International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

Robust's Ranking Function for Generalized 4th Multiple Polygonal Fuzzy Number

Jitendra Singh[1], Deepak Kumar[2]

[1]Department of Mathematics, R.R. College, Alwar- 301001, Rajasthan, India
Email: *drjsrrcalwar[at]gmail.com*

[2]Raj Rishi Bhartrihari Matsya University Alwar -301001, Rajasthan, India
Email: *deepakvaishnav565[at]gmail.com*

Abstract: ***We generalized an arithmetic operation of 4th multiple polygonal fuzzy numbers with its membership function. In this paper, we suggested Robust's ranking function for 4th multiple polygonal fuzzy numbers by using alpha- cut. The complexity in solving this type of problems has reduced to easy computation.***

Keywords: 4th multiple polygonal fuzzy numbers, Ranking function, α-cut, Membership function

1. Introduction

Fuzzy logic is an advanced research field of logical reasoning in the artificial intelligence of fuzzy optimization. The representation techniques of knowledge are required to be more flexible and powerful to perform fuzzy reasoning. Fuzzy production rules have been used to represent the imprecise knowledge in the real world in the various view of point. The fuzzy logic has been used in two senses such as narrow sense and a broad sense. In the narrow sense the fuzzy logic refers to a logical system that generalizes a classical logic for reasoning under uncertainty and in the broad sense, the fuzzy logic refers to all the theories and technique that employ fuzzy sets. In the fuzzy logical concepts, the degree of belonging to a set is introduced that is needed to capture the way of people think with membership function that allows various degree of membership for the elements of the given set. A ranking function is play important role in the fuzzy environment and describing a ranking function is the best way to order the elements that maps every fuzzy number in to the real line where the natural order exists. A lot of many researchers have developed different type of ranking functions based on ordering relation of a fuzzy numbers.

2. Literature Survey

Zadeh [18] be the first person who introduced the concept of fuzzy set with properties of membership function. The fuzzy sets gives out natural way of dealing with many problems just like control system and decision making in our real life. Many researchers and authors had been done work on the basic operation and the fundamental arithmetic operation by using arithmetic interval of α- cut for fuzzy numbers such as Trapezoidal fuzzy number (Rezvani) [14], Octagonal fuzzy number (Raju and Jaygopal) [11], Dodecagonal fuzzy number (Felix and Devadoss)[2], Hexadecagonal fuzzy number (Sudha and Gokilamani) [16], Icosagonal fuzzy number(Raju and Jaygopal)[12], Icosikaitetragonal fuzzy number(Raju and Jaygopal)[13] and Icosikaioctagonal fuzzy number(Raju and Jaygopal)[10]. A new ranking function be purposed by Pavithra and Jenita[8] for solving the fuzzy assignment problem in which assignment costs are in form of dodecagonal fuzzy number. Robust's ranking function can be used to convert this fuzzy assignment problem into crisp assignment problem by using and to obtain an optimal solution by traditional method. Kumar and Singh[5] introduced Generalized 4thmultiple polygonal fuzzy number with its membership function. They also defined fundamental operations including addition, subtraction and multiplication of alpha cut. Amutha and Uthra[1] introduced a method for solving Intuitionistic fuzzy assignment problem in which cost values are in form of Symmetric octagonal fuzzy numbers and solved by Hungarian method. A new method be purposed by Jayalakshmi and Mohana[4]to solve fuzzy transportation problem using hexadecagonal fuzzy numbers and be obtained optimal solution by using Vogel's approximation method. The ranking function of fuzzy numbers was first introduced by Sasikumar and Raju[15] purposed a technique for two person zero sum game in which pay off matrices represented to Icosikatetragonal fuzzy numbers and solved oddment method. Jain[3] first introduced the ranking function of fuzzy numbers for decision making and logic programming etc. Kumar and Subramanian[6] developed a new method for solving trapezoidal fuzzy transport problem and obtained optimal solution same as by using other traditional method. Raja and Raju[7] introduced the Robust's ranking method for solving the elucidation of fuzzy assignment problem in which assignment costs represented Icosikaioctagonal fuzzy number and obtained optimal solution by Hungarian method. Singh et al[17] introduced the theory of soft and fuzzy soft relations based on soft set and fuzzy soft set with their applications in decision making problems. Raju and Vathana[9] developed a method for solving fuzzy critical path in which activity duration times represented in the form of the Icosagonal fuzzy number. In the fuzzy project network, this fuzzy network problem converted into the crisp network problem using the ranking function and obtained critical path form the initial point to terminal point. In this paper, we extended this work for Generalized 4th multiple polygonal fuzzy numbers to the Robust's ranking function. The different type of fuzzy optimization problems in form of 4th multiple polygonal fuzzy numbers can be converted into crisp problem by using the proposed ranking function.

