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Determination and Analysis of Height Differences between Orthometric and Ellipsoidal Heights for Engineering Applications

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Abstract: ***This study compares and analyses orthometric and ellipsoidal heights differences of points on the earth's surface for engineering applications. Differential levelling was used to determine orthometric heights of 72 points and Global Positioning System (GPS) was used to determine three-dimension coordinates of the same points with a period of thirty (30) minutes at each GPS point. The reduced levels from differential levelling were computed using rise and fall method while the 3D coordinates determined using GPS was adjusted using Least Squares Solution software. The accuracy of the height differences was determined using standard deviations. The standard deviation of ellipsoidal height difference was 53.59cm while the standard deviation for orthometric height was 53.07cm respectively. A Root Mean Square Error value of 0.131m was obtained as the accuracy of the change between the two height differences while the correlation coefficients for the observations for differential levelling and GPS observations were 0.999 and 0.998 respectively. The result also indicated that there is a very small height difference between the two reference systems.***

Keywords: Differential levelling, GPS, Ellipsoidal height, Orthometric height

1. Introduction

Engineering surveys are the types of surveys required for construction and engineering projects such as route mapping, leveling, laying out different types of buildings, and deformation monitoring of structures, among others. One of the common methods used to determine heights for engineering surveys is differential leveling. Differential levelling procedure is quite accurate, and the equipment used is relatively cheap, however, differential levelling over long distances/routes, it requires a lot of manpower, labor, and the field processes are tiresome and prone to human, systemic, and random errors. In addition, if the weather is not favorable, it is very difficult to execute this process. (Badejo, et al., 2016). According to Badejo, this technique is less appealing for high-profile research proposals because of how common place it is. (Badejo, et al., 2016).

In recent years, most professional engineers have adopted satellite positioning systems for various applications. The Global Positioning System (GPS) is one of the most widely utilized satellite positioning systems in the world. GPS is based on the World Geodetic System of 1984, and it determines three – dimension positions (X, Y, Z) quickly and effectively from WGS 84 reference ellipsoid. Heights from GPS are commonly known as ellipsoidal heights (h), however, the accuracy and reliability of heights determined by GPS have not been extensively investigated. Orthometric heights (H) are heights frequently and commonly used for mapping, engineering projects, navigation, and other geophysical purposes and, orthometric heights (H) are more commonly used and useful heights. However, orthometric heights determined by differential levelling is a time-consuming and expensive. The height disparities along the leveling lines are added up to determine the heights of all other points in the network. In general, the location of the current sea level is just approximate because the exact location may change due to the current global warming or climate change. Therefore, the geoid is assumed to be at the same elevation as the mean sea level.

To determine heights of points on the earth surface, one must determine the distance between the ellipsoid and the geoid to fully utilize the three-dimensional coordinates (X, Y Z) determined by GPS. In engineering applications, the reduced levels are used to construct roads, railroads, canals, sewage systems, water supply systems, and other facilities that best fits the topography of that area. (Ghazal & Saleh, 2021). The global coordinates X, Y, and Z are converted into ellipsoidal coordinate's ϕ, λ, and h and then into local horizontal coordinates n, e, and u for various practical applications. Practically, the heights determined from GPS observations must be converted into orthometric heights to be used engineering for applications.

Meyer, et al., 2005a; Kumar 2005) investigated the ellipsoidal heights determined using GPS and found that ellipsoidal heights are almost never suitable surrogates for orthometric heights because equipotential ellipsoids are not, in general, suitable surrogates for the geoid.

Zarko & Sinisa (2014) investigated the reliability of spirit levelling which is based on the fact that the axis of the spirit level is perpendicular to the plumb line and the height difference between two points is obtained as the difference of readings on the level rods settled on those two points. They finally concluded that spirit levelling is a vital operation through which elevation of points or differences in elevation are determined to produce necessary data for mapping, engineering design, and construction.

Badejo, et al., (2016) investigated on the application of ellipsoidal heights (h) rather than orthometric heights for engineering surveys. In their study, GPS and differential levelling observations were made to determine the ellipsoidal and orthometric heights over a distance of

139.114 km, whereby a mean accuracy of 13.2 ppm was obtained, satisfying the third order accuracy requirement for engineering application. Audu and Tijani (2017) examined the elevation discrepancies derived from the total station and the automatic level instrument in a different investigation. The height difference determined by the two instruments had a maximum and minimum difference of 62mm and 20mm, respectively. The height disparities determined using the differential leveling method and GPS method were compared and found that a root mean square error of 0.98mm was determined and conclusion that there is no significant difference between the two leveling procedures. Although it would show that the height differences determined using Global Positioning System (GPS) and differential leveling cannot be compared, it is possible to say that under some circumstances it is possible to do the comparison.

Tata, H and Olatunji, R.A (2020) compared the change between orthometric and ellipsoidal heights differences and found that GPS and Spirit levelling nan be used for engineering projects.

1.2 Relationship Between Ellipsoidal and Orthometric Heights

Considering the fundamental differences between the two methods and instrumentation, it is possible within the same environment and constraints, one can confident argue that the height differences obtained between differential levelling and GPS methods are comparable. According to a first approximation, the fundamental relationship between heights regarding a vertical geodetic datum generated using gravity and spirit-leveling data and heights acquired from GPS measurements is given by the following mathematical expression.

