An Investigation into the Effects of Secondary Mathematics Trainee Teachers' Use of Mathematical Language in Reducing Mathematical Errors: A Jamaican Context

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Abstract: ***The purpose of the study was to evaluate the use of the mathematics language in the classroom among secondary mathematics trainee teachers at a selected teachers' college in the corporate area of Jamaica, and its impact on students' understanding of mathematical concepts. The study took mainly a qualitative approach using classroom observations, focus group discussions, pre-test, post-test, and document analysis in the data collection and analysis process. The study was conducted with a convenient sample of fifty-four ($n = 54$) trainee mathematics teachers from the target population of one hundred and fifteen ($N = 115$). The findings from the research showed that majority of the mathematics trainees inappropriately used the mathematics language which strongly correlates to them making mathematical errors as they work to discuss mathematical concepts; which resulted in poor academic performance. Most of them shared that facilitated group discussions have helped them in understanding mathematics concepts. A vast majority of the participants have inferred inadequate familiarity of mathematics concepts caused them to struggle with the mathematics language. The researcher also found that the intervention given after the pre-test, focusing on comprehension and explanation by the appropriate usage of mathematical language, helped students to improve their mathematical skills and competencies, and by extension their academic performance. The results showed that students' mathematical comprehension had improved after the employment of games, role-play and other activities such as creating posters and participating in expositions. There is a need for students to be encouraged in reading and understanding mathematical notation and language until common conventions are fully understood. Also, trainee mathematics teachers need to develop the action of self-reflection and spend time to properly review their work.***

Keywords: Mathematical Errors, Mathematical Notation and Language, Self-reflection

1. Introduction and Literature Review

The perpetual cry from the society at large is a sound that conveys the perennial underachievement in mathematics nationally. This is a real and heart-breaking issue continuously on the agenda of key advocates of national development [constantly on the horizon year after year]. The headlines after every Caribbean Secondary Examination Council (CSEC) Mathematics and the Primary Exit Profile (PEP) students' results reveal how poorly the students are performing, especially because of the pandemic. Take for instance the CSEC results released September 2022 which indicated a 5% decline in the mathematics results in comparison to 2021, with only a 37% pass **[1]**. Even though the students performed better in 2016, where the results showed a 47.7% pass rate, the Minister of Education at that time also expressed concern of a 14.3% decline when compared to 2015 **[2]**. Similarly, according to an article titled "PEP students struggled in maths, did better in language arts" **[3]** written in 2022 reported that half of the students who sat PEP performed poorly in mathematics, though most of them performed better in the language arts exam. In response to the issue, the Ministry of Education and Youth (MOEY) made steps to rectify this by establishing a special mathematics unit. This unit developed a programme where Mathematics coaches and specialists came on the pipeline. These specialists and coaches were greatly needed, and since their intervention among others factors, in-service mathematics teachers seemed to have gotten more excited about teaching Mathematics especially using concrete and pictorial models to overcome the barrier of abstractness of mathematics in the classroom. In 2014, the Ministry of Education and Youth added $390 million Jamaican dollars to the mathematics programme budget which brought it up to $440 million Jamaican dollars with the intention to increase Grade 4 students' numeracy mastery up to 85% at least by 2018 **[4]**. This move by the former Minister of Education, Rev Ronnie Thwaites [he] was intended to provide support to the primary teachers which included an online mathematics training programme, mathematics coaches and mathematics resource teachers **[4]**. He shared in a parliamentary session that the aim was to use the mathematics coaches to raise the teaching standards as well as the teaching achievement in mathematics **[4]**. Back in 2016, the MOEY decided to send 100 coaches to underperforming secondary schools across Jamaica to provide support **[2]**. Also, efforts to improve these standards led the MOEY in collaboration with the Mico University College to seek the help of Dr. Jennifer Bay-Williams [she], a Professor in Mathematics Education at the University of Louisville, who facilitated at least ten training sessions for in-service mathematics teachers, mathematics coaches and mathematics teacher educators over a two-year period from 2018 to 2019.In June 2019, she facilitated a three-day workshop focusing on the mathematics practices (MPs) as seen in **Figure 1** which are now embedded in the National Standards Council (NSC) document to guide the teaching profession of mathematics in Jamaica **[5]**.

Mathematics Practices (MPs)
1) Make sense of problems & persevere in solving them
2) Reason abstractly & quantitatively
3) Construct viable arguments & critique the reasoning of others
4) Model with mathematics
5) Use appropriate tools strategically
6) Attend to precision
7) Look for & make use of structure
8) Look for & express regularity in repeated reasoning

Figure 1: CCSS Mathematical Practice Standards by NCTM in USA

This three-day session allowed mathematics educators to become familiar with each mathematics practice by interacting with activities that supported each practice to develop their competencies to facilitate teaching practices and shifts in classroom practices **[5]**. She shared that these teaching practices and shifts would better enable the trainee teachers to clearly communicate their expectations for learning, allow students to move away from routine tasks to reasoning [non-routine] tasks, amongst many other ideas; thus, promoting better comprehension, critical thinking, and problem-solving skills of the students **[5]**.

However, this still was not as fruitful as intended since in 2015, 111 fully trained teachers migrated, and several untrained and inadequately trained teachers were left in the system **[6]**. The Jamaica Teachers' Association (JTA) did a survey in 2022 with 140 schools which revealed that 13% of the teachers retired, while 43% resigned **[6]**. Back in October 2022, at least 67 mathematics teachers were needed due to teacher migration, this gap however was filled with newly trained mathematics teachers who graduated from the teachers' colleges **[7]**. Imagine now in 2023, when that number of teacher migrations still seem alarming. The Minister of Education Mrs. Fayval Williams announced in May 2023 that there is a shortage of mathematics and science teachers for the academic year 2023/2024 **[8]**. It was found that the teachers have been leaving for many reasons which included the lack of resources impacting on their teaching abilities along with little to no psychosocial support, inability to properly provide for their needs based on the unattractive remuneration package amidst the inflation rates, and poor leadership and management abilities of the school administrators **[9]**.