3. Problem definition

3.1 Fuzzy set: Let X be real number set. A fuzzy set A of X is defined as A= {(x, $\mu_A(x)$)/ x∈ X}, where $\mu_A(x)$: X → [0, 1] is the membership function of x in A.

3.2 Fuzzy number: Let A be a fuzzy set of X then A is called fuzzy number if its membership function $\mu_A(x)$ satisfies the following characteristics

a) A is normal, that is $\exists x_0 \in X$ such that $\mu_A(x_0)$ =1.

b) $\mu_A(x)$ is piecewise continuous.

c) A is convex for membership function $\mu_A(x)$ satisfying the property $\mu_A(\lambda x_1 + (\lambda-1)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$ for each $x_1, x_2 \in X$ & $\lambda \in [0, 1]$.

d) The support of A, that is $S(A) = \{x \in X / \mu_A(x) > 0\}$ is bounded in X.

3.3 Alpha cut The α-cut of fuzzy set A is defined as $[A]_\alpha = \{x \in X / \mu_A(x) \geq \alpha\}$.

4. 4th Multiple Polygonal Fuzzy Number

We define the Generalized 4th multiple polygonal fuzzy number $A_m = (a_1, a_2, \dots, a_{4n})$, m=n, n ∈ N whose membership function is defined as

$$\mu_{A_m}(x) = \begin{cases} 0 & for \quad 0 < a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right) & for \quad a_1 \leq x \leq a_2 \\ k_1 & for \quad a_2 \leq x \leq a_3 \\ k_1 + (k_2-k_1)\left(\dfrac{x-a_3}{a_4-a_3}\right) & for \quad a_3 \leq x \leq a_4 \\ k_2 & for \quad a_4 \leq x \leq a_5 \\ .. & \\ .. & \\ k_{n-2} & for \quad a_{2n-4} \leq x \leq a_{2n-3} \\ k_{n-2} + (k_{n-1}-k_{n-2})\left(\dfrac{x-a_{2n-3}}{a_{2n-2}-a_{2n-3}}\right) & for \quad a_{2n-3} \leq x \leq a_{2n-2} \\ k_{n-1} & for \quad a_{2n-2} \leq x \leq a_{2n-1} \\ k_{n-1} + (1-k_{n-1})\left(\dfrac{x-a_{2n-1}}{a_{2n}-a_{2n-1}}\right) & for \quad a_{2n-1} \leq x \leq a_{2n} \\ 1 & for \quad a_{2n} \leq x \leq a_{2n+1} \\ k_{n-1} + (1-k_{n-1})\left(\dfrac{a_{2n+2}-x}{a_{2n+2}-a_{2n+1}}\right) & for \quad a_{2n+1} \leq x \leq a_{2n+2} \\ k_{n-1} & for \quad a_{2n+2} \leq x \leq a_{2n+3} \\ k_{n-2} + (k_{n-1}-k_{n-2})\left(\dfrac{a_{2n+4}-x}{a_{2n+4}-a_{2n+3}}\right) & for \quad a_{2n+3} \leq x \leq a_{2n+4} \\ k_{n-2} & for \quad a_{2n+4} \leq x \leq a_{2n+5} \\ .. & \\ .. & \\ k_2 & for \quad a_{4n-4} \leq x \leq a_{4n-3} \\ k_1 + (k_2-k_1)\left(\dfrac{a_{4n-2}-x}{a_{4n-2}-a_{4n-3}}\right) & for \quad a_{4n-3} \leq x \leq a_{4n-2} \\ k_1 & for \quad a_{4n-2} \leq x \leq a_{4n-1} \\ k_1\left(\dfrac{a_{4n}-x}{a_{4n}-a_{4n-1}}\right) & for \quad a_{4n-1} \leq x \leq a_{4n} \\ 0 & for \quad a_{4n} \leq x \end{cases}$$