$h - H - N = 0$………………………..............(1)

where:
h is the ellipsoidal height.
H is the orthometric height and
N is the geoidal undulation derived from a regional gravimetric geoid model or a global geopotential model.

There are many causes that resulted into disparities in equation (1) above. The causes of these disparities include derivations of the parameters h, H, and N, datum discrepancies, because each parameter is referring to a different reference surface. Also, other disparities are systematic errors and distortions due to tropospheric refraction. However, systematic errors and datum disparities are the major factors for the above-mentioned discrepancies. The evaluation of the height data determined from GPS observations is another important undertaking which will help to determine the accuracy of the ellipsoidal heights determined by post-processing (Fotopoulos, 2003). The ellipsoidal surface which is referred to by the parameter (h) . while the normal vertical line to the Geoid, which is measured with a level, is used to determine the orthometric height (H) of a distance from a location on the surface of the Earth to the distance that points to the Geoid. The height from any reference ellipsoid to the location on the ground is represented by the ellipsoidal height measured using GPS. Geoid height, also known as Geoid Undulation (N), is the distance between the ellipsoid and the Geoid surface. There are various sources of errors in GPS which they impact satellite signals as they travel from the satellite toward the receiver(s) positioned on the surface of the Earth are caused by atmospheric inaccuracies. The ionosphere and the neutral atmosphere are the two regions of the atmosphere through which the signal passes. This neutral region, which is thought to be a crucial degrading factor for height determination, is located between 0 km and 40 km above the surface of the Earth. Signals passing through the troposphere specifically experience the effects of attenuation, delay, and short-term fluctuations in the troposphere (scintillation).

The sizes of these effects depend on the satellite's altitude as well as the temperature, pressure, and relative humidity of the atmosphere at the time the signal is being propagated. In addition. the troposphere is a non-dispersive medium for GPS frequencies, tropospheric range errors are not frequency dependent and cannot be eliminated by using dual-frequency measurements technique. The relative tropospheric bias, which results from inaccuracies in tropospheric refraction at one of the stations in a baseline arrangement, is the most harmful component. The troposphere can be modelled using water vapour radiometers and ground meteorological measurements, or the tropospheric parameters. These parameters can be modelled simultaneously with the other GPS parameters such as clock, latitude, longitude, height, and ambiguities.

2. Material and Methods

Description of the study area

This study is conducted on a road segment at the Botswana International University of Science and Technology (BIUST), Palapye in the central part of Botswana. The geographic location lies approximately on latitude and longitude 22.5946 ° S; 27.1239 ° E. Figure 7.0 shows the study area of this study while Figure 8.0 shows the configuration of points to be established. The methodology used in this study is a combination of differential levelling using automatic level and GPS observation using GPS receivers. A total of seventy (70) points were selected and marked along the road were observed using GPS receivers an automatic level at an approximately 1.2 km stretch. The points were spaced at 20m interval to minimize effects caused by the curvature and refraction of the earth.

Figure 9: Satellite image of the road segment and established points along the road. (Google Earth, 2022)

Data Acquisition

The data acquired using differential levelling and computed using the rise and fall method to determine the orthometric heights on the Microsoft Excel spreadsheet while the GPS observed data was processed using the least squares adjustment software to obtain the final ellipsoidal heights. Finally, a comparative analysis of the observations between the orthometric height and the ellipsoidal height differences was made.

3.2.1 Differential levelling observations

A levelling operation was conducted with an automatic level instrument together with two levelling staffs of three meters long. The automatic level instrument was set-up midway between the established survey benchmark BMJ1 (PT1) and PT2, the back sight observation was taken on the benchmark with the levelling staff held over the benchmark point and another levelling staff held over PT2 for the foresight reading. The automatic level instrument was then moved to a point beyond PT2 at random intervals, take the back sight observation on PT2 and the foresight observation on PT3. Observations were made at AN interval of 20m at the centre of the road segment. This procedure was repeated until the point of emphasis which is the centre line of the road segment at junction 1 (J1) was reached. At this point, the staff will be placed on the centre line of the road at point RD-CL 1 and the automatic levelling instrument moved from the last point where it was set up. Observations were made at 20m intervals on the centre line of the road and recordings were made on the levelling booking sheet. The procedure was repeated until reaching the last point of emphasis RD-CL 58 at junction 3 (J3) of the road segment.

3.3.2 GPS Observations

The GPS surveys was carried out in order to obtain the ellipsoidal heights of the 70 points marked within the study area. The GPS measurements were taken with Hi-target DGPS receivers using the Real Time Kinematic method. The reference receiver was set up on a known reference station BIUST 1 as a set base. The base station GPS antenna was set up on a level tripod directly over the known surveyed point MJ. As a standard procedure, the height of the base station antenna phase centre above the surveyed point was measured. The GPS rover was positioned on a survey point with a radio receiver to receive correction data from the base station. The rover was placed over the marked points to collect the data of every single point from the base station until the closing benchmark. Figure 12 shows a typical configuration of the Base and antenna. The obtained data was further processed using the least squares adjustments software to get the final heights.