A drawback, however, is that teachers have been mainly focused on academic performance due to societal pressure under the ministries of education across the world. Thus, teachers in Jamaica have been [test centered] focused, [with written tests administered on an average of every six weeks in most secondary schools] on teaching to the test; just trying to get students to know how to be prepared to pass tests and examination because students' performance is important. So, teachers spend time going through past paper questions with students as much as possible teaching them how to look for patterns, highlighting repeat questions and memorized answers for each. There is little use of authentic assessments that could help students become more acquainted (or intimate) with mathematics applications in their daily operations, to move away from routine tasks to non-routine ones. Consequently, this provides an ideal environment for students to lack creativity, proper problem-solving and critical thinking skills. Ultimately, this is a recipe for disaster and thus, students leave high schools not knowing mathematics but how to pass exams. Therefore, by the time they reach the teachers' colleges they come with the very same mentality expecting to be taught to do exams but have found a new revelation. It is not that the teachers' colleges are not concerned about the mathematics trainee teachers' academic performance as an issue, but it has been realized that their greatest issue is poor use of mathematics language and evidence of misconceptions (and / or other errors). Some of these include, but are not limited to:

1) Trainee teachers verbally explained incorrect interpretation of mathematics concepts and properties.
2) Trainee teachers, when required to write explanations as they systematically work through questions, struggled to explain and at times seemed very confused.
3) Most trainee teachers in the research sample have inadequate or no mathematical vocabulary.

The researcher [she] while teaching trainee teachers in an introductory lesson on simplifying exponential functions, reviewed laws of indices with them first. A trainee teacher [he] was explaining one of the exponential laws and stated an erroneous result which reflected a misconception. The researcher had presented the mathematics statement $(a^m)^n$ to a trainee teacher and asked him to explain what it is implying (or saying) and make the decision to write down what the trainee said. His response is presented in **Figure 2** below.

$$(a^m)^n = a^m \times a^n$$

Figure 2: Image showing a trainee teacher's verbal expression written by his lecturer

The researcher told the trainee teacher to look at what was said, to which the trainee objected saying that was not what he said (see **Figure 2**). The other students unanimously responded, *"that's what you told Miss"*. He, however, said that was not what he meant and asked if he could write what he meant and he provided $(a^m)^n = a^{mn}$ instead. This seemed a little puzzling to the teacher-researcher that the trainee teacher could write but struggled to explain. As the researcher continued working with the trainee teachers, quite a few communicated errors during the discussion. Throughout the next few classes, the researcher noticed that some students either could write the mathematical language but struggled to speak it, or they could neither speak nor write the language. It was evident, however, that with this class approximately half of them performed above the 50% pass rate required though they showed signs of weak mathematical content knowledge. This became evident firstly, during revision of concepts that trainee teachers should have been equipped with through prior knowledge coming from high school. Note that similar issues have been seen across all four groups, Years 1-4 spanning all mathematics courses. Furthermore, even when they have good ideas, they struggle to effectively communicate them hence they were frequently misunderstood. A study done on students' using mathematics language in the classroom **[10]** indicated that oral dialogue should be considered when teaching mathematics because it is very important and must begin with providing young children mathematics vocabulary which should be developed as they progress in their high school years. Both **[10]** and **[11]** have shared that students [trainee teachers] need to be engaged in learning activities which will expose them to various teaching strategies and methods needed to help them understand the mathematics language that afford them the opportunity to become independent learners. Lee's work **[10]** suggested that some teaching methods and strategies the trainee teachers can be exposed to are making a record on the board in writing of what they verbalized, as well as to ask them to engage with diverse texts on different concepts and draft their ideas about them on posters, in created videos and Power Points as well as practice construction of their own knowledge of concepts under the guidance of the teachers [lecturers].

Now it is very important for the teachers' college to address the trainee teachers' deficiencies to avoid the spill over effects as much as possible when they enter the classroom. The present National Mathematics Coordinator, Dr. Tamika Benjamin, shared that teachers lacked the pedagogical knowledge needed to properly communicate the mathematical content **[12]**. Consequently, this echoes the former Minister of Education Mr. Ruel Reid's **[2]** and Asihia's study **[13]** that revealed teachers lacking in qualifications and methods of teaching. It has been observed that the trainee teachers usually tend to make three types of errors while learning mathematics. They are silly mistakes, misconceptions and misunderstandings. Silly mistakes are defined as errors made while students rush their work and are not focused which are often minor and can be avoided if students slow down while working and read carefully. Misconceptions, however, results from students' inability to apply mathematical knowledge and skills due to lack of conceptual understanding. Thus, they struggle to recognize, give examples, explain, and compare and contrast mathematical principles and concepts **[14]**. Two experienced teachers and researchers **[15, 16]** noticed that students mainly display the following errors: communication, algebra, confusion of notation, reasoning and unwarranted generalizations. The Caribbean Advanced Proficiency Examination (CAPE) reports on Pure Mathematics Unit 1 has revealed the same issues referred to by the experienced teachers and researchers **[17]**. Studies have also revealed that the inappropriate use of language has been linked to mathematical errors **[18]**. Take for instance, a study done with students at a public research university with a sample of 9 university students **[18]** where students' verbalized, written and pictorial representation of the Intermediate Value Theorem (IVT) was recorded in **Figures 3** and **4**. It is evident that Cori [she] could correct herself along the way and made minimal errors; resulting in a slight mathematical difference between what was spoken and written (see **Figure 3**). Bella [she], on the other hand, made quite a few errors and was not able to compare what she wrote and spoke to distinguish the errors made (see **Figure 4**). So, while Cori showed slight communication errors, Bella struggled with misconception of what the IVT is about.

