Split Delivery Vehicle Routing Problem for Transportation – A Case Study

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Abstract: ***The Split Delivery Vehicle Routing Problem (SDVRP) is a modification of the classical Vehicle Routing Problem (VRP), which aims to establish optimal routes for a fleet of vehicles serving customers with multiple delivery splits. In this study, we develop an exact solution MILP model for solving the SDVRP with a specific case study of a company operating a sugar factory in Bien Hoa, Dong Nai, Vietnam. This study contributes a MILP SDVRP model to help the company to construct a delivery system with the optimal delivery routes for their vehicle fleet with the objective to minimize the total distance traveled. The computational results prove that the proposed MILP is superior in solving a large number of customer problems and produces optimal solutions which is much better than the current system of the company. Moreover, the proposed MILP also outperforms an existing method in a publication for solving the same problem. The numerical results emphasize the contribution of our proposed MILP in both practical and academic aspects.***

Keywords: Mixed integer linear programming, Split delivery, Vehicle routing problem, Transportation

1. Introduction

The Split Delivery VRP (SDVRP) is a variant of the classical VRP that pursues to determine optimal routes for a vehicle's fleet serving customers with multiple delivery splits. The objective is to minimize either the total distance traveled or the total cost while satisfying specified constraints. The SDVRP is useful for real-life applications as well as attracts the attention of many researchers [1]-[8]. The SDVRP is a complex optimization problem that has not been fully studied, and there are opportunities for further research to improve existing solutions and provide new insights into this problem. Studying the SDVRP gives opportunities to develop and evaluate new optimization algorithms and techniques. In this study, we develop an exact solution MILP model for solving the SDVRP with a specific case study of a company operating a sugar factory in Bien Hoa, Dong Nai, Vietnam. Currently, this company relies on delivery service providers without possessing a delivery system of its own and especially does not build any effective methods for independently planning vehicle routes to satisfy delivery service requirements from customers in Ho Chi Minh City, Vietnam. This study contributes a MILP SDVRP model to help the company to construct a delivery system with the optimal delivery routes for their vehicle fleet to minimize the total distance traveled. The computational results prove that the proposed MILP is superior in solving a large number of customer problems and produces optimal solutions which is much better than the current system of the company. Moreover, the proposed MILP is also proven to outperform an existing method from the literature for dealing with the same problem. The numerical results emphasize the contribution of our proposed MILP in both practical and academic aspects.

2. Mathematical Model

Our proposed mathematical model of the SDVRP is based on [1] and [4]. The notation used in the model is first summarized. Set $N = \{0, 1, ..., n\}$ of all nodes including factory and customer nodes. Set $V = \{1, 2, ..., m\}$ consists of the number of vehicles used to satisfy all the delivery requirements. Parameters c_{ij}, q_i and Q represent the distance between nodes i and j, the demand of customer i, and the maximum vehicle capacity, respectively. Binary variable x_{ijv} is equal to 1 if vehicle v leaves node i to visit node j ($i \neq j$), otherwise equal to 0. Decision variables u_{iv} and y_{iv} indicates the sub-tour elimination constraints and the percentage of customer i demand delivered by vehicle v. Some assumptions are presented as follows:

Assumptions:

- The demands of all customers are required to be fulfilled.
- Vehicles start and end their routes at the factory.
- The vehicle's quantity in assigned routes does not exceed the vehicle's capacity.
- All vehicles have the same capacity assumed to be 15 tons per truck.
- The number of vehicles in the fleet is adequate to fulfill the total demand of customers.
- Each vehicle visits exactly each customer once.
- Each customer can be visited multiple times by different vehicles.

The proposed MILP model for our studied problem is shown as follows:

Objective function: Minimize the total distance traveled.

$$Minimize \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{m} c_{ij}x_{ijv}$$

Constraint 1: Ensure that each vehicle visiting any customer location must also leave it.

$$\sum_{i=0}^{n} x_{ikv} - \sum_{j=0}^{n} x_{kjv} = 0,\ \ k = 0,..,n;\ v = 1,...,m \qquad (1)$$

Constraint 2: Guarantee that deliveries satisfy each customer's demand.

$$\sum_{v=1}^{m} y_{iv} = 1,\ i = 1, ..., n \qquad (2)$$

Constraint 3: All loads on each route must respect the capacity of the vehicle.

$$\sum_{i=1}^{n} q_i y_{iv} \leq Q,\ v = 1, ..., m \qquad (3)$$

Constraint 4: If customer i is visited by vehicle v, then the same vehicle leaves that customer.