3. Results

Geoidal Undulation N

Table 1: Geoidal undulation (N) of 70 points

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	GEOIDAL Undulation N (m)
BM1	978,979	978,625	0,354
P2	980,26	979,765	0,495
P3	978,387	978,445	-0,058
P4	975,402	975,465	-0,063
P5	973,045	973,383	-0,338
P6	970,226	970,593	-0,367
CH0	968,688	969,013	-0,325
CH20	968,897	969,128	-0,231
CH40	969,236	969,338	-0,102
CH60	969,566	969,666	-0,1
CH80	969,903	970,003	-0,1
CH100	970,261	970,372	-0,111
CH120	970,71	970,827	-0,117
CH140	971,314	971,407	-0,093
CH160	972,04	972,144	-0,104
CH180	972,792	972,894	-0,102
CH200	973,523	973,619	-0,096
CH220	974,035	974,124	-0,089
CH240	974,406	974,503	-0,097
CH260	974,744	974,834	-0,09
CH280	975,147	975,244	-0,097
CH300	975,599	975,696	-0,097
CH320	976,151	976,256	-0,105
CH340	976,878	976,971	-0,093

CH360	977,635	977,726	-0,091
CH380	978,425	978,532	-0,107

Table 1(continuation): Geoidal undulation (N) of 70 points.

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	Geoidal Undulation N (m)
CH400	979,145	979,262	-0,117
CH420	979,713	979,821	-0,108
CH440	980,105	980,211	-0,106
CH460	980,302	980,408	-0,106
CH480	980,213	980,326	-0,113
CH500	980,098	980,201	-0,103
CH520	979,924	980,021	-0,097
CH540	979,633	979,736	-0,103
CH560	979,292	979,391	-0,099
CH580	978,856	978,947	-0,091
CH600	978,782	978,873	-0,091
CH620	978,475	978,578	-0,103
CH640	978,26	978,35	-0,09
CH660	978,098	978,198	-0,1
CH680	977,901	978,008	-0,107
CH700	977,746	977,848	-0,102
CH720	977,661	977,758	-0,097
CH740	977,495	977,599	-0,104
CH760	977,332	977,439	-0,107
CH780	977,187	977,289	-0,102
CH800	977,058	977,154	-0,096
CH820	976,956	977,044	-0,088
CH840	976,881	976,984	-0,103
CH860	976,807	976,901	-0,094
CH880	976,772	976,874	-0,102
CH900	976,699	976,804	-0,105
CH920	976,635	976,744	-0,109

Table 1(continuation): Geoidal undulation (N) of 70 points.

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	Geoidal Undulation N (m)
CH940	976,599	976,704	-0,105
CH960	976,533	976,639	-0,106
CH980	976,49	976,589	-0,099
CH1000	976,411	976,514	-0,103
CH1020	976,358	976,469	-0,111
CH1040	976,319	976,405	-0,086
CH1060	976,177	976,28	-0,103
CH1080	976,052	976,145	-0,093
CH1100	975,922	976,014	-0,092
CH1120	975,764	975,859	-0,095
CH1140	975,589	975,699	-0,11
CH1160	975,427	975,519	-0,092
P7	976,267	976,839	-0,572
P8	976,695	976,995	-0,3
P9	976,91	977,034	-0,124
P10	977,318	977,534	-0,216
P11	977,996	978,844	-0,848

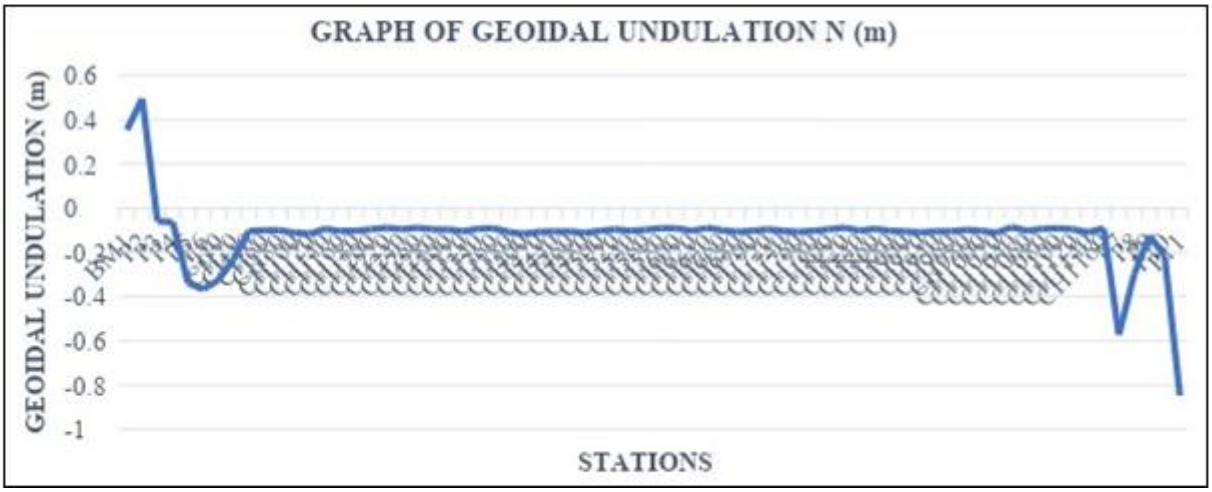

Figure 13: Graph of Geoidal Undulation (N)

Difference in height

Table 2: The difference in height of 70 points obtained by GPS and differential levelling