where $0 \leq k_1 \leq k_2 \leq \dots \leq k_{n-2} \leq k_{n-1} \leq 1$.

4.1 Alpha cut For $\alpha \in [0, 1]$, then α- cut of 4thmultiple polygonal fuzzy numbers $A_m = (a_1, a_2, \dots, a_{4n})$, m=4n, n ∈ N is defined as

$$[A_m]_\alpha = \begin{cases} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1), \quad a_{4n} - \frac{\alpha}{k_1}(a_{4n} - a_{4n-1})] & for \alpha \in [0, k_1] \\ [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3), \quad a_{4n-2} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{4n-2} - a_{4n-3})] & for \alpha \in [k_1, k_2] \\ .. & \\ .. & \\ [a_{2n-3} + (\frac{\alpha - k_{n-2}}{k_{n-1} - k_{n-2}})(a_{2n-2} - a_{2n-3}), \quad a_{2n+4} - (\frac{\alpha - k_{n-2}}{k_{n-1} - k_{n-2}})(a_{2n+4} - a_{2n+3})] & for \alpha \in [k_{n-2}, k_{n-1}] \\ [a_{2n-1} + (\frac{\alpha - k_{n-1}}{1 - k_{n-1}})(a_{2n} - a_{2n-1}), \quad a_{2n+2} - (\frac{\alpha - k_{n-1}}{1 - k_{n-1}})(a_{2n+2} - a_{2n+1})] & for \alpha \in [k_{n-1}, 1] \end{cases}$$

For all $\alpha \in [0, 1]$, and here $[k_1, k_2, \dots, k_{n-1}, k_n] = [\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n-1}{n}, 1] n \in N$ is given by

$$[A_m]_\alpha = \begin{cases} [a_1 + n\alpha(a_2 - a_1), \quad a_{4n} - n\alpha(a_{4n} - a_{4n-1})] & for \alpha \in [0, \frac{1}{n}] \\ [a_3 + (n\alpha - 1)(a_4 - a_3), \quad a_{4n-2} - (n\alpha - 1)(a_{4n-2} - a_{4n-3})] & for \alpha \in [\frac{1}{n}, \frac{2}{n}] \\ .. & \\ .. & \\ [a_{2n-3} + (n\alpha - n + 2)(a_{2n-2} - a_{2n-3}), \quad a_{2n+4} - (n\alpha - n + 2)(a_{2n+4} - a_{2n+3})] & for \alpha \in [\frac{n-2}{n}, \frac{n-1}{n}] \\ [a_{2n-1} + (n\alpha - n + 1)(a_{2n} - a_{2n-1}), \quad a_{2n+2} - (n\alpha - n + 1)(a_{2n+2} - a_{2n+1})] & for \alpha \in [\frac{n-1}{n}, 1] \end{cases}$$

