Analysis of Mechanical Error in Spherical Geneva Mechanism - A Stochastic Approach

Himanshu Suthar, Mihir Raj

Abstract: The report presents the analysis of mechanical error in Spherical Geneva systems, with the inclusion of behavior of kinematic properties with respect to the errors incorporated. The report consists of acceleration and jerk values of a spherical Geneva for a uniform input in terms of the dimensions of the system. The values of these components were calculated for around 40 positions, over 20 error values. The analysis is done separately for constant values of radius of the driver wheel, and length between centres of driver wheel and driven wheel while having tolerance in the other. The physical means and standard deviations for these kinematic elements can help in understanding the properties and optimizing functioning for real-life purposes. For the purpose, the errors (tolerance values of the link lengths) were taken normally distributed and can be calculated for the errors incorporated using the methods described here. This report also helps in understanding the changes over a complete cycle and can be used for optimal synthesis of Geneva systems, considering strength and damages.

1. Introduction

1.1 Background

Geneva Mechanism is very commonly found in devices around us. The primary use of Geneva Mechanisms relevant to the common household is in analog watches. Due to its unique property of converting a continuous circular motion into intermittent rotary motion, it has a wide spectrum of industrial applications.

Figure 1: Planar Geneva Mechanism

Figure 2: A Video Camera that uses Geneva Mechanism

Geneva Drive or Maltese Cross (a. k. a. Geneva Stop) is a gear which translates continuous rotational movement into intermittent rotary motion. The rotating wheel is usually equipped with a pin that reaches the slot of the driven wheel that advances one step at a time. Historically, the Geneva Mechanism was extensively used in manufacturing watches and it gets its name from Geneva, which was the classical origin of watchmaking industries. When the driven wheel has four spokes, it is also referred to as the Maltese Cross Mechanism due to its visual resemblance with the Maltese Cross.

The driven wheel can have n number of spokes but the most common version has four spokes which rotate 90° with each full rotation of the drive wheel. If the driven wheel has n slots, it advances by $360°/n$ per full rotation of the drive wheel.

Lowest practical spokes in the Geneva Mechanism is 3 but more than 18 slots are seldom used. If one of the slots is uncut, then the number of rotations is limited. Originally, it was invented by a Swiss watchmaker to prevent over-winding of spring in watches but later it found a huge number of applications in the industrial revolution.

1.2 Applications

The Geneva mechanism played a crucial role during the industrial revolution. It has its application in a wide range of areas. Apart from watchmaking industries, it is extensively used in low and high-speed automatic machinery. Its use revolutionized the film making industries. It is used in projectors where a film is pulled and exposed to projectors light with periodic starts and stops. As the film advances frame by frame, each frame stands still for typically 1/24th of a second and rapidly accelerates for the rest of the cycle. This mechanism also provides a precisely repeatable stop position, which is critical in minimizing jitter in successive images. Figure 2 shows a movie projector with a hand crank and Geneva drive. The indexing nature of this mechanism finds several other applications: plotters, machine tools,
pressing machinery, packaging and automation industries, transfer lines, etc.

1.3 Types of Geneva Mechanism

Generally, Geneva Mechanisms are of two types:
- Planar Geneva Mechanism
- Spherical Geneva Mechanism

Planar Geneva can also be divided into two types based on their working and arrangement:
- Internal Geneva Mechanism
- External Geneva Mechanism

1.4 Planar Geneva Mechanism

External

In this type of mechanism, the Geneva cross is connected with the cam drive externally. This is the most popular Geneva mechanism and can also withstand higher mechanical stress. The driver grooves lock the driven wheel pins during the dwell. During movement, the driver pin mates with the drive wheel slot.

Internal

In this type of mechanism, the Geneva cross and cam drive are connected internally in the closed box. The driver and the driven wheel rotate in the same direction. The duration of dwell is more than 180° of driver rotation.

1.5 Spherical Geneva Mechanism

This type of Geneva mechanism is most rare. In this type of mechanism, the Geneva cross is in a spherical shape and cam drive is connected externally. The driver and the driven wheel are on perpendicular shafts. The duration of dwell is exactly 180° of driver rotation. We will be discussing mainly the Spherical Geneva mechanism in this report.