Station	Eastings (m)	Northings (m)	Ellipsoidal Height (m)	Orthometric Height (m)
BM1	-12454.959	2499123.037	978.979	978,625
P2	-12376.440	2499192.690	980.260	979,765
P3	-12295.079	2499274.453	978.387	978,445
P4	-12233.042	2499341.346	975.402	975,465
P5	-12159.870	2499412.652	973.045	973,383
P6	-12094.527	2499480.143	970.226	970,593
CH0	-12027.042	2499542.371	968.688	969,013
CH20	-12040.957	2499527.965	968.897	969,128
CH40	-12054.849	2499513.574	969.236	969,338
CH60	-12068.784	2499499.241	969.566	969,666
CH80	-12082.804	2499484.828	969.903	970,003
CH100	-12096.727	2499470.520	970.261	970,372
CH120	-12110.675	2499456.245	970.710	970,827
CH140	-12124.624	2499441.933	971.314	971,407
CH160	-12138.590	2499427.645	972.040	972,144
CH180	-12152.493	2499413.301	972.792	972,894
CH200	-12166.388	2499399.002	973.523	973,619
CH220	-12180.355	2499384.669	974.035	974,124
CH240	-12194.279	2499370.373	974.406	974,503
CH260	-12208.150	2499355.977	974.744	974,834
CH280	-12222.173	2499341.686	975.147	975,244
CH300	-12236.114	2499327.341	975.599	975,696
CH320	-12250.051	2499313.025	976.151	976,256
CH340	-12263.968	2499298.695	976.878	976,971

CH360	-12277.836	2499284.375	977.635	977,726
CH380	-12291.718	2499270.008	978.425	978,532

Table 2 (continuation): The difference in height of 70 points obtained by GPS and differential levelling.

Station	Eastings (m)	Northings (m)	Ellipsoidal Height (m)	Orthometric Height (m)
CH400	-12305.668	2499255.746	979.145	979,262
CH420	-12319.598	2499241.435	979.713	979,821
CH440	-12333.469	2499227.081	980.105	980,211
CH460	-12347.496	2499212.738	980.302	980,408
CH480	-12361.368	2499198.408	980.213	980,326
CH500	-12375.294	2499184.123	980.098	980,201
CH520	-12389.178	2499169.788	979.924	980,021
CH540	-12403.005	2499155.353	979.633	979,736
CH560	-12416.464	2499140.676	979.292	979,391
CH580	-12430.088	2499126.118	978.856	978,947
CH600	-12446.553	2499109.270	978.782	978,873
CH620	-12431.466	2499096.024	978.475	978,578
CH640	-12416.597	2499082.924	978.260	978,35
CH660	-12401.518	2499069.374	978.098	978,198
CH680	-12386.569	2499056.177	977.901	978,008
CH700	-12372.093	2499042.129	977.746	977,848
CH720	-12362.453	2499024.435	977.661	977,758
CH740	-12357.446	2499005.566	977.495	977,599
CH760	-12352.895	2498986.126	977.332	977,439
CH780	-12348.241	2498966.577	977.187	977,289
CH800	-12343.701	2498947.600	977.058	977,154
CH820	-12339.064	2498928.073	976.956	977,044
CH840	-12332.809	2498909.107	976.881	976,984

CH860	-12323.439	2498891.376	976.807	976,901
CH880	-12311.142	2498875.657	976.772	976,874
CH900	-12298.320	2498860.330	976.699	976,804
CH920	-12285.507	2498844.979	976.635	976,744

Table 2 (continuation): The difference in height of 70 points obtained by GPS and differential levelling.

Station	Eastings (m)	Northings (m)	Ellipsoidal Height (m)	Orthometric Height (m)
CH940	-12272.779	2498829.615	976.599	976,704
CH960	-12260.842	2498813.554	976.533	976,639
CH980	-12250.111	2498796.709	976.490	976,589
CH1000	-12240.562	2498779.163	976.411	976,514
CH1020	-12231.879	2498761.109	976.358	976,469
CH1040	-12224.788	2498742.378	976.319	976,405
CH1060	-12218.827	2498723.306	976.177	976,28
CH1080	-12214.265	2498703.840	976.052	976,145
CH1100	-12210.992	2498684.090	975.922	976,014
CH1120	-12208.791	2498664.235	975.764	975,859
CH1140	-12207.283	2498644.319	975.589	975,699
CH1160	-12206.205	2498624.399	975.427	975,519
P7	-12218.807	2498702.944	976.267	976,839
P8	-12244.756	2498777.199	976.695	976,995
P9	-12301.790	2498857.174	976.910	977,034
P10	-12348.196	2498946.347	977.318	977,534
P11	-12375.658	2499039.375	977.996	978,844