5. Robust's Ranking Function

The Robust's raking function of the Generalized 4^{th} multiple polygonal fuzzy number $A_m = (a_1, a_2, \dots, a_{4n})$, m=4n, $n \in N$ is defined as below

$$R(A_m) = 0.5 \int_0^1 (\alpha^L, \alpha^U) d\alpha$$

$$= \frac{1}{2} \int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1), \; a_{4n} - \frac{\alpha}{k_1}(a_{4n} - a_{4n-1})] d\alpha$$

$$+ \frac{1}{2} \int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3), \; a_{4n-2} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{4n-2} - a_{4n-3})] d\alpha + \cdots$$

$$+ \frac{1}{2} \int_{k_{n-2}}^{k_{n-1}} [a_{2n-3} + (\frac{\alpha - k_{n-2}}{k_{n-1} - k_{n-2}})(a_{2n-2} - a_{2n-3}), a_{2n+4} - (\frac{\alpha - k_{n-2}}{k_{n-1} - k_{n-2}})(a_{2n+4} - a_{2n+3})] d\alpha$$

$$+ \frac{1}{2} \int_{k_{n-1}}^{1} [a_{2n-1} + (\frac{\alpha - k_{n-1}}{1 - k_{n-1}})(a_{2n} - a_{2n-1}), a_{2n+2} - (\frac{\alpha - k_{n-1}}{1 - k_{n-1}})(a_{2n+2} - a_{2n+1})] d\alpha$$

For all $\alpha \in [0, 1]$, and here $[k_1, k_2, \dots, k_{n-1}, k_n] = [\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n-1}{n}, 1] n \in N$ is given by

$$R(A_m) = \frac{1}{2} \int_0^{\frac{1}{n}} [a_1 + n\alpha(a_2 - a_1) + a_{4n} - n\alpha(a_{4n} - a_{4n-1})] d\alpha$$

$$+ \frac{1}{2} \int_{\frac{1}{n}}^{\frac{2}{n}} [a_3 + (n\alpha - 1)(a_4 - a_3) + a_{4n-2} - (n\alpha - 1)(a_{4n-2} - a_{4n-3})] d\alpha + \cdots$$

$$+ \frac{1}{2} \int_{\frac{n-2}{n}}^{\frac{n-1}{n}} [a_{2n-3} + (n\alpha - n + 2)(a_{2n-2} - a_{2n-3}) + a_{2n+4} - (n\alpha - n + 2)(a_{2n+4} - a_{2n+3})] d\alpha$$

$$+ \frac{1}{2} \int_{\frac{n-1}{n}}^{1} [a_{2n-1} + (n\alpha - n + 1)(a_{2n} - a_{2n-1}) + a_{2n+2} - (n\alpha - n + 1)(a_{2n+2} - a_{2n+1})] d\alpha$$

$$= \frac{a_1 + a_2 + a_{4n-1} + a_{4n}}{4n} + \frac{a_3 + a_4 + a_{4n-3} + a_{4n-2}}{4n} + \dots + \frac{a_{2n-3} + a_{2n-2} + a_{2n+4} + a_{2n+3}}{4n} + \frac{a_{2n-1} + a_{2n-1} + a_{2n+2} + a_{2n+1}}{4n}$$

$$= \frac{a_1 + a_2 + a_3 + a_4 + \dots + a_{2n-5} + a_{2n-4} + a_{2n-3} + a_{2n-2} + a_{2n-1} + a_{2n} + a_{2n+1} + a_{2n+2} + \dots + a_{4n-1} + a_{4n}}{4n}.$$

Therefore,

$$R\,(A_m) = \sum_{i=1}^{4n} \frac{a_i}{4n}.$$

5.1 Ranking function of Trapezoidal fuzzy number

Let fuzzy number $A_4 = (a_1, a_2, a_3, a_4)$ is Trapezoidal fuzzy number (for n=1, m=4) then the Ranking Function of Trapezoidal fuzzy number is defined as fellows

$$R(A_4) = 0.5 \int_0^1 (\alpha^L, \alpha^U)\, d\alpha$$

$$= \frac{1}{2}\int_0^1 [a_1 + \alpha(a_2 - a_1) + a_4 - \alpha(a_4 - a_3)]\, d\alpha$$