Above Figure shows the diagram of a Spherical Geneva Mechanism. Driver 1 rotates about fixed axis A and carries pin a which consecutively engages slots b in spherical Geneva wheel 2. Geneva wheel 2 rotates about fixed axis B and its slots b are located symmetrically at angles of 90° between adjacent slots. Driver 1 has concentric member ‘d’ which engages concave surfaces e of Geneva wheel 2 to prevent its unintentional rotation during its idle periods. When driver 1 rotates continuously at uniform velocity, Geneva wheel 2 rotates intermittently at non-uniform velocity.

1.6 Errors in Mechanisms

The error caused by a mechanical defect such as clearance at the joints or inaccuracy in length is called mechanical error. In analyzing the kinematic characteristics of mechanisms follow -

1.7 Introduction to Problem

Errors in Geneva Mechanism

Errors in manufacturing mechanisms in the practical world are inevitable. Error is the difference between the estimated or computed value and the accepted true or theoretically
correct value. It is the difference between desired and actual performance or behavior of system or object.

In designing any system or machine, errors in measurement and manufacturing are inevitable.

Engineers tend to minimize (or preferably avoid) these errors. These errors may produce wear and tear in a machine over years of use, hence reducing the life of a product.

The errors are generally caused by various reasons. Some of them are:
1) Compliance
2) Property Variation
3) Motion errors
4) Setup errors
5) Programming errors errors

In the manufacturing of the Geneva Mechanism, its performance deviates from the desired one due to certain errors, for example, tolerances on link lengths and clearances in various joints, errors due to misalignment, variation in input speed, and power, etc. These errors not only affect kinematic accuracy but also have an effect on dynamic performance, noise levels, and the lifetime of mechanism. A designer should keep these errors and their effects in mind while fabricating any mechanism.

Here, we will analyze these errors for Spherical Geneva Mechanism, which will help us for a better understanding of working and synthesis of the same. The idea is to induce tolerance in the link lengths which are an inevitable error for every practical mechanism or machine.

2. Literature Review

2.1 Literature

This report deals with the analysis of the probabilistic approach of the mechanical error of the Spherical Geneva mechanism.

1) Reference for the mechanical error of the Geneva mechanism is taken from the report, ‘Analysis and Synthesis of Mechanical Error in Geneva Mechanism’. This error analysis was done using the kinematics of the mechanism. This report from S. S. Gavane, deals with the errors and probabilistic approach of the errors in the Planar Geneva system. The aim included the expression derivation for the mean and variance of acceleration and jerk in the mechanism. With the planar mechanism model, the probabilistic model considering tolerances and clearances is derived. The probability distribution of random variables is assumed normally distributed throughout the zone. The paper also deals with expressions of acceleration and jerk, in terms of geometrical measurements of the mechanism. The above-mentioned methods are used in this report and methodology for the Spherical Geneva Error Analysis proceeds in the same way. The thesis mentioned above by Gavane was insightful as it talks about the possibilities about the analysis as well as, the further scope in the field using these.

2) For the Spherical systems, we referred to the book, “Kinematics of Spherical Mechanism” by C H Chiang, for the kinematics of the Spherical Geneva system. From there, we proceeded for kinematics of the Spherical Geneva system. The kinematics can be used for similar equations of angular acceleration and jerk in terms of geometrical values, which then further be used for error analysis when there is deflection in those values. Since the scope of the project doesn’t carry out the derivations of these formulae, we will not be dealing with the derivations of the formulae mentioned in the previously mentioned report. Angular acceleration and Jerk formulae are expressed here, thus by, without including the tolerances and clearances. Though these formulae can further be used for a little deflection or error from the original or computed value. Since, the case in study is about Spherical Geneva, roots of the formulae used later can be found in this book. One of the most important insights about dealing the link lengths in terms of angle values for the Spherical system, instead of the lengths as it was taken in the report mentioned previously, helps significantly in the ease of understanding and calculation.