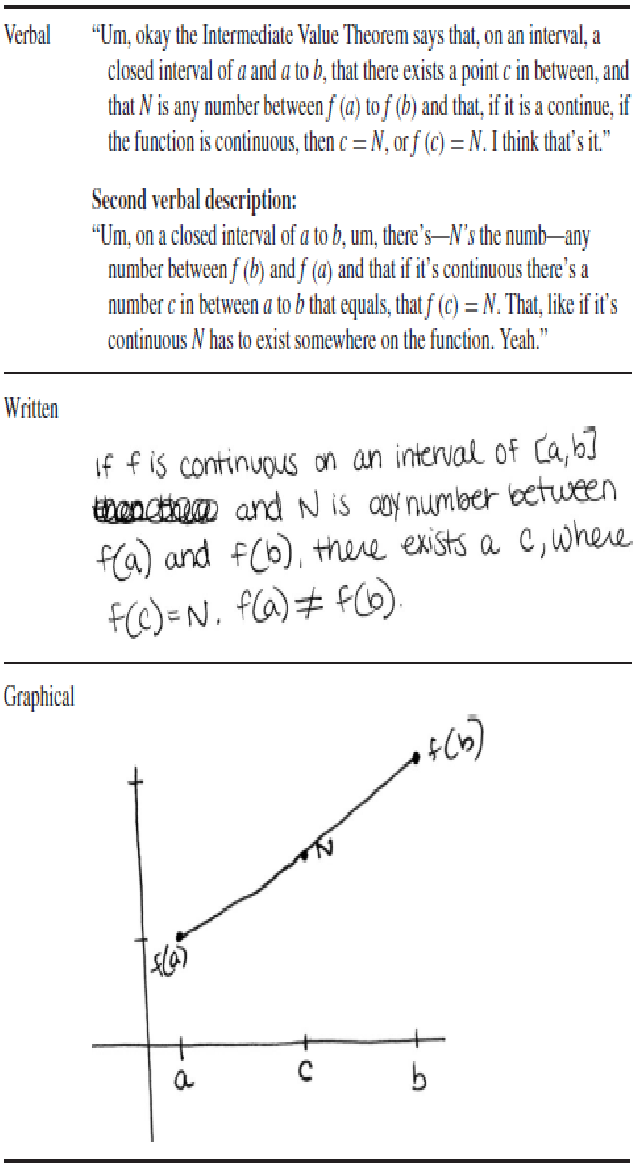
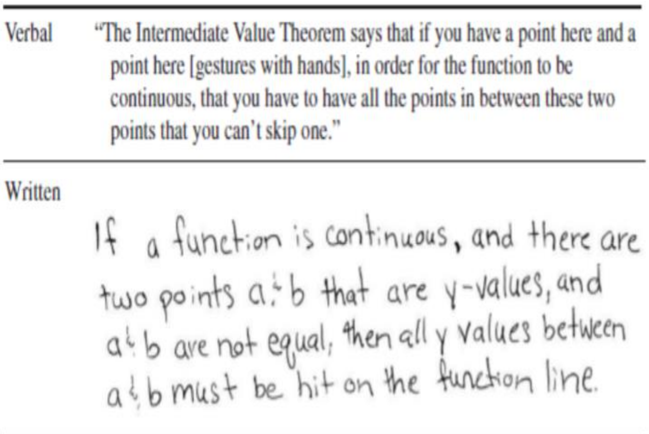


Figure 3: Image showing Cori's explanation of the Intermediate Value Theorem (IVT) **[18]**



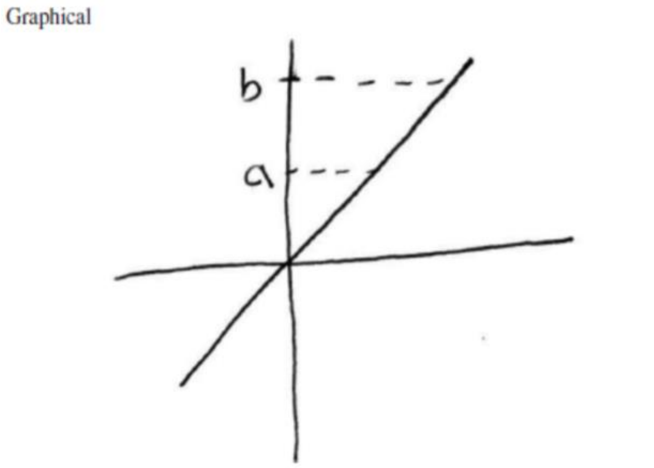


Figure 4: Image showing Bella's explanation of the Intermediate Value Theorem (IVT) **[18]**

The theorist, Gergen **[19]**, stated that communication is key in understanding mathematics; the more you communicate the better you become fluent as a student [trainee teacher] and can problem solve using mathematics. Lee **[10]** posited that the more students can "describe their reasoning and patterns that they observed in a conventional style" using suitable mathematics then eventually they will create their own knowledge. This knowledge **[10]** will allow them to be able to better function as they progress through their mathematics experience. Consequently, the researcher desires that trainee teachers will minimize the errors made as they become involved in their learning, recognizing the errors they have made and addressing them by using appropriate mathematical language. With this in mind, the following research questions were examined:

1) To what extent does inappropriate use of mathematical language impact mathematical errors made in the mathematics classroom?
2) How effective is the use of appropriate mathematical communication (through the usage of written and oral expressions) in reducing trainee teachers' mathematical errors?
3) Did the use of appropriate mathematical communication (through the usage of written and oral expressions) by trainee teachers have an impact on their academic performance?

1.1. Definition of Terms

Mathematical Errors - these are mistakes made by students [trainee teachers] which are usually evident in their calculations and/or thought processes as they learn mathematical concepts. These errors are likely to be made due to miscalculations, silly mistakes or misunderstanding of mathematical concepts.

Mathematical Language – this is defined as a system or academic language which requires students [trainee teachers] to learn, use and apply mathematical ideas, concepts, theories and theorems. This system involves vocabulary and language structures similar to other languages but are more abstract in nature because it requires the use of symbols, notations and numbers.

2. Methodology

2.1 Study Design

This study is an action research developed using convenient sampling. The trainee teachers who showed signs of mathematical errors were the selected subjects for the research.

2.2 Participants

The research was done with fifty-four (54) secondary trainee teachers between the ages of 17 - 47 years from a teachers' college in the corporate area of Jamaica. These trainee teachers consented to participate from a total of one-hundred and fifteen (115) trainee teachers.

2.3 Instruments and Data Analysis

The instruments used throughout the study were: observation schedule, lesson plans, field notes, worksheets, lesson reflections, questionnaire, interviews, focus group discussions, pre-test and post-test. Data was analyzed using coding and thematic analysis, descriptive statistics, and Microsoft Excel.

3. Results

The results of this study are organized, presented and discussed according to the research questions.