$$= \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

5.2 Ranking function of Octagonal fuzzy number

Let fuzzy number $A_8 = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is Octagonal fuzzy number (for n=2, m=8) then the Ranking function of Octagonal fuzzy number is defined as fellows

$$R(A_8) = 0.5 \int_0^1 (\alpha^L, \alpha^U)\, d\alpha$$

$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_8 - \frac{\alpha}{k_1}(a_8 - a_7)]\, d\alpha + \frac{1}{2}\int_{k_1}^1 [a_3 + (\frac{\alpha - k_1}{1 - k_1})(a_4 - a_3),\ a_6 - (\frac{\alpha - k_1}{1 - k_1})(a_6 - a_5)]\, d\alpha.$$

Here $k_1 = \frac{1}{2} = 0.5$, therefore

$$R(A_8) = \frac{1}{2}\int_0^{0.5} [a_1 + 2\alpha(a_2 - a_1) + a_8 - 2\alpha(a_8 - a_7)]\, d\alpha + \frac{1}{2}\int_{0.5}^1 [a_3 + (2\alpha - 1)(a_4 - a_3) + a_6 - (2\alpha - 1)(a_6 - a_5)]\, d\alpha$$

$$= \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8}{8}.$$

5.3 Ranking function of Dodecagonal fuzzy number

Let fuzzy number $A_{12} = (a_1, a_2, \dots, a_{12})$ is Dodecagonal fuzzy number (for n=3, m=12) then the Ranking function of Dodecagonal fuzzy number is defined as fellow

$$R(A_{12}) = 0.5 \int_0^1 (\alpha^L, \alpha^U)\, d\alpha$$

$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_{12} - \frac{\alpha}{k_1}(a_{12} - a_{11})]\, d\alpha + \frac{1}{2}\int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3),\ a_{10} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{10} - a_9)]\, d\alpha + \frac{1}{2}\int_{k_2}^1 [a_5 + (\frac{\alpha - k_2}{1 - k_2})(a_6 - a_5),\ \ a_8 - (\frac{\alpha - k_2}{1 - k_2})(a_8 - a_7)]\, d\alpha.$$

Here $k_1 = \frac{1}{3}$ and $k_2 = \frac{2}{3}$

$$R(A_{12}) = \frac{1}{2}\int_0^{\frac{1}{3}} [a_1 + 3\alpha(a_2 - a_1) + a_{12} - 3\alpha(a_{12} - a_{11})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{1}{3}}^{\frac{2}{3}} [a_3 + (3\alpha - 1)(a_4 - a_3) + a_{10} - (3\alpha - 1)(a_{10} - a_9)] d\alpha$$
$$+\frac{1}{2}\int_{\frac{2}{3}}^{1} [a_5 + (3\alpha - 2)(a_6 - a_5) + a_8 - (3\alpha - 2)(a_8 - a_7)]\, d\alpha.$$

Therefore,

$$R(A_{12}) = \frac{a_1 + a_2 + a_3 + a_4 + \cdots + a_{11} + a_{12}}{12}.$$

5.4 Ranking function of Hexadecagonal fuzzy number

Let fuzzy number $A_{16} = (a_1, a_2, ..., a_{16})$ is Hexadecagonal fuzzy number (for n=4, m=16) then the Ranking function of Hexadecagonal fuzzy number is defined as fellows

$$R(A_{16}) = 0.5 \int_0^1 (\alpha^L, \alpha^U) d\alpha$$

$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_{16} - \frac{\alpha}{k_1}(a_{16} - a_{15})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3),\ \ a_{14} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{14} - a_{13})] d\alpha$$
$$+\frac{1}{2}\int_{k_2}^{k_3} [a_5 + (\frac{\alpha - k_2}{k_3 - k_2})(a_6 - a_5),\ \ a_{12} - (\frac{\alpha - k_2}{k_3 - k_2})(a_{12} - a_{11})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_3}^{1} [a_7 + (\frac{\alpha - k_3}{1 - k_3})(a_8 - a_7),\ \ a_{10} - (\frac{\alpha - k_3}{1 - k_3})(a_{10} - a_9)]\, d\alpha$$