2.2 Conclusion

The thesis is concluded with results that can be used for synthesis at optimized values. It dealt with the computation of the standard of acceleration and jerk, considering the possible tolerance and clearance values.

For the mechanical error of any system, errors can be induced and expressed in terms of tolerances on link lengths and clearances in various joints, also the variations in speed and errors due to the misalignment in the assembly. These errors are important to keep in check, as besides affecting the kinematic precision, they affect dynamic performance too, thus, the life of the mechanism.

The nature of movement and dynamics can also be understood from a single slot spherical Geneva system with movement freedom of ninety degrees, i.e. one - fourth of a four - slot spherical Geneva. The model with uniform angular velocity to the input link results in the Harmonic motion of output link in the direction of movement, which tells about the nature of the motion.

From the literature, there are two important insights involved, that is going to be used. The methodology for the work proposed and done was based on the previously mentioned reports. For the values and calculations for Spherical systems rather than planar systems, the conversion was based on the reference from the given book mentioned above.

3. Project Objectives and Work Plan

3.1 Problem Definition

The motivation of this project is to identify sources of errors in the Geneva Mechanism and find out their effect on acceleration and jerk using analytical methods.
3.2 Objective

Our objective in this project is to calculate the deviation of acceleration and jerk due to tolerances in link length. We also want to determine the orientation of the mechanism when the error has its maximum effect on acceleration and jerk of various components of the Geneva Mechanism.

The aim of this project is to use a probabilistic approach to estimate the effect of errors due to tolerance in link lengths on output response, i.e., acceleration and jerk of the driven wheel of the Geneva Mechanism and visualize it using graphs.

When any mechanism is fabricated and assembled, its performance differs from desired outputs due to certain errors. Errors in the fabrication of a component are inevitable by nature and can only be reduced to a certain limit. In most of the cases, such errors occur due to tolerances in link lengths and clearance in various joints. Such errors may result in a deviation of output from expected results. Such errors not only affect kinematic accuracy, but also have dynamic effects such as building up stress, noise levels, and overall life of the mechanism.

Considering these errors, our final objective is to analyse this problem such that the results can be further used for synthesis optimizations and further studies.

3.3 Methodology

Deviation from expected output is the result of unexpected errors due to tolerance and clearances in link lengths and joints. The effect of such errors can be estimated analytically. Using kinematics, we can find the relationship between input angular velocity and its response in terms of output acceleration and jerk. In the Geneva Mechanism, neither acceleration nor jerk is constant but it depends on various factors like link lengths, input angular velocity, and orientation of both driver wheel and driven wheel.

In the Geneva Mechanism, a major source of errors is inaccuracy in measurements of link lengths and joints while fabrication of components. This inaccuracy is also termed as tolerances and clearances. Hence, tolerance in link lengths and clearance in joints is the major source of errors in acceleration and jerk in the Geneva Mechanism. Other sources of error due to tolerance and clearance may arise due to a change in temperature, which may result in the expansion and contraction of material using which mechanism was fabricated. Although errors due to change in temperature can be considered negligible, it can be included in our analysis. Errors may be included due to the dynamic behavior of material such as bending and deformation of link lengths and holes. But, in this experiment, such sources were ignored as these sources would not contribute at small angular velocities. In this report due to the constraint of time, we had considered errors induced only due to the tolerances of the link lengths and clearances.

The effect of errors on jerk and acceleration due to tolerances can be estimated analytically. Using kinematics, we can derive an expression of the ideal behavior of the Geneva Mechanism. Assuming link lengths are distributed normally between tolerance limits, we can estimate the output response i.e. acceleration and jerk of the driven wheel.

Assuming tolerance limits to be some percentage of ideal link length, we can assume different values in that range. Such values would be distributed normally between tolerance range and therefore, can be used to estimate jerk and acceleration. Values of jerk and acceleration would also depend on parameters of mechanism i.e. angular orientation of both the driven wheel and drive wheel. By estimating values at different angles for all combinations of link lengths, we can estimate the individual effect of errors in both components on both acceleration and jerk. Using the data generated, we can visualize the deviation of output response due to errors with respect to the angular positions of both components.

