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Exploring Properties and Applications of the F-Structure Equation $F^{4k}+F^{k}=0$ in Differential Manifolds

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Abstract: ***In this paper, we have studied various properties of the F- structure equation $F^{4k}+F^{k}=0$, k being a positive integer. Nijenhuis tensors, metric F- structure, kernel, tangent and normal vectors have also been discussed.***

Keywords: DM, PO, ACS, NT, Ker, TVS, NVS

Notations Through the paper we use the following abbreviations in the place of standard technical terms.
DM- Differential manifold
PO- Projection operator
ACS- Almost complex structure
NT- Nijenhuis tensor
Ker- Kernel
TVS- Tangent vectors
NVS- Normal Vectors

1. Introduction

Various authors and researchers have studied differentiable manifolds, real and complex manifolds, and the F- structure equations from time to time. After the reviewed literature mentioned in the references [1], [2], [3], [4] …………,[14] we find that currently, this field is alive for academicians and researchers. So, we posed a sequel of [6], [7],[8], and [9]. Let M^n be a differentiable manifold of class C^∞ and F be a $(1,1)$ tensor of class C^∞ defined on M^n by-

$$F^{4k}+F^{k}=0 \qquad (1.1)$$

We define the operators l and m on M^n, satisfying-

$$l=-F^{3k}, m=I+F^{3k}, I \text{ denotes identity operator} \qquad (1.2)$$

From (1.1) and (1.2) we have,

$$l+m=I, l^2=l, m^2=m, lm=ml=0, F^k l=lF^k=F^k, F^k m=mF^k=0 \qquad (1.3)$$

Theorem (1.1): Let the $(1,1)$ tensor α and β satisfy-

$$\alpha=m+F^k, \beta=m-F^{2k} \text{ then,} \qquad (1.4)$$

$$\alpha^3=m-l, \alpha^6=I=\alpha\beta \qquad (1.5)$$

Proof: Using (1.2), (1.3) and (1.4) we get the results.

Theorem (1.2): Let the $(1,1)$ tensor p and q satisfy-

$$p=m+F^{3k}, q=m-F^k \text{ then,} \qquad (1.6)$$

$p^2 = q^3 = I$ (1.7)

Proof: Using (1.2), (1.3) and (1.6) we get (1.7).

Theorem (1.3): Let k be even and rank $((F)) = n$ then,

$l = I, m = 0$ (1.8)

and $F^{3k/2}$ acts as an almost complex structure.

Proof: From the fact

$rank((F)) + nulity((F)) = \dim M^n = n$ (1.9)

We have

$rank((F)) = 0 \Rightarrow \ker F = \langle 0 \rangle$

Thus $FX = 0 \Rightarrow X = 0$.

Let $FX_1 = FX_2 \Rightarrow F(X_1 - X_2) = 0 \Rightarrow X_1 = X_2$ or F is $1-1$, moreover M^n being finite dimensional F is onto also. Thus F and hence F^k is invertible.

Operating F^{-k} on $F^k l = lF^k = F^k$ and on $F^k m = mF^k = 0$ we get $l = I, m = 0$. Operating F^{-k} on (1.1) we have $F^{3k} + I = 0 \Rightarrow F^{3k/2}$ acts as an almost complex structure.

2. NT

Let $\underset{F}{N}, \underset{l}{N}$ and $\underset{m}{N}$ denote the Nijenhuis tensors corresponding to the operators F, l and m respectively. Then,

$\underset{F}{N}(X,Y) = [FX, FY] + F^2[X,Y] - F[FX,Y] - F[X,FY]$ (2.1)

$\underset{l}{N}(X,Y) = [lX, lY] + l^2[X,Y] - l[lX,Y] - l[X,lY]$ (2.2)

$\underset{m}{N}(X,Y) = [mX, mY] + m^2[X,Y] - m[mX,Y] - m[X,mY]$ (2.3)

Theorem (2.1): Let F, l and m Satisfy (1.1), (1.2) and (1.3) then,

$\underset{F^k}{N}(mX, mY) = F^{2K}[mX, mY]$ (2.4)

$F^k \underset{F^k}{N}(mX, mY) = -l[mX, mY]$ (2.5)

$F^k \underset{F^k}{N}(mX, mY) + \underset{l}{N}(mX, mY) = 0$ (2.6)

$\underset{m}{N}(lX, mY) = 0$ (2.7)

Proof: Using (1.2), (1.3), (2.1), (2.2) and (2.3) we get the results.