Here $k_1 = \frac{1}{4}, k_2 = \frac{2}{4}$ and $k_3 = \frac{3}{4}$, therefore

$$R(A_{16}) = \frac{1}{2}\int_0^{\frac{1}{4}} [a_1 + 4\alpha(a_2 - a_1) + a_{16} - 4\alpha(a_{16} - a_{15})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{1}{4}}^{\frac{2}{4}} [a_3 + (4\alpha - 1)(a_4 - a_3) + a_{14} - (4\alpha - 1)(a_{14} - a_{13})] d\alpha$$
$$+\frac{1}{2}\int_{\frac{2}{4}}^{\frac{3}{4}} [a_5 + (4\alpha - 2)(a_6 - a_5) + a_{12} - (4\alpha - 2)(a_{12} - a_{11})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{3}{4}}^{1} [a_7 + (4\alpha - 3)(a_8 - a_7) + a_{10} - (4\alpha - 3)(a_{10} - a_9)]\, d\alpha.$$

Thus

$$R(A_{16}) = \frac{a_1 + a_2 + a_3 + a_4 + \cdots + a_{15} + a_{16}}{16}.$$

5.5 Ranking function of Icosagonal fuzzy number

Let fuzzy number $A_{20} = (a_1, a_2, ..., a_{20})$ is Icosagonal fuzzy number (for n=5, m=20) then the Ranking function of Icosagonal fuzzy number is defined as fellows

$$R(A_{20}) = 0.5 \int_0^1 (\alpha^L, \alpha^U) d\alpha$$

$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_{20} - \frac{\alpha}{k_1}(a_{20} - a_{19})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3),\ a_{18} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{18} - a_{17})]d\alpha$$
$$+\frac{1}{2}\int_{k_2}^{k_3} [a_5 + (\frac{\alpha - k_2}{k_3 - k_2})(a_6 - a_5),\ \ a_{16} - (\frac{\alpha - k_2}{k_3 - k_2})(a_{16} - a_{15})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_3}^{k_4} [a_7 + (\frac{\alpha - k_3}{k_4 - k_3})(a_8 - a_7),\ \ a_{14} - (\frac{\alpha - k_3}{k_4 - k_3})(a_{14} - a_{13})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_4}^{1} [a_9 + (\frac{\alpha - k_4}{1 - k_4})(a_{10} - a_9),\ \ a_{12} - (\frac{\alpha - k_4}{1 - k_4})(a_{12} - a_{11})]\, d\alpha .$$

Here $k_1 = \frac{1}{5}, k_2 = \frac{2}{5}, k_3 = \frac{3}{5}$ and $k_4 = \frac{4}{5}$, therefore

$$R(A_{20}) = \frac{1}{2}\int_0^{\frac{1}{5}} [a_1 + 5\alpha(a_2 - a_1) + a_{20} - 5\alpha(a_{20} - a_{19})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{1}{5}}^{\frac{2}{5}} [a_3 + (5\alpha - 1)(a_4 - a_3) + a_{18} - (5\alpha - 1)(a_{18} - a_{17})]d\alpha$$
$$+\frac{1}{2}\int_{\frac{2}{5}}^{\frac{3}{5}} [a_5 + (5\alpha - 2)(a_6 - a_5) + a_{16} - (5\alpha - 2)(a_{16} - a_{17})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{3}{5}}^{\frac{4}{5}} [a_7 + (5\alpha - 3)(a_8 - a_7) + a_{14} - (5\alpha - 3)(a_{14} - a_{13})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{4}{5}}^{1} [a_9 + (5\alpha - 4)(a_{10} - a_9) + a_{12} - (5\alpha - 4)(a_{12} - a_{11})]\, d\alpha .$$