Following approach was considered while analysing the effect of errors induced due to tolerance:

1) We started with one of the most important and basic steps to have in-depth knowledge about the working of the mechanism of the Geneva System, as well as understanding the need of error analysis and the way of proceeding for the same. For this we studied the research paper “Analysis and Synthesis of Mechanical Errors in Geneva Mechanisms” and gained insights from it.

2) After proceeding, our main aim was to deal with the errors of the system. Thus our aim was to identify the sources of errors. Also, since our plan included analysing these errors with the help of the kinematic equations and inducing these errors in them, we had proceeded in the same manner.

3) With these kinematic equations including the error induced, we proceeded with deriving acceleration and jerk equations in the above mentioned terms. The derivation included multiple variables and parts for derivation.

4) Having equations for acceleration and Jerk in hand, we found the values and generated data for the same at different angular positions during the rotation and for different error values. These error values are taken with probability normally distributed. The weightage for this probability is counted separately, over 20 data points, taking required parameters in consideration.

5) With the data for acceleration and Jerk calculated separately while keeping the values of ‘L’ and ‘R’ constant, values of mean and variance were calculated over the different angular positions. This shows the changes due to the errors and their effect over these positions. Graphs were formulated for the Mean, Standard deviation and Range while keeping the positions in X-axis.

After this we worked on the results and towards the completion of the project. The graphs in Figure (9 - 16) can be used for further analysis and is able to help in Synthesis and Optimisation of Spherical Geneva Mechanism.
4. Work Progress and Results

4.1 Calculation of terms of Angular Acceleration and Angular Jerk for Planar and Spherical Geneva

Planar Geneva Mechanism
As mentioned in methodology, effect of induced errors can be measured by inducing errors in variables of ideal kinematic expression. Such errors would result due to tolerance in link lengths. Before inducing tolerance errors for the analysis, kinematic terms like angular velocity, acceleration and jerk were calculated in terms of link dimensions and uniform input velocity applied for the movement of the system.

Figure 6 shows the driven wheel for 4 slots. Figure 7 shows its geometrical representation where (P) is the center of the pin. R is the distance between the center of the driven wheel and point of contact. G is the center of the driven wheel. D is the center of the drive wheel. r is the distance between the pin and center of the driven wheel. L is the distance between centers of driven wheel and drive wheel. Therefore, at any moment, if PDG is $\theta$ then angle $\phi$ would be:

$$\tan(\phi) = \frac{R \sin(\theta)}{L - R \cos(\theta)}$$

Using this relationship we can derive the relationships of angular velocities, angular acceleration, and jerk.

Angular velocity of the driven wheel is given by:
$$\frac{d\phi}{dt} = -\frac{R (L \cos(\theta) - R)}{R (2L \cos(\theta) - R) - L^2 \omega}$$

Angular acceleration is given by:
$$\frac{d^2 \phi}{dt^2} = \frac{RL (R - L) (R + L) \sin(\theta)}{(R (2L \cos(\theta) - R) - L^2)^2 \omega^2}$$

Jerk is given by:
$$\frac{d^3 \phi}{dt^3} = \frac{RL (R - L) (R + L) (2LR \sin^2(\theta) - (R^2 + L^2) \cos(\theta) + 2LR)}{(R (2L \cos(\theta) - R) - L^2)^3 \omega^3}$$

To visualize these relationships, Figure 10 shows graphs of angular velocity, the angular acceleration with jerk with respect to angular position. Here we can clearly see that angular acceleration starts with non-zero value when it starts to move. This produces a jerk during the start and end of the cycle. Commonly four spokes are used in Geneva Mechanisms where the radii of both wheels are the same. In such cases:

$$\tan(\phi) = \frac{\sin(\theta)}{\sqrt{2 - \cos(\theta)}}$$
where,

\[ C = (3v^2 - 2\mu^2)\cos^2\theta \]