$$\sum_{j=0}^{n} x_{ijv} \geq y_{iv},\ i = 1, ..., n,\ v = 1, ..., m \qquad (4)$$

Constraint 5: Guarantee that each customer will be visited at least once.

$$\sum_{v=1}^{m} \sum_{j=0}^{n} x_{ijv} \geq 1,\ i = 1,\ ..., n \qquad (5)$$

Constraint 6: If vehicle v does not serve customer i, no travel of this vehicle from this customer location is conducted.

$$\sum_{j=0\, j\neq i}^{n} q_i x_{ijv} \geq y_{iv} q_i,\ i = 1, ..., n;\ v = 1, ..., m \qquad (6)$$

Constraint 7: Fractional cycle elimination constraints.

$$x_{ijv} \leq \sum_{p=0\, p\neq i}^{n} x_{jpv}, \qquad (7)$$
$$i = 1, ..., n; j = 1, ..., n;\ v = 1, ..., m$$

Constraint 8: Eliminate subtours.

$$u_{iv} - u_{jv} + n x_{ijv} \leq n - 1, \qquad (8))$$
$$i, j = 1, ..., n;\ i\neq j;\ v = 1, ..., m$$

Constraint 9: Eliminate two node subtours.

$$x_{ijv} + x_{jiv} \leq 1, \qquad (9)$$
$$i, j = 1, ..., n; i\neq j;\ v = 1, ..., m$$

Constraint 10: The route's direction by getting the shortest arc traversed first of the two arcs adjacent to the factory.

$$\sum_{j=1}^{n} c_{0j} x_{0jv} \leq \sum_{i=1}^{n} c_{i0} x_{i0v},\ v = 1, ..., m \qquad (10)$$

Constraint 11: No delivery to the same node.

$$x_{iiv} = 0,\ i = 0, ..., n;\ v = 1, ..., m \qquad (11)$$

$$x_{ijv} \in \{0, 1\}, \qquad (12)$$
$$i, j = 0, ..., n;\ i\neq j,\ v = 1, ..., m$$

3. Experimental Results

The customers whose company's factory needs to be served are spread around Ho Chi Minh City. The data is collected from the customer database of the company's Logistics Department. The number of customers who need to be satisfied is 21. All vehicles in the fleets have the same capacity, which is 15 tons per truck, and the number of used vehicles is computed by dividing total customer demand by vehicle capacity. The solutions are acquired by solving by CPLEX optimization software in 7200 seconds. Next, **Table 1** is used to give the results of the route for each truck that delivers goods to assigned customers. While **Figure 1** illustrates the found optimal solution with vehicle routes on a map

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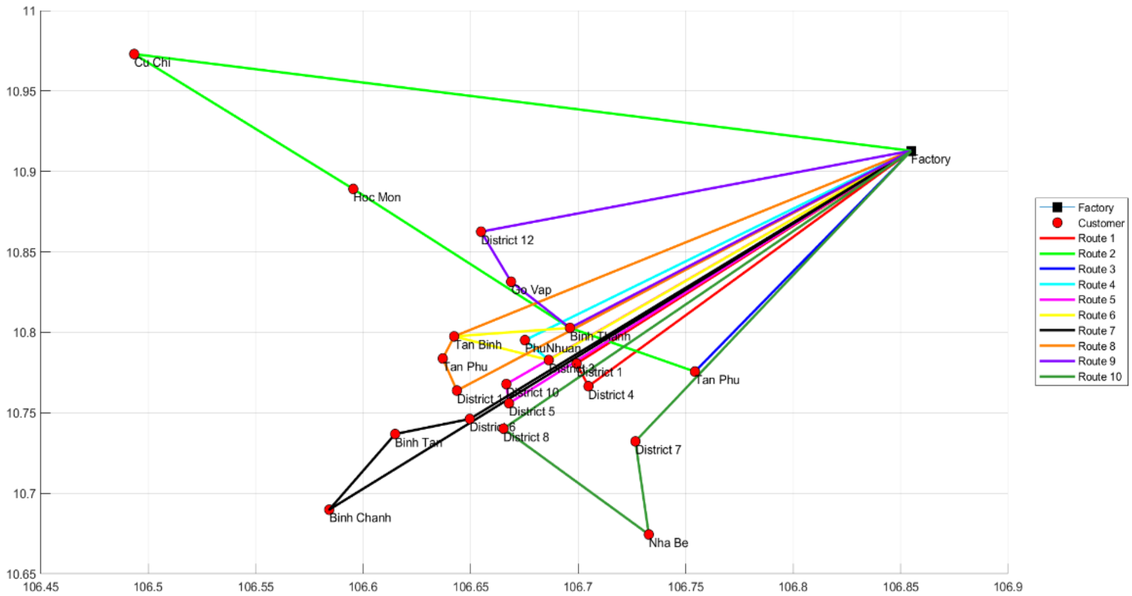