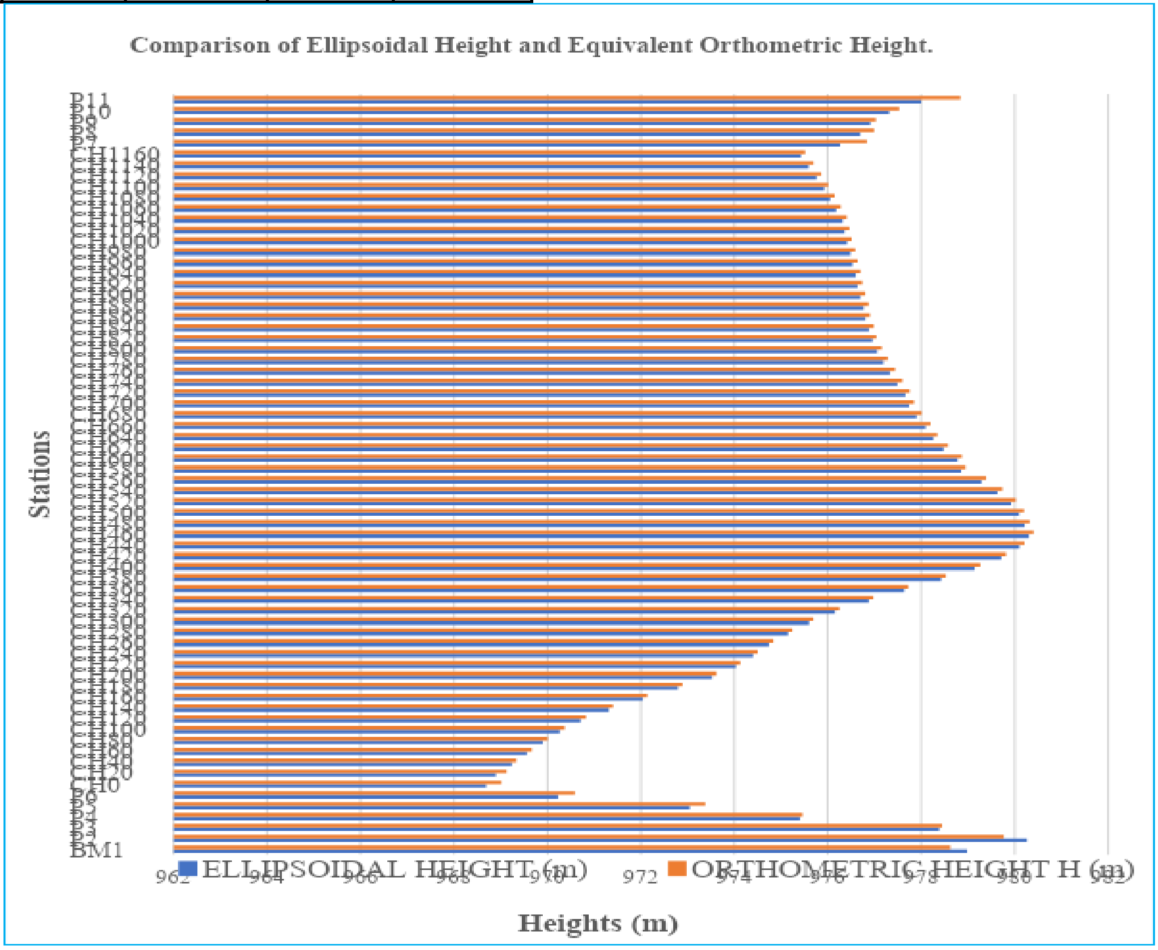

Figure 14: Comparison of Ellipsoidal Height and Equivalent Orthometric Height.

Table 3: Change between Ellipsoidal and equivalent Orthometric height difference of the 70 points.

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	Δh (m)	ΔH (m)	(Δh-ΔH) m
BM1	978,979	978,625			
P2	980,260	979,765	1,281	1,14	0,141
P3	978,390	978,445	-1,87	-1,32	-0,553
P4	975,402	975,465	-2,98	-2,98	-0,005
P5	973,045	973,383	-2,36	-2,082	-0,275
P6	970,226	970,593	-2,82	-2,79	-0,029
CH0	968,688	969,013	-1,54	-1,58	0,042
CH20	968,897	969,128	0,21	0,115	0,094
CH40	969,236	969,338	0,34	0,21	0,129
CH60	969,566	969,666	0,33	0,328	0,002
CH80	969,903	970,003	0,34	0,337	0
CH100	970,261	970,372	0,36	0,369	-0,011
CH120	970,71	970,827	0,45	0,455	-0,006
CH140	971,314	971,407	0,60	0,58	0,024
CH160	972,04	972,144	0,73	0,737	-0,011
CH180	972,792	972,894	0,75	0,75	0,002
CH200	973,523	973,619	0,73	0,725	0,006
CH220	974,035	974,124	0,51	0,505	0,007
CH240	974,406	974,503	0,37	0,379	-0,008
CH260	974,744	974,834	0,34	0,331	0,007
CH280	975,147	975,244	0,40	0,41	-0,007
CH300	975,599	975,696	0,45	0,452	0
CH320	976,151	976,256	0,55	0,56	-0,008
CH340	976,878	976,971	0,73	0,715	0,012
CH360	977,635	977,726	0,76	0,755	0,002
CH380	978,425	978,532	0,79	0,806	-0,016
CH400	979,145	979,262	0,72	0,73	-0,01

Table 3 (continuation): Change between Ellipsoidal and equivalent Orthometric height difference of the 70 points.

