta-1)r}{T^{\frac{1}{n}}}\Bigg\{\frac{n}{n+1}t_1^{\frac{1}{n}+1}$$

$$-\frac{\theta_0\alpha n^2}{(1+2n)(1+3n)}t_1^{\frac{1}{n}+3} + \frac{\theta_0^2\alpha^2 n(2n-1)}{8(1+2n)(1+5n)}t_1^{\frac{1}{n}+5}\Bigg\}$$

$$-\frac{C_hQ}{T}\left\{t_1 - \frac{\theta_0\alpha}{6}t_1^3 + \frac{\theta_0^2\alpha^2}{40}t_1^5\right\}$$

$$+\frac{C_h(2n+1)r}{(n+1)T^{\frac{1}{n}}}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) - \frac{C_h\theta_0\alpha r}{2(1+2n)T^{\frac{1}{n}}}\Bigg\{\frac{12n^2-n-2}{3(1+3n)}T^{\frac{1}{n}+3}$$

$$-\frac{2n^2}{1+3n}t_1^{\frac{1}{n}+3} - \frac{1+2n}{3}t_1^3T^{\frac{1}{n}} - t_1T^{\frac{1}{n}+2}\Bigg\} - \frac{A}{T} \qquad ...(11)$$

Now to maximize the total profit we take $\frac{\partial P}{\partial t_1} = 0 = \frac{\partial P}{\partial T}$.

And calculate stationary values of t_1 and T as t_1^*, T^*. At the above calculated point we find $\frac{\partial^2 P}{\partial t_1^2}, \frac{\partial^2 P}{\partial T^2}, \frac{\partial^2 P}{\partial t_1 \partial T}$. If

$\frac{\partial^2 P}{\partial t_1^2} \times \frac{\partial^2 P}{\partial T^2} > \left(\frac{\partial^2 P}{\partial t_1 \partial T}\right)^2$ then the profit is maximum.

4. Numerical Example with Graph

Effect of parameter α on Total cost, sales revenue and profit with values of others taken as β = 10, r = 1, n = 2, θ_0 = 1, t_1*, T* Place table titles above the tables.

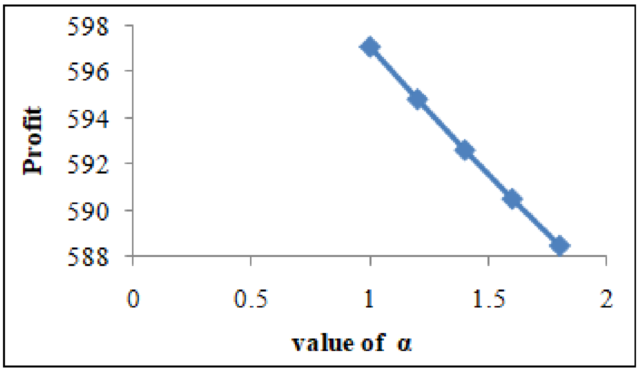
α	Total Cost	Sale Revenue	Profit
1	502.81	1099.90	597.08
1.2	498.20	1093.01	594.81
1.4	494.28	1086.90	592.61
1.6	490.84	1081.34	590.50
1.8	487.78	1076.25	588.47

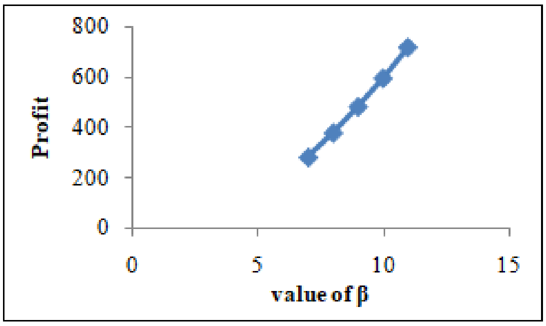
β	Total Cost	Sale Revenue	Profit
1	599.85	1319.82	719.97
1	502.81	1099.90	597.08
1	430.46	914.91	484.44
1	378.55	758.86	380.30
1	340.45	623.72	283.26

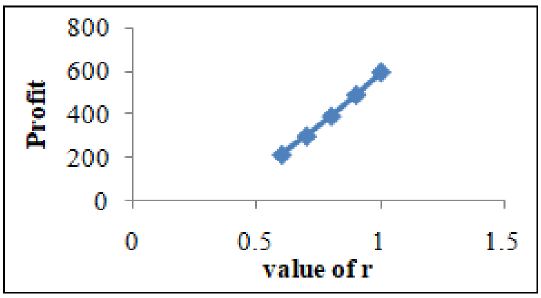
r	Total Cost	Sale Revenue	Profit
1	502.81	1099.90	597.08
0.9	438.74	930.60	491.86
0.8	386.35	779.64	393.28
0.7	343.01	644.01	301.00
0.6	306.00	520.88	214.87

