Theorem 2.1 [1] If $V$ is a closed, simply connected three-dimensional smooth manifold, then $V$ is diffeomorphic to the three-sphere $S^3$, as Poincaré Conjecture holds.

3. Elementary Proof

Theorem 3.1[5] If $M$ is a closed, orientable 3-manifold, then it can be represented as a branched cover over the three-sphere $(S^3)$ with branching index at most two. This means there exists a smooth map $f: M \rightarrow S^3$ and a link $L \subset S^3$ such that the map $f: M \rightarrow f^{-1}(L) \rightarrow S^3 \approx L$ is a covering, and when restricted to a neighborhood of any component of $f^{-1}(L)$, $f$ is either an embedding or a standard branched double covering map of $D^2 \times S^1$ by $D^2 \times S^1$. Additionally, the link $L = \{L_1, \ldots, L_n\}$ is an unknotted and unlinked, meaning that each $L_i$ bounds an embedded disk and these disks do not intersect each other.

In layman terms, the above states that any closed, three-dimensional shape can be transformed into the shape of a sphere with some special conditions. It’s like stretching and bending the shape without tearing it. The shape’s points might overlap or branch out, but it remains connected.

Proof of Theorem 2.1: The theorem states that if we have a closed, simply connected 3-dimensional space, denoted as $M$, and we have a map $f: M \rightarrow S^3$, then $M$ can be smoothly stretched and folded over $S^3$ in a way that every point in $M$ is covered by exactly one point in $S^3$, except for a special set of points on $S^3$. These special points are called the branching points. The map $f$ is called a branched cover map because it covers most of $S$ smoothly, but at the branching...
To prove this theorem, we start with a map $g_1: M \to S^3$ that is a “homotopy equivalence”. This means that it preserves the basic shape of $M$ while mapping it to $S^3$. Then, we find a map $g: S^3 \to M$ that is the “inverse” of $g_1$. Next, we combine $f$ and $g$ to get a new map $h: S^3 \to S^3$. We show that $h$ can be homotoped (smoothly deformed) into a branched cover map $h_0: S^3 \to S^3$, such that $h_0$ behaves like $f$ on a neighborhood of the branching points. Finally, we use $h_0$ to construct a diffeomorphism $h: S^3 \to M$, which means that we can smoothly stretch and fold $S^3$ onto $M$ in a way that preserves all the shapes and structures. This proves the Theorem 2.1.

References