By differentiating above equation, we can also calculate the expression of jerk:

\[ \frac{d^3\phi}{dt^3} = \frac{\mu (D + E + (F + G + H) \sin^2(\theta) + I + J + K)}{\left(\mu^2 \sin^2(\theta) + \nu^2 \cos^2(\theta) - 2\nu \cos(\theta) + 1\right)^3} \omega^3 \]

where,

\[ D = (2\mu^2v^3 - 2\mu^4v) \sin^6\theta \]

\[ E = (12\nu^4 + 16\nu^2v^2 - 8\nu^4v) \cos^2\theta + (12\nu^4 - 16\mu^2v^2 + 5\mu^4) \cos\theta - 6v^4 + 2\mu^4v) \sin^4\theta \]

\[ F = (-8v^5 + 14\nu^2v^3 - 6\mu^4v) \cos^4\theta \]

\[ G = (20v^6 - 24\nu^2v^2 + 6\mu^4) \cos^3\theta \]

\[ H = (6v^6 - 12\nu^2v^4) \cos^2\theta + (4\mu^2 - 4v^2) \cos\theta + 4v \]

\[ I = (2\mu^2v^3 - 2\nu^3) \cos^5(\theta) \]

\[ J = (7\nu^4 + 6\mu^2v^2) \cos^5(\theta) + (4\mu^2v - 8v^3) \cos^4(\theta) \]

\[ K = (2\nu^2 - 2\mu^2) \cos^3(\theta) + 2\nu \cos^2(\theta) - \cos(\theta) \]

After finding the acceleration and Jerk of movement of Spherical Geneva Mechanism in terms of angles and geometry of the mechanism, for processing and statistical analysis we have done using the same approach as used by S. Rao and S. Gavane for Planar Geneva Mechanism. We have found the values for acceleration and Jerk for different positions of the driver wheel. Assuming link lengths to be distributed normally in the tolerance range, we have assumed the range varying with an error of ±5%, and we have taken 20 discrete value over this error range. Assuming angular velocity of the driver wheel to be constant, we found the values of angular acceleration and angular jerk at any given position and for the error values. After this, for statistical analysis and analyse the variations of angular acceleration and angular jerk in respect to the attributes mentioned above, we have found the variance, standard deviation and mean for a set of angular positions considering the errors mentioned. Also, to find the effect of errors induced due to variation in link lengths, we need to find the effect of error due to one link by keeping the length of the other link constant. Following graphs show the variation by keeping one of the variables as constant.

4.2 Generating Data

Generating values of acceleration and jerk require a huge amount of computing power and there are huge possibilities of errors while writing these equations in MATLAB. This can be handled by iteratively differentiating the expression of relationship of derivatives of angular velocity with respect to the attributes giving the acceleration and angular jerk.

Distribution of link lengths is assumed to be a Gaussian Distribution within the tolerance range. Gaussian distribution is chosen because there is more probability that link lengths are close to mean than having a larger deviation. Hence, the probability decreases as we move away from mean. Such distribution is naturally observed in manufacturing processes. The parameters of distribution are chosen in such a way that tolerance range consists of ±3σ. Here, the mechanical error is analyzed for the three - sigma band of confidence level. The probability that a link’s lengths lies between this range is approximately 0.997. Hence almost all of the cases would lie in this range and...
By storing values of angular position at different values of then using First Principal of Derivatives, values of angular angular velocity of the driven wheel can be generated. Similarly, by repeating the same process, values of angular acceleration and angular jerk can be calculated too. Such generated values would be really close to actual values if the difference between successive values of \( \theta \) is kept very small. Such an approach to calculate angular acceleration and angular jerk would be very efficient in terms of computational resources. Also, this approach takes care of boundary conditions too. 