3. M – Structure

Let ${}^{/}F(X,Y) = g(FX,Y)$ is skew symmetric then,

$g(FX,Y) = -g(X,FY)$ (3.1)

Theorem (3.1): Let F satisfies (1.1) then,

$g(F^k X, F^{2k} Y) = (-1)^{k+1}[g(X,Y) - {}^{/}m(X,Y)]$ (3.2)

where,

${}^{/}m(X,Y) = g(X, mY)$ (3.3)

Proof: Using (1.2), (1.3), (3.1) and (3.3) we get-

$$g(F^k X, F^{2k} Y) = (-1)^k g(X, F^{3k} Y)$$
$$= (-1)^k g(X, -lY)$$
$$= (-1)^{k+1} g(X, lY)$$
$$= (-1)^{k+1} g(X, (I-m)Y)$$
$$= (-1)^{k+1}[g(X,Y) - g(X, mY)]$$
$$= (-1)^{k+1}[g(X,Y) - {}' m(X,Y)] \qquad (3.4)$$

4. Ker, tangent and normal vectors

We define-

$$\ker F = \langle X : FX = 0 \rangle \qquad (4.1)$$
$$\tan F = \langle X : FX \parallel X \rangle = \langle X : FX = \lambda X \rangle \qquad (4.2)$$
$$NorF = \langle X : g(X, FY) = 0, \forall Y \rangle \qquad (4.3)$$

Theorem (4.1): Let F satisfies (1.1) then,

$$\ker F^k = \ker F^{4k} \qquad (4.4)$$
$$\tan F^k = \tan F^{4k} \qquad (4.5)$$
$$NorF^k = NorF^{4k} \qquad (4.6)$$

Proof: Using (1.1), (4.1), (4.2) and (4.3) we get the results. We proved only (4.6).

Let $X \in NorF^k \Rightarrow g(X, F^k Y) = 0$

$$\Rightarrow g(X, -F^{4k} Y) = 0$$
$$\Rightarrow g(X, F^{4k} Y) = 0$$
$$\Rightarrow X \in NorF^{4k}$$

Thus,

$$NorF^k \subseteq NorF^{4k} \qquad (4.7)$$

Again let $X \in NorF^{4k} \Rightarrow g(X, F^{4k} Y) = 0$

$$\Rightarrow g(X, -F^k Y) = 0$$
$$\Rightarrow g(X, F^k Y) = 0$$
$$\Rightarrow X \in NorF^k$$
$$NorF^{4k} \subseteq NorF^k \qquad (4.8)$$

From (4.7) and (4.8) we get (4.6).

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References

[1] Das L.S. 2004. On CR-structures and $F(2K+1,1)$ −structure satisfying $F^{2k+1}+F=0$, JTSl, India. 22, 1-7.
[2] Das, L.S., Nivas, R. and Singh, A., 2008, On CR-structures and F-structures satisfying $F^{4n}+F^{4n-1}+\cdots+F^2+F=0$, Tesor N.S. 70. 255-260.
[3] Das. L.S. Nikic, J. and Nivas. R., 2005, Distributions parallelism and geodesic in $F(a_1, a_2,, \ldots.., a_{n)}$ structure Lagrangian manifolds. Differential Geometry Dynamitical Systems, Romania, 7, 82-89.
[4] E. Scholz, 1999, The concept of manifold, 1850-1950, in: History of Topology, I.M. James (ed.), Elsevier Science B.V., Amsterdam, pp. 25-64.
[5] Goldberg, S.I. 1972, On the existence of manifold with an F-structure, Tensor (N.S.), 26, 323-329.
[6] Lakhan Singh, 2020, On The Cyclic Group related to the F- structure equation $\sum_{k=1}^{p} F^k = 0$, Naveen Shodh Sansar Oct. to Dec.Vol. I, issue XXXII ISSN 2320-8767.
[7] Lakhan Singh, 2017, On Some Properties of Metric F- structure satisfying$F^{2k+1}+F=0$, International Journal of mathematics and its Applications, Vol. 5, issue 2-A, 63- 66, ISSN 2347-1557.
[8] Lakhan Singh, 2016, On the Structure Equation$F^{3k}+F^k=0$, International Journal of Current Research, Vol. 8, issue 09, 38792- 38795, ISSN 0975-833X.
[9] Lakhan Singh,2015, On CR- structure F- structure satisfying$F^7+F=0$, International Journal of mathematics Trends and Technology, Vol. 26, No.1, 04- 08, ISSN 2231-5373.
[10] R. Narasimhan, 1973, Analysis on Real and Complex Manifolds, North-Holland Publishing Comp., Amsterdam.
[11] Ram Nivas and Surendra Yadav, 2012, On CR-structures and $F_\lambda(\,2v+3{,}2)-HSU-$ structure satisfying $F^{2v+3}+\lambda^r F^2=0$, Acta Ciencia Indica, Vol. XXXVIII M, NO. 4, 645.
[12] S.S. Chern, 1955, Differentiable Manifolds, mimeographed lecture notes (spring 1952), Department of Mathematics, University of Chicago Press.
[13] Upadhyay, M.D. and Gupta, V.C. 1977. Integrability conditions of a structure f_λ satisfying $f^3+\lambda^2 f=0$, Publications Mathematics, 24 (3-4), 249-255.
[14] Yano, K. 1963. On structure defined by a tensor Reid f of type $(1,1)$ satisfying $f^3+f=0$, Tensor (N.S.), 14,99-109.