3.1 Results Based on Research Question 1

To what extent does inappropriate use of mathematical language impact mathematical errors made in the mathematics classroom?

Table 1 reveals the challenges seen across all the mathematics content courses regardless of year groups. One main issue seen across all year groups is their inability to recognize functions coupled with correctly identifying and properly using the properties of functions and expressions. It is mainly evident that whenever the participants were working with functions which involve not just polynomials but other functions such as trigonometric functions, exponential and logarithmic functions, they seemed to struggle a lot in making sense of the mathematics related to each. The study showed that the participants were able to recall these functions and their properties but the moment they are required to use them in different context it becomes very troublesome and erroneous results start to pop-up. For instance, while discussing functions in courses such as College Algebra, Pre-Calculus, and Calculus courses (in focused group discussions) where the participants were given examples such as $P(t) = P_0(e^{kt})$, $\cos(x^2 + x)$ and $\log xy$, they struggled to recognize the inputs of these functions, and at times were challenged with establishing the relationship of the functions with their inverses. When the trainee teachers were asked to read aloud each function, the response given by most were as follows: $\cos(x^2 + x)$ means *'cos multiplied by* $x^2 + x$*'*; while *'*$\log xy$ *means log multiplied by x and y'*. Moreover, when the researcher wrote down their explained product and provided them with actual numerical values for each variable [input values] and then asked a volunteer to show the multiplication process mentioned they asked, *"Miss, what are you talking about?"* Incidentally, when they were provided with the input values, students often got the correct answer when they wrote down the result(s). The study uncovered that after the participants were insistently told to examine the products using the given input values and match against the results obtained, they then stated *"Miss, you know what we meant, Miss, you know"*. This revealed that what they say is not necessary what they meant based on written work; so, there were clear disconnections between the verbal and written interpretations. It grew even more alarming when these participants when required to use the properties of logarithms to aid in the differentiation process in a Calculus course; quite a few of them wrote erroneous responses especially when using the form $\log xy$ where they interpreted it as $log\ x \times log\ y$. It was found that the function was regarded as more of a product of two functions instead of a function having inputs x and y. While working with another group of participants who were using $P(t) = P_0(e^{kt})$ to solve a population problem which involved them re-expressing the function using logarithmic representation, as seen in **Figure 5,** they also provided a similar interpretation conveying multiplication. These participants, however, were able to correctly use the logarithmic property, although in explaining, the use of mathematics language employed was faulty.

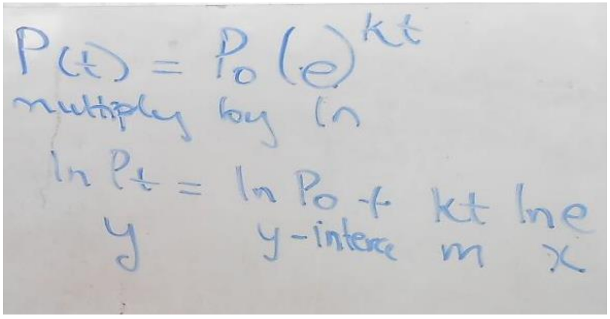


Figure 5: Inability to recognize functions

Figure 6 also provides two full questions in which a student lacked the ability to properly use properties of functions and expressions. Take for example, line 2 of the solution in the first question (on the left-hand side) where the participant is integrating the function $\frac{3x}{16x^2+1}$; the participant [she] factored out $3x$ instead of "3" as she treated it as a constant rather than part of the integrand which should be integrated; consequently, the resulting error seen in the 4th line. This type of error made by the student is referred to as a misconception because she had done it a few times with similar questions. On the right-hand side, in lines 2 and 4, use of brackets is lacking but she was able to generate the correct value for **A** (see **Figure 6**). The pre-test revealed the same issues seen in the focused group discussion. Mathematics statement shared by four students on a test item for which the participants of the study were required to circle the statements they agreed with and to justify their thinking, only a few participants, however, overlooked the erroneous statements made by Ronnie (see **Figure 7**). Nevertheless, it was good that the students could at least identify that Tom's statement was erroneous and give explanations (see **Figure 7**).

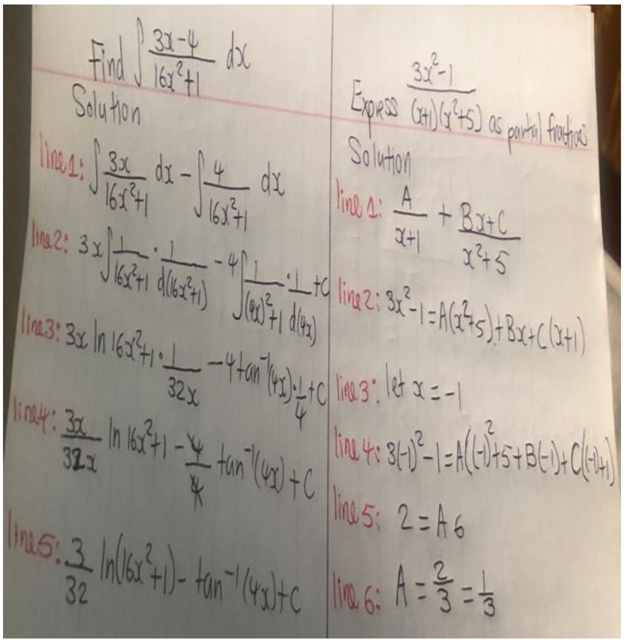


Figure 6: Inability to correctly use properties of functions and expressions

Table 1: Challenges students displayed with understanding mathematics language and errors they made

Challenges	Indicators
Inability to recognize functions	Not clearly understanding the meaning of a given function; what the input truly is to make a function work
Problems identifying and properly utilizing the properties of functions and expressions	Incorrectly using the law of indices properties such as a^{m+n} and a^{mn}; *logxy* is treated as $\log x \times \log y$
Trainee teachers not seeing mathematics as a language	Struggling in reading and interpreting worded problems; improperly writing mathematical problems
Cementing prior knowledge for future use	Difficulty in seeing the relevance of prior knowledge going forward in the teaching and learning process; challenge in transferring prior knowledge to new concept development
Unable to distinguish routine problems from non-routine problems	Lack of comprehension skills; inability to develop non-routine problems from routine problems; struggling with finding a strategy to solve non-routine problems; creativity issues
Difficulty linking mathematics to their daily life	The constant struggle to identify real life examples for any given topic that relates to real life as well as instruments used to convey mathematical processes