First Principal of Derivatives is given by:

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

### 4.3 Calculation of Mean and Variance

Using the data generated, mean and variance at different values of \( \theta \) can be found. One of the things to notice while calculating the means and variance, is to consider the probability of tolerances to be distributed normally. All these calculations were done in MS Excel, taking the values and their weightages considering the probabilities normally distributed. The mean (a. k. a average) of a data set is found by adding all numbers in the data set and then dividing by the number of values in the set. Calculating mean is not enough to get information from any kind of data set. Hence, calculating standard deviation and variance of distribution of angular acceleration and angular jerk of the driven wheel is necessary too. Since the data - sets in the MS Excel were discrete, thus the values are not exact, though due to enough data points, they are calculated to be precise enough for further procedures.

Variance is defined as the average of squared differences from the mean and standard deviation is square root of variance. Standard deviation relates to the spread of values in the data set. Higher spread in values would result in higher standard deviation. This property can be used to determine the effect of errors in link lengths on values of angular acceleration and angular jerk. Also, since Standard deviation being more fundamentally used during the calculations, the graphs were calculated for Standard deviation instead of variance. Also, the graphs included the range for each of these values.

**Figures 11: 18** show variance of mean error and standard deviation of angular acceleration

![Figure 9: Mean of Angular Acceleration with Constant L](image)

![Figure 10: Standard Deviation of Angular Acceleration with Constant L](image)

Here,

- \( R \) = arc length of link of driver 
- \( L \) = arc length between centre of driver wheel and driven wheel

These graphs show the effect of inducing tolerance in ideal system and its effect on angular jerk and angular acceleration.
Figure 11: Mean of Angular Acceleration with Constant R

Figure 12: Standard Deviation of Angular Acceleration with Constant R

Figure 13: Mean of Angular Jerk with Constant L

Figure 14: Standard Deviation of Angular Jerk with Constant L

Figure 15: Mean of Angular Jerk with Constant R

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5. Results

For the problem of finding the effect of error on spherical Geneva Mechanism, angle at which the variation due to error induced would be a point of interest. Trends of these effects over the rotation are shown here. Exact values can be calculated for provided velocity from the formulae.

In this case, acceleration spreads out most around angle, $\theta = \pm 11.5^\circ \pm 0.202$ radians when $R$ is kept constant and around angle $\theta = \pm 8.7^\circ \pm 0.152$ rad) when $L$ is kept constant. Similarly, in case of angular jerk, errors induced have most effect around angle, $\theta = 0^\circ$ in case of both constant $L$ and constant $R$. It can also be noticed that there is a sudden spike in values of jerk when the drive wheel makes contact with the driven wheel and when the drive wheel loses contact with the driven wheel. This sudden jerk could also be helpful while considering the wear and tear possibilities during the mechanism.

6. Conclusion and Future Work

6.1 Conclusion

The analysis of mechanical error in any system is important for any of the mechanisms as it aids in understanding the practical applications and commercial usability. Analysing this for Spherical Geneva does not only help us in the given mechanism only. But, taking the same procedure and similar methodology, this work could be a reference to error analysis for different spherically oriented mechanisms. In order to have error analysis, kinematic analysis also has been done, all of which can be used for synthesis optimisation for the same.

The approach of calculating errors using computational methods may provide a deeper insight into analysing errors in different mechanisms. Kinematic relationships in components of different mechanisms can be used to predict the deviation of performance from an ideal situation. Using iterative methods to calculate derivatives can prove to be beneficial and require a lot less amount of computational powers. Possibility of errors due to input of complex equations may be eliminated by (in this case) differentiating using iterative methods.

6.2 Future Work

Errors are not induced only due to tolerance but clearances in joints also have a significant contribution in mechanical error of mechanism. As clearances don’t have a direct role in kinematic equations, a different approach would be needed. Such approach is discussed by Prof. S. Rao and S. Gavane in their research paper for planar Geneva Mechanism. Similar approach can be used for Spherical Geneva Mechanisms too.

Similarly, many other factors which do not have significant effect but may be considered if precision is priority while manufacturing have been neglected. Such factors include change in temperature in surrounding, frictional forces due to relative motion, localised change in temperature due to friction, deformation due to sudden jerk, etc. Our results would be much precise if such factors are considered too. These factors can be included by using sophisticated computer programs to simulate the mechanism.

References