Figure 1: Optimal vehicle routes on a map found for the case study

Table 1: Truck assignment on each customer node and total distance

Vehicle	Routes	Distance
1	Factory - District 1 - District 4 - Factory	53.7
2	Factory - Thu Duc - Binh Thanh - Hoc Mon - Cu Chi - Factory	111.7
3	Factory - Thu Duc - Factory	42.1
4	Factory - Phu Nhuan - District 3 - Factory	54.8
5	Factory - District 10 - District 5 - Factory	60.4
6	Factory - Binh Thanh - Tan Binh - District 3 - Factory	62.4
7	Factory - Binh Chanh - Binh Tan - District 6 - Factory	87.3
8	Factory - Tan Binh - Tan Phu - District 11 - Factory	68.4
9	Factory - Binh Thanh - Go Vap - District 12 - Factory	60.1
10	Factory - District 7 - Nha Be - District 8 - Factory	84.8
Total		**685.7**

Besides, **Figure 2** indicates the comparison between the proposed model's solution and [6]'s MILP model solution in terms of the total distance traveled based on the same benchmark. As we can see, the proposed model's solution has the advantage of solving a considerable number of customer problems which can reach 40 customers, whereas the model of [6] only can solve the 16-customer problem. Moreover, in the range of 10 to 12 customers, the proposed model's solution provides a shorter total distance traveled, while [6] generates a better solution for the 14-customer and 16-customer problems. Therefore, the proposed solution is more appropriate to deal with large customers' problems and produces more optimal solutions than publication solutions in some cases.

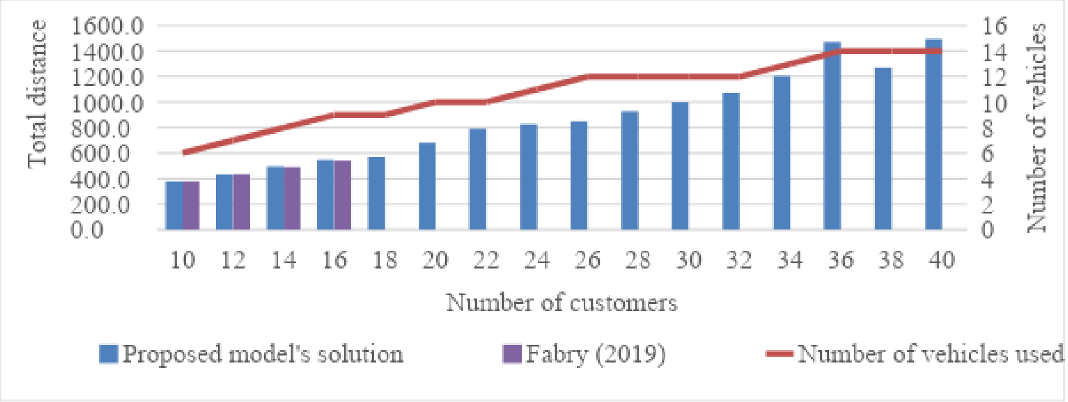

Figure 2: Comparison of a publication and proposed model solution

Furthermore, the sensitivity analysis is conducted to understand the effect of adjusting input parameters (namely vehicle capacity and customer demand in each customer node) on the proposed method's output. Figure 3 and Figure 4 are used to demonstrate the changes in total distance traveled, and the number of vehicles used to fulfill customer demands in Ho Chi Minh City when adjusting vehicle capacity and when customer demands at each customer node, respectively. It is shown that an increasing trend of total distance traveled and the number of vehicles required occurs when vehicle capacity reduces while the increase of vehicle capacity leads to the reduction of total distance traveled and the number of vehicles used. Also, a decreasing trend of total distance traveled and the number of vehicles needed happens when customer demands go down while an increase in customer demands leads to the rise of the total distance traveled and the number of vehicles used.