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	Δh (m)	ΔH (m)	(Δh-ΔH) m
CH420	979,713	979,821	0,57	0,559	0,009
CH440	980,105	980,211	0,39	0,39	0,002
CH460	980,302	980,408	0,20	0,197	0
CH480	980,213	980,326	-0,09	-0,082	-0,007
CH500	980,098	980,201	-0,12	-0,125	0,01
CH520	979,924	980,021	-0,17	-0,18	0,006
CH540	979,633	979,736	-0,29	-0,285	-0,006
CH560	979,292	979,391	-0,34	-0,345	0,004
CH580	978,856	978,947	-0,44	-0,444	0,008
CH600	978,782	978,873	-0,07	-0,074	0
CH620	978,475	978,578	-0,31	-0,295	-0,012
CH640	978,26	978,35	-0,22	-0,228	0,013
CH660	978,098	978,198	-0,16	-0,152	-0,01
CH680	977,901	978,008	-0,20	-0,19	-0,007
CH700	977,746	977,848	-0,15	-0,16	0,005
CH720	977,661	977,758	-0,09	-0,09	0,005
CH740	977,495	977,599	-0,17	-0,159	-0,007
CH760	977,332	977,439	-0,16	-0,16	-0,003
CH780	977,187	977,289	-0,14	-0,15	0,005
CH800	977,058	977,154	-0,13	-0,135	0,006
CH820	976,956	977,044	-0,10	-0,11	0,008
CH840	976,881	976,984	-0,08	-0,06	-0,015
CH860	976,807	976,901	-0,07	-0,083	0,009
CH880	976,772	976,874	-0,03	-0,027	-0,008
CH900	976,699	976,804	-0,07	-0,07	-0,003
CH920	976,635	976,744	-0,06	-0,06	-0,004
CH940	976,599	976,704	-0,04	-0,04	0,004

Table 3 (continuation): Change between Ellipsoidal and equivalent Orthometric height difference of the 70 points.

Station	Ellipsoidal Height h (m)	Orthometric Height H (m)	Δh (m)	ΔH (m)	(Δh-ΔH) m
CH960	976,533	976,639	-0,07	-0,065	-0,001
CH980	976,49	976,589	-0,04	-0,05	0,007
CH1000	976,411	976,514	-0,08	-0,075	-0,004
CH1020	976,358	976,469	-0,05	-0,045	-0,008
CH1040	976,319	976,405	-0,04	-0,064	0,025
CH1060	976,177	976,28	-0,14	-0,125	-0,017
CH1080	976,052	976,145	-0,13	-0,135	0,01
CH1100	975,922	976,014	-0,13	-0,131	0,001
CH1120	975,764	975,859	-0,16	-0,155	-0,003
CH1140	975,589	975,699	-0,17	-0,16	-0,015
CH1160	975,427	975,519	-0,16	-0,18	0,018
P7	976,267	976,839	0,84	1,32	-0,48
P8	976,695	976,995	0,43	0,156	0,272
P9	976,91	977,034	0,21	0,039	0,176
P10	977,318	977,534	0,41	0,5	-0,092
P11	977,996	978,844	0,68	1,31	-0,632

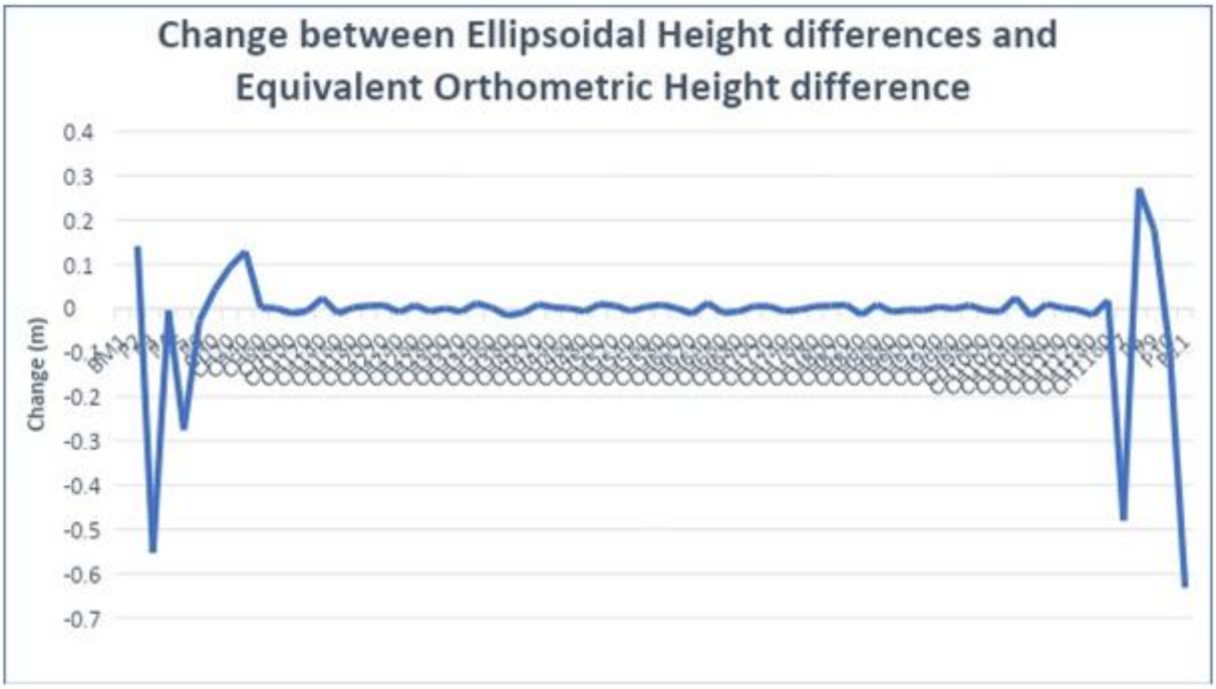

Figure 15: Change between Ellipsoidal Height differences and Equivalent Orthometric Height difference