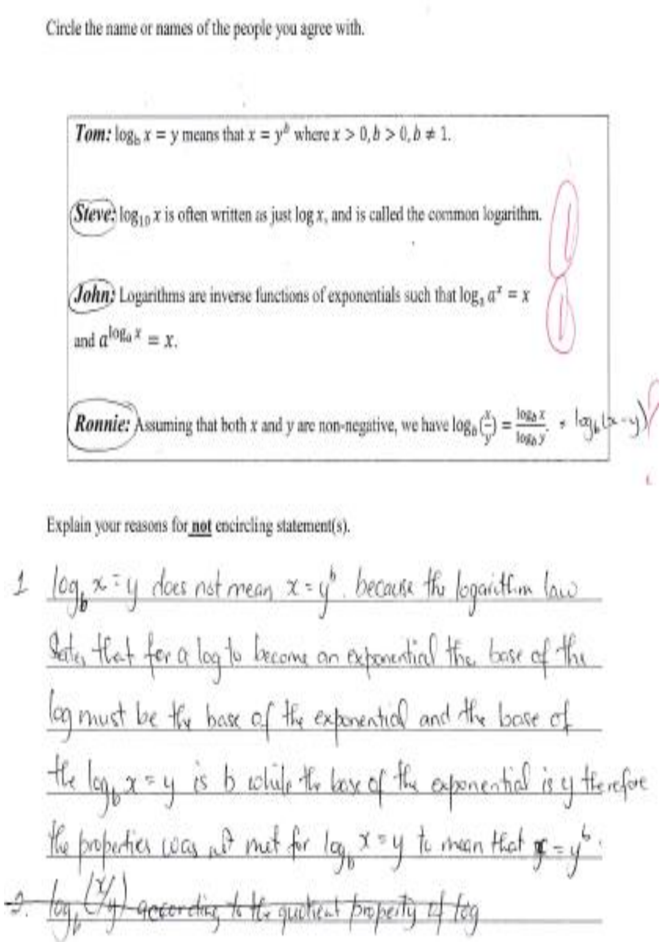


Figure 7: Inability to fully recognize erroneous statements

When these participants were given a questionnaire, 46.7% of them revealed that they have been taught mathematical misconceptions at some point in time (see **Figure 8**). The data revealed from the focus group discussions that their exposure to mathematics at the teachers' college was different than during their high school years; they expressed that more was expected of them at college, and they have difficulty "swatting" [remembering all the content in a short time frame to reproduce it on graded activities, instead of actually studying to understand the content]. The study also found that these participants never truly understood the mathematics they were taught because their focus was on remembering the content to pass (rote learning) an assessment rather than spending time to become acquainted with the concepts during their high school years. It was further uncovered that all of them shared different views of how prior knowledge has impacted their mathematical language. Twenty-six percent (26%) revealed that it helped them to become critical thinkers, 27% indicated that it is the foundation for explaining and understanding mathematics and 17% shared it makes the language understandable (see **Figure 9**).

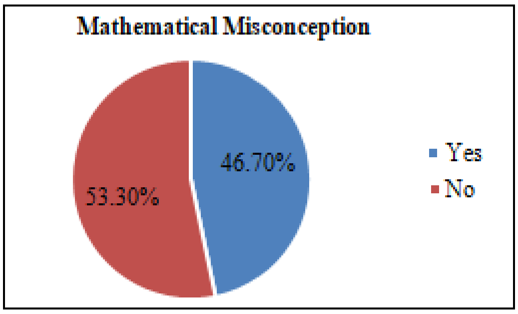


Figure 8: Proportion of student-teachers having been taught mathematical misconceptions (item from Students' Questionnaire)

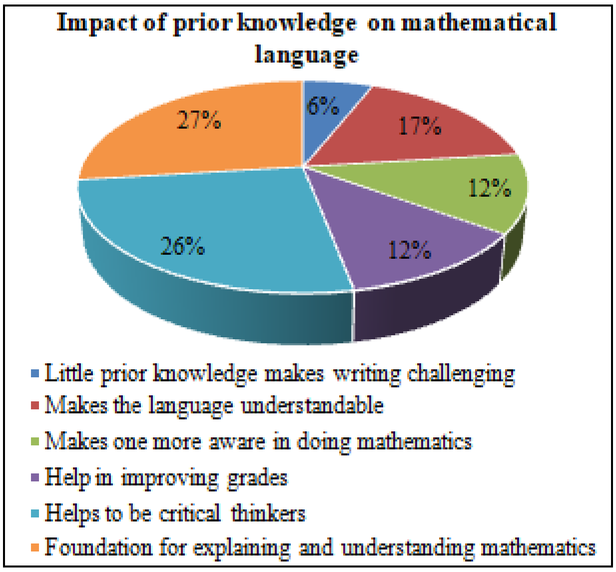


Figure 9: Beliefs in the ways prior mathematical knowledge impacts mathematical language (item from Students' Questionnaire)

Even though they knew they needed prior knowledge in order to understand the mathematics, they still struggled when discussing a new topic requiring them to transfer the prior knowledge to the new concept. These participants have not yet understood how to assimilate the new information learnt into their schema, so instead they compartmentalize their knowledge (see **Table 1**). This was observed on occasions where students learnt something in Year 1 that becomes pre-requisite knowledge for their Year 2 content knowledge, they usually have already either forgotten what was learnt or locked it away in memory unable to make connections with new topic; ever so often they would say, *"Miss, we never know we needed to remember this"*. Here is it seen that the students often failed to comprehend the depth of the mathematics and make it their own experience. The head of department (HOD) [he] who also taught these participants concurred that the reasons they continuously make these errors are because they *"lacked good comprehend skills in mathematics"*. He further stated that they try to remember question types and forms and solution layout but have not learnt to read and understand questions to work through the solution process. He went on to say that the students lacked the prior exposure and practice of mathematical concepts before entering the teachers' college. Data from the same interview, with the HOD, uncovered that most of the participants have not yet seen mathematics as a language which is central to them *"understanding the processes and* *decisions to take in mathematical problem-solving"*. The findings showed that since the participants lacked the mathematics experience where they could create their own knowledge and understanding of mathematics then they cannot make the link between the real-life examples and related mathematics concepts; especially since they have been *"drilled into working routine mathematics problems so non-routine problems are big stumbling blocks during their high school years"* as stated by the HOD.