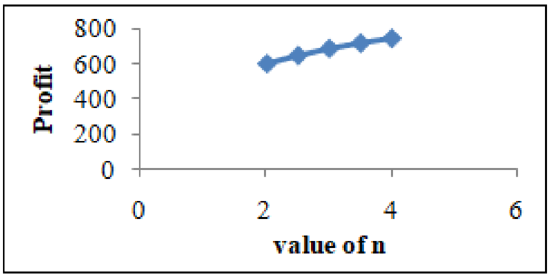
n	Total Cost	Sale Revenue	Profit
2	502.81	1099.90	597.08
2.5	432.10	1077.44	645.33
3	400.62	1085.67	685.04
3.5	382.11	1099.41	717.29
4	369.19	1113.09	743.90

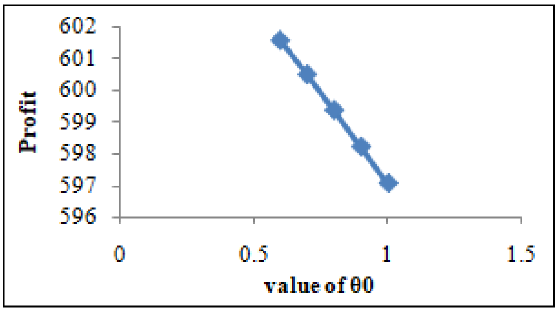
θ_0	Total Cost	Sale Revenue	Profit
1	502.81	1099.90	597.08
0.9	505.46	1103.69	598.23
0.8	508.44	1107.82	599.38
0.7	511.79	1112.30	600.50
0.6	515.72	1117.31	601.58











5. Conclusions and Remarks

In this research paper the production rate of items to replenish the goods is proportional to power demand. It is obvious that production increases as demand increases. And increasing sale increases the profit. From the above numerical calculation and study of graph it is clear that the increasing value of α, θ0 are decrease the total cost, total sale revenue and total profit. Increasing value of β, r are increases the total cost, total sale revenue and total profit. Increasing value of n increase the sale revenue and the profit but decreases the total cost. On behalf of above results we can optimize our inventory system and make large amount of profit. In practical condition these are not only reasons to affect production, cost, sale and profit. Our production depends on labours cost, man power, machinery system and government taxes. Some other reasons are also possible to affect the our inventory system. We will introduce these in our future research.

6. References

[1] A. Goswami and K. S. Chaudhuri, "EOQ model for an inventory with a linear trend in demand and finite rate of replenishment considering shortages", Int. J. Syst. Sci. (1991), 22(1), 181-187.
[2] C. F. Hsueh, "An inventory control model with consideration of remanufacturing and product life cycle", Int. J. Prod. (2011), Econ. 133(2), 645-652.7
[3] C. Y. Dye, "A note on an EOQ model for items with weibull distributed deterioration, shortages and power demand pattern", Int. J. Inf. Manage. Sci. (2004), 15(2), 81-84.
[4] Chakrabarti T. and Chaudhuri K. S. [1997]: An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles. Int. J. Prod. Econ. 49(3), 205-213.
[5] M. Tripathy, G. Sharma, and A. K. Sharma, "An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility", Opsearch, (2022), 59(4), 1215-1243.
[6] P. Ghasemi, K. K. Damghani, "A robust simulation-optimization approach for pre-disaster multi-period location–allocation–inventory planning, Mathematics and Computers in Simulation, (2021), 179, 69-95.
[7] Rajeswari N. and Vanjikkodi T. [2011]: deteriorating inventory model with power demand and partial backlogging. Int. J. Math. Arch. 2(9), 1495-1501.
[8] Rajeswari N. and Vanjikkodi T. [2012]: An inventory model for items with two parameter weibull distribution deterioration and backlogging. Am. J. Oper. Res. 2(2), 247-252.
[9] S. Bose, A. Goswami and K. S. Chauduri,"An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. J. Oper. Res. Soc. (1995), 46(6), 771-782.
[10] S. S. Mishraand P. K. Singh, "Partial backlogging EOQ model for queued customers with power demand and quadratic deterioration: computational approach", Am. J. Oper. Res. (2013), 3(2), 13-27.
[11] Sakaguchi M. [2009]: Inventory model for an inventory system with time varying demand rate. Int. J. Prod. Econ. 122(1), 269-275.
[12] Sharma A. K. and Preeti [2011]: Optimum ordering interval for random deterioration with selling price and stock dependent demand rate and shortages. Ganita Sandesh, 25(2), 147-159.
[13] Singh T. J., Singh S. R. and Dutt. R. [2009]: An EOQ model for perishable items with power demand and partial backordering. Int. J. Oper. Quant. Manage. 15(1), 65-72.
[14] T. K. Datta and A. K. Pal,"Order level inventory system with power demand pattern for items with variable rate of deterioration", Indian J. Pure Appl. Math.(1988), 19(11), 1043-1053.
[15] V. K. Mishra and L. S. Singh, "Deteriorating inventory model with time dependent demand and partial backlogging", Appl. Math. Sci. (2010), 4(72), 3611-3619.
[16] W. A. Donalson, "Inventory replenishment policy for a linear trend in demand: an analytical solution",Oper. Res. Q. (1977), 28(3), 663-670.