Root Mean Square Error

This study uses the following equation to determine the root mean square error (RMSE) to determine the accuracy of the change between ellipsoidal height differences and comparable orthometric height disparities.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\Delta h_1 - \Delta H_1)^2}{n}} \ldots.(2)$$

Where;
n is the total number of points.
Δh is the ellipsoidal height difference of point *i*
ΔH is the orthometric height difference of point

$$RMSE = \sqrt{\frac{1.194604}{70}}$$

$RSME = 0.130636$ **m**

Standard Deviation for orthometric and Ellipsoidal Heights

The mean and standard deviation is given by the following equations respectively.

$$X = \frac{\sum X}{n} \ldots.(3)$$

$$S_x = \sqrt{\frac{\sum(X - X^2)}{n-1}} \ldots.(4)$$

$$S_y = \sqrt{\frac{\sum(Y - Y^2)}{n-1}} \ldots.(5)$$

Where:
S – Standard Deviation X – Ellipsoidal Height Y – Orthometric height

$$S_x = \sqrt{\frac{39.6716}{70-1}}$$

$$S_x = 75.8255\ cm$$

$$S_y = \sqrt{\frac{38.3445}{70-1}}$$

$S_y = 74.5465\ cm$

A crucial measure of accuracy is the standard deviation. For ellipsoidal height differences, the standard deviation is equal to 75.8255 cm, but for similar orthometric height differences, standard deviation is equal to 74.5465 cm. The difference between the ellipsoidal height differences and the equivalent orthometric height difference *(Δh-ΔH)* was calculated to have a standard deviation of 19.6377 cm. This suggests that both heights can be applied equally to surveying measures in the research area.

The Correlation Coefficient

The correlation coefficient, denoted by r, shows on how closely data in a scatter plot fall along a straight line. The closer that the absolute value of r is to one, the better that the data are described by $\hat{a}$ linear equation. If ρ =1 or $\hat{\rho}$ = -1 then the data set is perfectly aligned. Data sets with values of r close to zero show no straight-line relationship.

$$\hat{\rho} = \frac{(n\sum xy - \sum x \sum xy)}{\sqrt{(n\sum x^2 - (\sum x)^2\ )(n\sum y^2 - (\sum y)^2)}}$$
…………………..(6)

$$\hat{\rho} = \frac{(70 \times 66.7 \times 10^6 \ - 4.67 \times 10^9)}{\sqrt{(70\ X\ 66.7\ X\ 10^6 - 4.7\ X\ 10^9)(70\ X\ 66.7\ X\ 10^6 - 4.7\ X\ 10^9)}}$$

Therefore, $\hat{\rho} = 0.998$

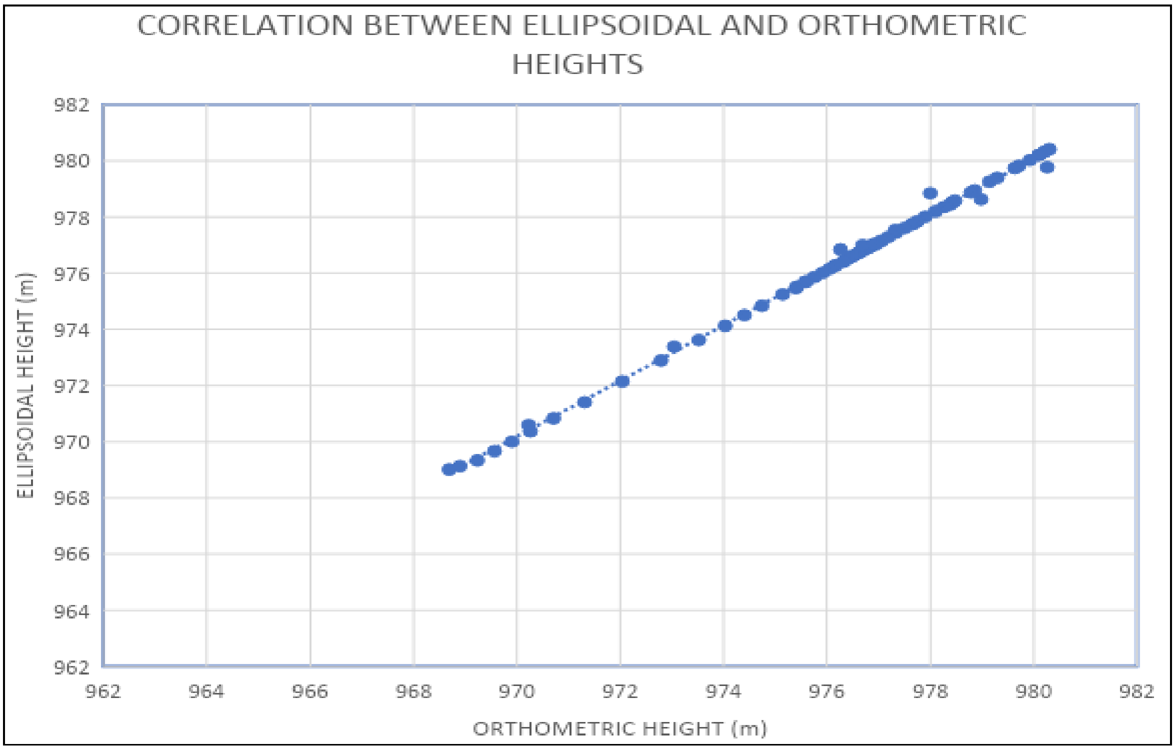

Figure 16: Scatter plot of the correlation between Ellipsoidal and Orthometric heights