Thus

$$R(A_{20}) = \frac{a_1 + a_2 + a_3 + a_4 + \cdots + a_{19} + a_{20}}{20}.$$

5.6 Ranking Function of Icosikaitetragonal Fuzzy Number

Let fuzzy number $A_{24} = (a_1, a_2, ..., a_{24})$ is Icosikaitetragonal fuzzy number (for n=6, m=24) then the Ranking function of Icosikaitetragonal fuzzy number is defined as fellows

$$R(A_{24}) = 0.5 \int_0^1 (\alpha^L, \alpha^U) d\alpha$$

$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_{24} - \frac{\alpha}{k_1}(a_{24} - a_{23})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3),\ a_{22} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{22} - a_{21})]d\alpha$$
$$+\frac{1}{2}\int_{k_2}^{k_3} [a_5 + (\frac{\alpha - k_2}{k_3 - k_2})(a_6 - a_5),\ \ a_{20} - (\frac{\alpha - k_2}{k_3 - k_2})(a_{20} - a_{19})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_3}^{k_4} [a_7 + (\frac{\alpha - k_3}{k_4 - k_3})(a_8 - a_7),\ \ a_{18} - (\frac{\alpha - k_3}{k_4 - k_3})(a_{18} - a_{17})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_4}^{k_5} [a_9 + (\frac{\alpha - k_4}{k_5 - k_4})(a_{10} - a_9),\ \ a_{16} - (\frac{\alpha - k_4}{k_5 - k_4})(a_{16} - a_{15})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_5}^{1} [a_{11} + (\frac{\alpha - k_5}{1 - k_5})(a_{12} - a_{11}),\ \ a_{14} - (\frac{\alpha - k_5}{1 - k_5})(a_{14} - a_{13})]\, d\alpha .$$

Here $k_1 = \frac{1}{6}, k_2 = \frac{2}{6}, k_3 = \frac{3}{6}$, $k_4 = \frac{4}{6}$ and $k_5 = \frac{5}{6}$, therefore

$$R(A_{24}) = \frac{1}{2}\int_0^{\frac{1}{6}} [a_1 + 6\alpha(a_2 - a_1) + a_{24} - 6\alpha(a_{24} - a_{23})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{1}{6}}^{\frac{2}{6}} [a_3 + (6\alpha - 1)(a_4 - a_3) + a_{22} - (6\alpha - 1)(a_{22} - a_{21})]d\alpha$$
$$+\frac{1}{2}\int_{\frac{2}{6}}^{\frac{3}{6}} [a_5 + (6\alpha - 2)(a_6 - a_5) + a_{20} - (6\alpha - 2)(a_{20} - a_{18})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{3}{6}}^{\frac{4}{6}} [a_7 + (6\alpha - 3)(a_8 - a_7) + a_{18} - (6\alpha - 3)(a_{18} - a_{17})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{4}{6}}^{\frac{5}{6}} [a_9 + (6\alpha - 4)(a_{10} - a_9) + a_{16} - (6\alpha - 4)(a_{16} - a_{15})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{5}{6}}^{1} [a_{11} + (6\alpha - 5)(a_{12} - a_{11}) + a_{14} - (6\alpha - 5)(a_{14} - a_{13})]\, d\alpha.$$

Thus

$$R(A_{24}) = \frac{a_1 + a_2 + a_3 + a_4 + \cdots + a_{23} + a_{24}}{24}.$$

5.7 Ranking Function of Icosikaioctagonal Fuzzy Number

Ranking function of Icosikaioctagonal fuzzy number is defined as fellows-

Let fuzzy number $A_{28} = (a_1, a_2, ..., a_{28})$ is Icosikaioctagonal fuzzy number (for n=7, m=28) then the

