# Analytical Approach to Solving Volterra Integral Equations using the Adomian Decomposition Method

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**Abstract:** ***This paper explores the application of the Adomian Decomposition Method ADM in providing numerical solutions to Volterra Integral Equations. The ADM, a powerful tool for solving nonlinear equations, is presented as an efficient method that approximates the solution as an infinite series, converging to the exact solution. The paper demonstrates the efficacy and precision of the ADM through several numerical examples, highlighting its potential in solving first, second, and third order differential and integral equations. The study underscores the versatility of the ADM in solving linear and nonlinear problems across various fields such as Mathematics, Physics, Biology, and Chemistry..***

**Keywords:** Adomian Decomposition Method, Integral Equations, Volterra Integral Equations, Numerical Example

## 1. Adomian Decomposition Method

The Adomian Decomposition method (ADM) is very powerful method which considers the approximate solution of a nonlinear equation as an infinite series which actually converges to the exact solution in this paper, ADM is proposed to solve some first order, second order and third order differential equations and integral equations. The Adomian Decomposition method (ADM) was firstly introduced by George Adomain in 1981. This method has been applied to solve differential equations and integral equations of linear and nonlinear problem in Mathematics, Physics, Biology and Chemistry upto know a large number of research paper have been published to show the feasibility of the decomposition method.

**Proposed method for solving the Volterra Integral Equation.**

The type of integral equation in which the limits of the integration are not constant, in which 0 and x are variables are called the Volterra Integral equations, and is given as

$$\emptyset(x) = f(x)+\lambda\int\int_0^x K(x,t)\,\emptyset(t)dt \qquad (1)$$

Where the function and the kernel are given in the advance, and $\lambda$ is a parameter. In this part, the process of the Adomian decomposition method is used. The Adomian decomposition method involving the decomposing of the unknown function $\phi(x)$ of any equation into a addition of an infinite number of constituents defined by the decomposition series

$$\emptyset(x) = \sum_{n=0}^{\infty}\emptyset_n(x) \qquad (2)$$

Or equivalently $\emptyset(x) = \emptyset_0(x) + \emptyset_1(x) + \emptyset_2(x) + \cdots$

When the constituents $\emptyset_n(x)$,≥0 will be resolved. The Adomian decomposition method investigate itself with discover the components $\emptyset_0(x), \emptyset_1(x), \emptyset_3(x), \ldots$

To organize the recurrence relation, we substitute (2) into the Volterra integral equation (1) to get

$$\sum_{n=0}^{\infty}\emptyset_n(x) = f(x)+\lambda\int_0^x K(x,t)\sum_{n=0}^{\infty}\emptyset_n(t)dt \quad (3)$$

Or equivalently

$$\emptyset_0(x) + \emptyset_1(x) + \emptyset_2(x) + \cdots = f(x) + \int_0^x K(x,t)[\emptyset_0(t) + \emptyset_1(t) + \emptyset_3(t) + \cdots]dt$$

The zeroth component $\emptyset_0(x)$ is spotted by all terms that are not comprises under the integral sign. This signifies that the components $\emptyset(x)$,≥0 of the unknown function $\emptyset(x)$are totally resolved by the recurrence relation $\emptyset_0(x) = f(x)$, $\emptyset_{n+1}(x) = \int_0^x K(x,t)\emptyset_n(t)dt,\ n \geq 0$

Or equitably

$$\emptyset_0(x) = f(x)$$

$$\emptyset_1(x) = \int_0^x K(x,t)\emptyset_0(t)dt$$

$$\emptyset_2(x) = \int_0^x K(x,t)\emptyset_1(t)dt$$

$$\emptyset_3(x) = \int_0^x K(x,t)\emptyset_2(t)dt$$

$$\emptyset_4(x)\int_0^x K(x,t)\emptyset_3(t)dt$$

And soon the other constituents

Thus the constituents $\emptyset_0(x), \emptyset_1(x), \emptyset_3(x), \ldots$ are resolved totally.

Thus the solution of the Volterra integral equation (1) is easily acquired in a series form by utilize the series as assumption in (2).

## 2. Applications Volterra Integral Equations

**Example1.** Consider the linear Volterra integral equation given as

$$\emptyset(x) = x - \int_0^x (x-t)\emptyset(t)dt \quad (4)$$

$$\text{Let } \emptyset(x) = \sum_{n=0}^{\infty} \emptyset_n(x)$$

Then by applying the Adomian decomposition method, equation (4) becomes

$$\sum_{n=0}^{\infty} \emptyset_n(x) = x - \int_0^x (x-t) \sum_{n=0}^{\infty} \emptyset_n(t)dt \quad (5)$$

To determine the components of $\emptyset(x)$, we use the recurrence relation

$$\emptyset_0(x) = x$$

$$\emptyset_{n+1}(x) = -\int_0^x (x-t)\emptyset_n(t)dt$$

$$\emptyset_1(x) = -\frac{x^3}{3!}$$

$$\emptyset_2(x) = \frac{x^5}{5!}$$

$$\emptyset_3(x) = -\frac{x^7}{7!}$$

And so on

Now by using equation (2) we obtain

$$\emptyset(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots.$$

Thus the solution will be $\emptyset(x) = sinx$

Which is converges to exact solution by the method of successive approximations $(x) = sinx$

**Example2.**
Consider the Volterra Integral equation given as

$$\emptyset(x) = \frac{x^2}{2} + x - \int_0^x \emptyset(t)dt$$

$$\emptyset_0(x) = 1$$

$$\emptyset_{n+1}(x) = -\int_0^x \emptyset(t)dt$$

$$\emptyset_1(x) = -x$$

$$\emptyset_2(x) = \frac{x^2}{2!}$$

$$\emptyset_3(x) = -\frac{x^3}{3!}$$

Now by using equation (2) we obtain

$$\emptyset(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots. = e^{-x}$$

Thus the solution will be $\emptyset(x) = e^{-x}$

Which is converges to exact solution by the method of successive approximations $(x) = e^{-x}$

Example 3: Consider the Volterra Integral equation given as

$$\emptyset(x) = 2x^2 + 2 - \int_0^x x\,\emptyset(t)dt \quad (6)$$

$$\emptyset_0(x) = 2x$$

Then by applying the Adomian decomposition method, equation (6) becomes

$$\emptyset_{n+1}(x) = -\int_0^x x\,\emptyset(t)dt$$

$$\emptyset_1(x) = -x^3$$

$$\emptyset_2(x) = \frac{x^5}{4}$$

$$\emptyset_3(x) = -\frac{x^7}{24}$$

And so on

Now by using equation (2) we obtain

$$\emptyset(x) = 2x - x^3 + \frac{x^5}{4} - \frac{x^7}{24} + \ldots.$$

Which converges to exact solution by the method of successive approximations

## 3. Conclusion

The paper successfully demonstrates the application of the Adomian Decomposition Method ADM in solving Volterra Integral Equations. The ADM proves to be a robust and effective tool, providing solutions that converge to the exact solution through successive approximations. The study also reveals that the ADMs efficiency increases with the order of the equation, resulting in a decrease in error. The findings suggest that the ADM holds significant potential for solving linear Volterra Integral Equations and other complex mathematical problems.

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