This in turn has had a negative impact on the teaching and learning process when they are out on teaching practice (in Year 2, 3 or 4). For instance, most of these participants while out on practicum showed signs of lacking the comprehension needed in their lesson deliver; this is mostly seen in their lesson plans where they struggle to identify suitable learning activities that work well together from the beginning to the ending of the lessons. One trainee teacher while on Practicum, when introducing the concept of perimeter began the lesson by asking students to brainstorm ideas related to Perimeter and wrote down their ideas, one student even told him area is related to perimeter. He proceeded to move to another activity without addressing the points shared by the students and from there the lesson started to have quite a few disconnections not clearly developing truly what perimeter is; and students constantly referred to perimeter as area. The findings showed that intervention in the lesson time had to be done by the supervisor and recommendation was made for this trainee teacher to re-teach the lesson after reviewing the issues and identifying ways to address them. Usually if the trainee teachers do not address the inappropriate use of the mathematical language as they teach, then it often leads to students having erroneous ideas or misconceptions.

3.2 Results Based on Research Question 2

How effective is the use of appropriate mathematical communication (through the usage of written and oral expressions) in reducing trainee teachers' mathematical errors?

Table 2 shows activities that have benefited the trainee teachers to help improve their usage of mathematics language and reduce the errors they made when learning mathematics concepts. The study revealed that even though the trainee teachers were reluctant to participate in the active learning strategies guided by the mathematics practices (MPs), they eventually participated. Thirty-seven percent (37%) had no issues working with partners to go through regular mathematics and problem-based questions during class sessions and take-home worksheet. Even though initially they complained that that they found the activities difficult; after researching and having discussions with their partners, the trainee teachers began to have a clearer understanding of mathematical concepts. These trainee teachers were able to share their answers on the board for their classmates' revision. Although only 44% of the trainee teachers enjoyed sharing what they understood, they confessed that by doing so they could more readily pin-point their misconceptions (and/ or other errors made) and gained more clarity on the concepts.

Data in **Table 2** that showed that 50% of them willing spent the time to work on words and phrases relating to the concepts. They expressed that they found it interesting and helpful. This was also evident where 3 of them decided to use a similar approach while on teaching practice called a word wall; one of them even completed an action research using the Word Wall Strategy with her students. Additionally, 57.4% of the trainee teachers found technology; manipulatives and poster presentations to be very helpful in learning mathematical concepts (see **Table 2**). In explaining the participants acknowledged that the use of mathematics software and manipulatives helped them to understand the

Table 2: Active learning strategies for comprehending mathematical concepts

Activities used	Indicators	Number of respondents who indicated that the activities were effective
Writing words on a concept and discuss it with your partner	It helped to give a clear understanding of mathematical concepts; I enjoy working with a partner in discussing mathematical ideas.	20
Correct fellow classmates' mathematical ideas on concepts	It cleared up misconceptions and gives more clarity.	24
Build a vocabulary of words and phrases relating to the concepts	I find it interesting and helpful to better understand the mathematics.	27
Use of technology and physical manipulatives along with poster presentations relating to the concepts	Using mathematics software such as graphs helped me to understand concepts adequately; the use of poster in constructing ideas based on concepts both individually and/or collaboratively as students help to enhance my conceptual understanding.	31
Use of role play to facilitate mathematics concepts and ideas	I learnt best by doing and discovering things.	35
Written descriptions have improved but still far exceed verbal ones	I get better grades on my test when I participated in oral and written class activities versus the teacher being the sole communicator.	30
Usage of mathematical interventions and tutorials (additional one-hour weekly) has allowed students to develop a greater appreciation of mathematics	Provided opportunities to clarify any misunderstanding of concepts; communicate computational errors for resolution; take home tasks and practice tasks when completed could be discussed at the weekly tutorial sessions to explore troubling mathematics concepts planned for 1 hour but extended and hence students were grateful.	38

mathematics concepts adequately which they had eventually adapted to use in their classroom. They went further to say that the mathematics poster and model presentations, though requiring intense preparation to share at the college department and national mathematics expos, have helped them greatly with conceptual understanding (see **Figures 10, 11** and **12**).

Figure 10: Model of road network showing students interacting with it to determine shortest travelling paths

While observing the participants at a regional mathematics exposition, it was evident that they enjoyed the opportunity to share with the attendees which included students and teachers from different schools across Kingston and St. Andrew (see **Figure 11** and **12**). This allowed them to share the mathematical concepts related to each designed chart (or model). Additionally, the experience has enabled them to become creative in their lesson planning ideas using items in the environment that students can associate to communicate the mathematical language.

Figure 11: Mathematics game allowing students to improve their multiplication skills

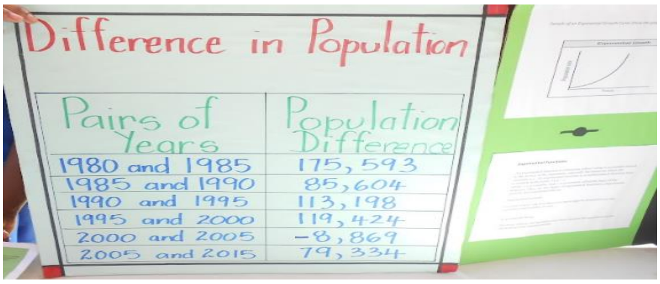


Figure 12: Poster presentation depicting real life examples related to exponential and logarithm functions

Moreover, in quite a few lessons with one in particular on domain and range the trainee teacher [she] gave her students a juice ingredient requiring them to identify the input(s). She facilitated discussions with them in using the ingredients to clearly distinguish domain and range; the lesson taught was good but the lesson development could be a little more organized so that the activities could be better integrated and more detailed explanations as well could have been considered.