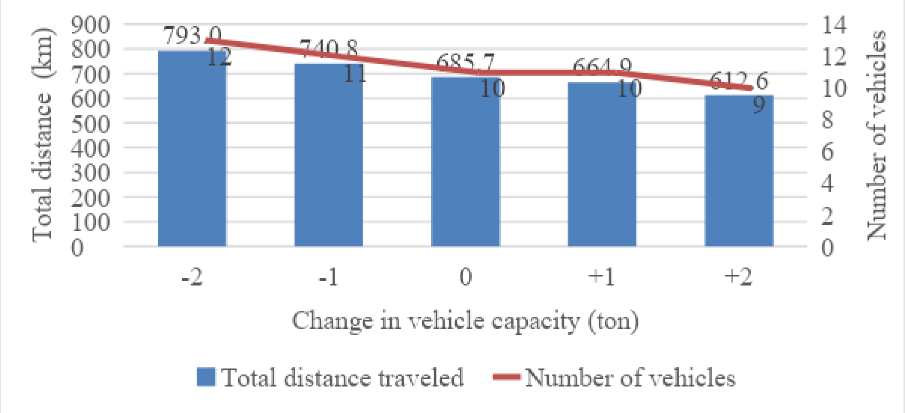

Figure 3: The changes in total distance and number of vehicles when adjusting vehicle capacity

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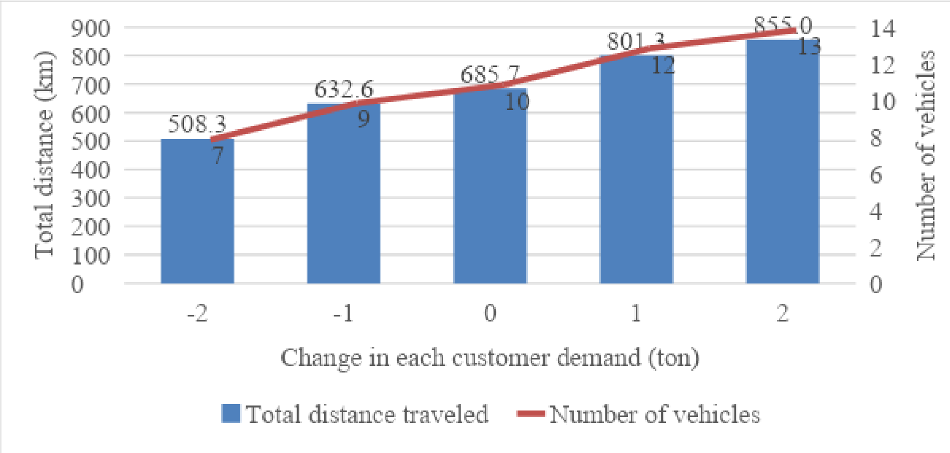

Figure 4: The changes in total distance and number of vehicles when adjusting customer demands

4. Conclusion

This study concentrates on solving the SDVRP of a case study by developing a MILP. The MILP is implemented by CPLEX optimization software and the computational results prove that the proposed MILP is superior in solving large-size problems and produces optimal solutions which is much better than the current system of the company. Moreover, the proposed MILP also outperforms an existing method in a publication for solving the same problem. In future research, some criteria can be included in the model to improve the model application in the real world. One of the factors is that the cost of a delivery system such as fixed cost and variable cost can be included in the model's objective function. Another factor is that vehicles with diverse capacities can be considered to improve the solution.

References

[1] Dror, M., Laporte, G. & Trudeau, P. Vehicle routing with split deliveries. *Discrete Appl Math* 50, 239–254 (1994).

[2] Archetti, C., Bianchessi, N. & Speranza, M. G. Branch-and-cut algorithms for the split delivery vehicle routing problem. *Eur J Oper Res* 238, 685–698 (2014).

[3] Jin, M., Liu, K. & Bowden, R. O. A two-stage algorithm with valid inequalities for the split delivery vehicle routing problem. *Int J Prod Econ* 105, 228–242 (2007).

[4] Wilck, J. & Cavalier, T. A Construction Heuristic for the Split Delivery Vehicle Routing Problem. *American Journal of Operations Research* 02, 153–162 (2012).

[5] Gschwind, T., Bianchessi, N. & Irnich, S. Stabilized branch-price-and-cut for the commodity-constrained split delivery vehicle routing problem. *Eur J Oper Res* 278, 91–104 (2019).

[6] Fábry, J. Transportation of Euro Pallets Solved As A Split Delivery Vehicle Routing Problem. *Acta logistica* 6, 49–53 (2019).

[7] Chen, P., Golden, B., Wang, X. & Wasil, E. A novel approach to solve the split delivery vehicle routing problem. *International Transactions in Operational Research* 24, 27–41 (2017).

[8] Le, T. D. C., Nguyen, D. D., Oláh, J. & Pakurár, M. Clustering algorithm for a vehicle routing problem with time windows. Transport 37, 17–27 (2022).

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