The correlation coefficient was calculated to be 0.998 and the scatter plot of ellipsoidal vs orthometric heights yielded a correlation coefficient of 0.999. There is very minimal difference between the values of the correlation coefficient, and they are both within the acceptable range of -1 to +1. The values are significantly close to 1 which implies that a perfect positive correlation of observations was determined.

4. Analysis of the Results

Geoidal Undulation

In this study, a comparison between ellipsoidal and orthometric heights was carried out. Differential levelling and GPS observations were made to obtain the heights of the selected points along the study area and the results are presented in Table 1 above with the corresponding Geoidal undulation (N) which is the height of the geoid relative to a given reference ellipsoid. Figure 13 presenting the Geoidal Undulation and most of the selected points on the study area appear to be giving a negative geoidal undulation. This implies that the geoid is lower than or beneath the reference ellipsoid. Practically, this could be associated with areas of lower elevations within the area of study.

Height Differences

Table 2 shows the difference in ellipsoidal and orthometric heights of the seventy (70) points. Figure 14 shows direct comparison of the two sets of data, ellipsoidal and orthometric heights of the selected points of study. The chart indicates a close relationship between the two data sets of every point because of the minimal changes in height of the bars on the chart. Furthermore, Figure 15 is a line graph of the changes between ellipsoidal height differences and equivalent orthometric height differences *(Δh-ΔH)*. The graph shows very minimal differences for points from CH0 to CH1160 except for the outliers (points before CH0 and after CH1160). The vast variations at the beginning and at the end of the graph were brought about the random selection of points at a non-uniform distance (not more than 50m) unlike the points of emphasis (CH0 to CH1160) which were selected at every 20m interval.

Correlation Coefficient

Figure 16 is a scatter plot of the correlation between ellipsoidal and orthometric heights which indicates a proportional relationship between the two data sets. The graph yielded a correlation coefficient of 0.999 while the calculated value is 0.998. There is very minimal difference between the values of the correlation coefficient, and they are both within the acceptable range of -1 to +1. The values are significantly close to 1 which implies a perfect positive correlation or a direct relationship between the two sets of data.

Standard Deviation and the Root Mean Square Error (RMSE)

For ellipsoidal height differences, the standard deviation is equal to 75.8255 cm, but for similar orthometric height differences, standard deviation is equal to 74.5465 cm. The difference between the ellipsoidal height differences and the equivalent orthometric height difference *(Δh-ΔH)* was calculated to have a standard deviation of 19.6377 cm whereas the root mean square error RSME was found to be

13.1 cm. This suggests that both heights can be applied equally or interchangeably to surveying measures in the research area.

5. Conclusions

This research compared the height difference between orthometric, and ellipsoidal height determined using differential levelling and GPS methods respectively. A total of seventy (70) points of the study area were observed using automatic level and GPS. The height difference between the orthometric and ellipsoidal heights was computed whereby the difference in heights between these methods was determined. The computed heights show a standard deviation of 19.6 cm and Root Mean Square Error of 0.131 m was obtained as the accuracy of the difference between the two heights. In addition, considering the result determined from two methods, it shows that the difference is minimum and that the two heights can be used for third order engineering applications such as road construction, property surveys, and contouring. However, further investigation must be conducted for second or higher order accuracies to determine the errors in differential levelling and GPS observations.

References

[1] Badejo, O., Aleem, K. & Olaleye, J., 2016. Repalcing Orthometric Heights with Ellipsoidal Heights in Engineering Surveys. *Nigerian Journal of Technology (NIJOTECH)*, 35(4), pp. 761-768.

[2] Ghazal, N. K. & Saleh, N., 2021. A Comparison of Orthometric Heights Calculated from (GPS/Leveling) and (EGM08) Methods Based–GIS. *Journal of Physics: Conference Series*, 3(1), pp. 2-3.

[3] Meyer, T., 2002. Grid, Ground and globe: Distances in the GPS Era. *Surveying and Land Information Science* , 62(3), pp. 179-202.

[4] Audu, H. & Tijani, M., 2017. Comparative Assessment of the Accuracy of the Elevation Differences Obtained from Different Geomatics Techniques and Instruments. *Nigerian Journal of Environmental Sciences and Technoogy (NIJEST)*, 1(1), pp. 136-145.

[5] Fotopoulos, G., 2003. *An analysis on the optimal combination of geoid, orthometric and ellipsoidal height data*, Calgary, Alberta: UNIVERSITY OF CALGARY

[6] Tata, H and Olatunji, R.A (2020). Determination of Orthometric Height using GNSS and EGM Data: a Scenario of the Federal University of Technology Akure.