Almost all trainee teachers, approximately 94% expressed that the more they communicated the mathematics verbally or in writing have greatly helped them in reducing the errors they made (see **Figure 13**). Moreover, it was found that 55.5% of the participants have shared that they obtained better test scores since participating in oral and written class activities versus having the teacher [lecturer] being the sole communicator (see **Table 2**). Also, 64.8% found the use of role plays to be effective in aiding their understanding of mathematical concepts; especially since they noticed they learnt more when they are active participants in their own learning (see **Table 2**). Similarly, it was observed that when the participants organized two such role plays, one on simple and compound interest in banking activities and the other on a courthouse activity, they understood the mathematics ideas much better. That is, the participants could better differentiate simple and compound interest; while the participants who participated in the courthouse activity could better recognize arguments and understand truth tables in identifying the validity of arguments. Even though all students were exposed to the weekly tutorial, not all of them were interested though they still attended the sessions. It was found that 70.4% of the participants who attended the tutorial sessions expressed that they were able to share what they did not understand from regular mathematics classes. They also stated that they were able to ask questions about mathematical concepts they were battling with as they struggled to identify where they went wrong and share their solutions to get clarifications (see **Table 2**).

Therefore, not only did the intervention minimize conceptual errors, it also served to motivate students to spend time on the take-home and additional practice activities to work through them before attending the sessions. They even attended the sessions with questions they found either on past papers or in textbooks that seemed troubling to them for exploration. The HOD also shared similar sentiments that he has seen an increase in their knowledge-base of mathematical concepts as the students began to read the mathematics with understanding. He noticed that they were now able to decode and deduce the necessary plan for a plausible decision to enable problem-solving.

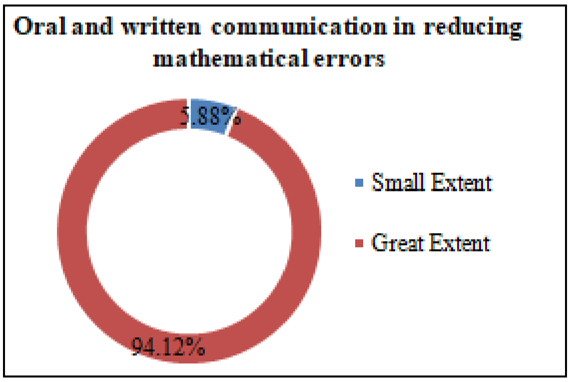


Figure 13: Extent to which student-teachers believe oral and written communication aid in reducing mathematical errors in the classroom (item from Students' Questionnaire)

3.3 Results Based on Research Question 3

Did the use of appropriate mathematical communication (through the usage of written and oral expressions) by trainee teachers have an impact on their academic performance?

During a focus group discussion, after the pre-test was administered, at least 50% of the participants shared their discomfort with understanding aspects of the courses; however, after revision and active learning activities were utilized their mathematics performance eventually improved. To put things into perspective; prior to interventions, 63.4% of the Year 3 participants failed the pre-test as seen in **Figure 14**. After the interventions, all passed the post-test with scores above 50% (see **Figure 14**). Therefore, it is safe to conclude that most participants saw a reduction in mathematical errors when they were consistent in learning and using the correct mathematics language. The improvement was visible even before the post-test, trainee teachers seemed more enthusiastic about learning and utilizing their mathematics vocabulary as all the participants started to arrive early for classes. During these times they met in the classrooms to discuss concepts (both orally and in writing). The students even willing invited the teacher-researcher to participate in their discussions as they utilized the board space; to review and guide them as they work through mathematical concepts. Some of participants went a step further to meet daily, to discuss mathematical concepts and work through questions in available classrooms. The Year 2 teachers, on the other hands, did not record the same level of improvements observed with the Year 3 trainee teachers. Approximately 27% of the Year 2 participants initially failed the pre-test on first year Calculus, while 63.63% also failed the post-test which included the same participants who failed the pre-test (see **Figure 15**). The slow progress with the Year 2 trainee teachers was partly attributed to them being away from studies for 3 weeks to engage in the Teaching Practicum experience. They expressed during focused group discussions and students' interviews that they did not have the time to practice and review. Consequently, the study revealed that these participants unlike the others did not get the opportunity to interact as much as possible with the strategy as originally intended. Another factor uncovered in the study as shared by the participants was that the one-week mid-term break in April, approximately one week back from practicum, was the time they spent to complete the "too many" assignments they had for other courses that would have been due in the following week. On the other hand, the study found that these participants who were struggling made no attempt to consult their lecturer[s] to help provide clarity where they had any misunderstanding of concepts.

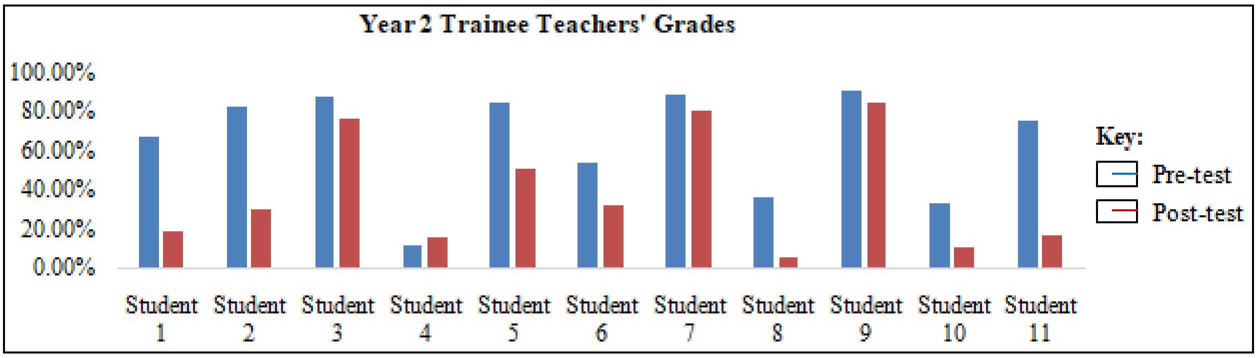


Figure 14: Bar Graph showing samples of Year 2 students' comparative test grades

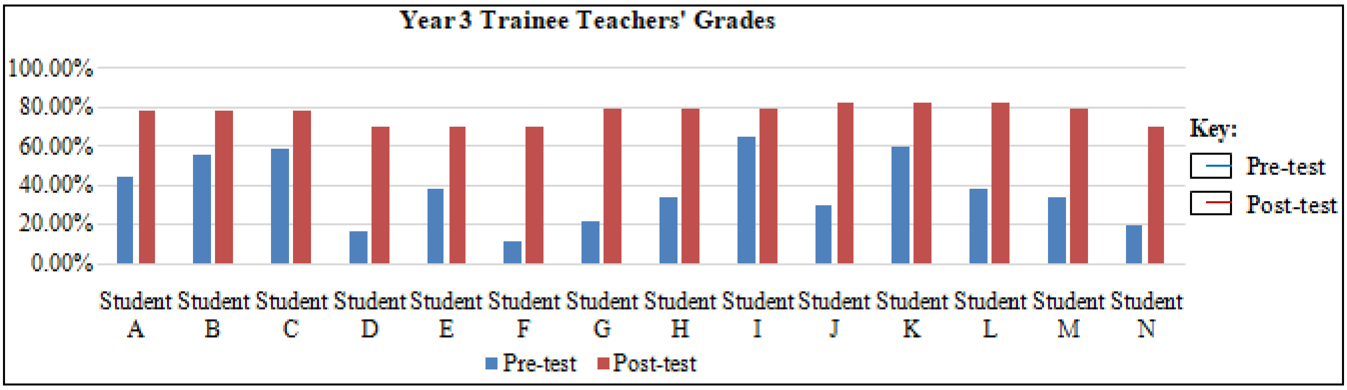


Figure 15: Bar Graph showing samples of Year 3 students' comparative test grades

3.4 Further Discussion and Implications

The study discovered that mathematics educators must become more aware of cultural impacts in the classroom, particularly, the use of language. For years, some Jamaicans have argued that Patois (Jamaican Creole) should be used by educators in the classroom to address language and communication barriers among students whose first language is not the commonly accepted English language. However, its use has negatively affected students' comprehension of mathematics discourse conveyed in Standard English and ultimately impacted students' understanding of some mathematical concepts. In fact, throughout history mathematics has never once been written in Patois (Creole). Whilst all teacher trainees in the study were engaged in mandatory creative and academic writing stipulated by the teachers' colleges; Patois has affected their interpretation and usage of words in the classroom. Words such as "number" and "amount" are often used incorrectly in the mathematics classroom because the trainee teachers are accustomed to those math words having a different meaning in Patois. Not surprising then is the correlation between the inappropriate use of mathematics language and math errors made by trainees. Just about 76% of the sample corroborated this finding indicating that their inappropriate use of mathematical language greatly impacted the errors they made. Language barriers such as these often result in students making mathematics errors (see **Figure 8**). Factors such as misconceptions learned in high school and a lack of prior knowledge to comprehend new concepts were most frequently highlighted by the participants as reasons for their math errors. As a result, the participants had to start unpacking what they learnt in order to unlearn misconceptions and to now assimilate their new mathematics knowledge, language and experiences. This study found that the active learning activities allowed the participants to take charge of their learning to become independent learners. They even have now learnt the importance of engaging in the learning process for each course where they could prepare posters to share at expositions which provided interacts with students and teachers from different schools. This new approach in turn allowed the participates to become creative and much better able to plan lessons to not only engage their students but provide them with similar learning opportunities at the teachers' college such as created exposure to project-based activities and microteaching sessions. Note, however, that even though the participants have been improving they still need to spend quality time to become more familiar with mathematics concepts as to avoid adverse issues in the classroom when they exit the teachers' college. The findings also showed that these errors if not addressed will spill over into the participants' teaching in the classroom; and if evidence of this was seen on practicum, it is the responsibility of the practicum supervisors to ensure they are addressed.

According to the book titled "Teaching Student-Centered Mathematics", teaching for relational understanding is paramount to help activate students' curiosity for them to want to develop a robust understanding of mathematics **[20]**. Thus, this is just in line with the study which uncovered that the active learning activities allowed the participants to become curious about the mathematical concepts they need to learn. The trainee teachers in turn can now spend the time to look at each activity incorporating their ideas as they desire and create similar activities with their students to design their ideal student-centred classroom to facilitate the teaching of mathematical [language] concepts. As seen in **Figures 10, 11** and **12**, the participants were found to become creative in finding examples (models) to facilitate explaining concepts, providing different representation of mathematical ideas that articulate what they understood in the classroom. These developed models by the participants **[20]** will in turn allow them to properly identify the required tools to facilitate instrumental understanding. The study revealed that 94% of the participants have expressed that since they are being exposed to writing and speaking of the mathematics, they have now been making less errors and have developed a better understanding of mathematics concepts. In addition, the more time the participants spent with active learning activities in and outside the classroom, not only did their use of the correct mathematical language reduce errors made but their academic performances also improved (see **Figures 14** and **15**).

The findings of this study could have the following implications:

a) More investment in mathematics intervention plans are needed in terms of time and finances;
b) There is a need to encourage students [trainee teachers] to do more self-reflection in a strategic way, and review their work properly; and
c) Trainee teachers need to invest the time needed to become more familiar with mathematics concepts [language] and have at least one mentor at the teachers' college that they can reach out to whenever they encounter challenges with understanding math concepts and tasks, especially while planning lessons for their students.

4. Conclusion

The following conclusions were made based on the findings of the study:

1) The trainee teachers had inadequate familiarity with mathematical concepts [language].
2) The usage of games, role plays, group discussions and creation of posters have helped in improving trainee teachers' use of mathematical language.
3) The study revealed the need for all the participants to become independent learners who spend time to self-reflect to ensure that they become more familiar with the mathematical language and thus the mathematical concepts.

5. Recommendations

It was recommended that both the lecturers and trainee teachers should be continuously engaged in using manipulatives and student-centered activities to encourage students' discourse in the classroom.

There is great need for trainee teachers to be engaged in self-reflective processes until it becomes second nature to them. They need to spend quality time editing their own work as they read and understand the mathematical notation and language until common conventions become fully understood by them.

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