$$R(A_{28}) = 0.5 \int_0^1 (\alpha^L, \alpha^U) d\alpha$$
$$= \frac{1}{2}\int_0^{k_1} [a_1 + \frac{\alpha}{k_1}(a_2 - a_1),\ \ a_{28} - \frac{\alpha}{k_1}(a_{28} - a_{27})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_1}^{k_2} [a_3 + (\frac{\alpha - k_1}{k_2 - k_1})(a_4 - a_3),\ a_{26} - (\frac{\alpha - k_1}{k_2 - k_1})(a_{26} - a_{25})]d\alpha$$
$$+\frac{1}{2}\int_{k_2}^{k_3} [a_5 + (\frac{\alpha - k_2}{k_3 - k_2})(a_6 - a_5),\ \ a_{24} - (\frac{\alpha - k_2}{k_3 - k_2})(a_{24} - a_{23})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_3}^{k_4} [a_7 + (\frac{\alpha - k_3}{k_4 - k_3})(a_8 - a_7),\ \ a_{22} - (\frac{\alpha - k_3}{k_4 - k_3})(a_{22} - a_{21})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_4}^{k_5} [a_9 + (\frac{\alpha - k_4}{k_5 - k_4})(a_{10} - a_9),\ \ a_{20} - (\frac{\alpha - k_4}{k_5 - k_4})(a_{20} - a_{19})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_5}^{k_6} [a_{11} + (\frac{\alpha - k_5}{k_6 - k_5})(a_{12} - a_{11}),\ \ a_{18} - (\frac{\alpha - k_5}{k_6 - k_5})(a_{18} - a_{17})]\, d\alpha$$
$$+\frac{1}{2}\int_{k_6}^{1} [a_{13} + (\frac{\alpha - k_6}{1 - k_6})(a_{14} - a_{13}),\ a_{16} - (\frac{\alpha - k_6}{1 - k_6})(a_{16} - a_{15})]d\alpha.$$

Here $k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}$ and $k_6 = \frac{6}{7}$, therefore

$$\mathrm{R}(\mathrm{A}_{28}) = \frac{1}{2}\int_0^{\frac{1}{7}} [a_1 + 7\alpha(a_2 - a_1) + a_{28} - 7\alpha(a_{28} - a_{27})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{1}{7}}^{\frac{2}{7}} [a_3 + (7\alpha - 1)(a_4 - a_3) + a_{26} - (7\alpha - 1)(a_{26} - a_{25})] d\alpha$$
$$+\frac{1}{2}\int_{\frac{2}{7}}^{\frac{3}{7}} [a_5 + (7\alpha - 2)(a_6 - a_5) + a_{24} - (7\alpha - 2)(a_{24} - a_{23})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{3}{7}}^{\frac{4}{7}} [a_7 + (7\alpha - 3)(a_8 - a_7) + a_{22} - (7\alpha - 3)(a_{22} - a_{21})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{4}{7}}^{\frac{5}{7}} [a_9 + (7\alpha - 4)(a_{10} - a_9) + a_{20} - (7\alpha - 4)(a_{20} - a_{19})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{5}{7}}^{\frac{6}{7}} [a_{11} + (7\alpha - 5)(a_{12} - a_{11}) + a_{18} - (7\alpha - 5)(a_{18} - a_{17})]\, d\alpha$$
$$+\frac{1}{2}\int_{\frac{6}{7}}^{1} [a_{13} + (7\alpha - 6)(a_{14} - a_{13}) + a_{16} - (7\alpha - 6)(a_{16} - a_{15})] d\alpha.$$

Thus

$$\mathrm{R}(\mathrm{A}_{28}) = \frac{\mathrm{a}_1 + \mathrm{a}_2 + \mathrm{a}_3 + \mathrm{a}_4 + \cdots + \mathrm{a}_{27} + \mathrm{a}_{28}}{28}.$$

We introduced the Robust's ranking function of 4thmultiple polygonal fuzzy numbers in Table 4.1 as below

Table 5.1: The Robust's ranking function of 4th multiple polygonal fuzzy numbers

S. No.	Type of fuzzy number m= 4n, n ∈ N		The Robust's ranking function of polygonal fuzzy numbers
1	n= 1	m=4	Trapezoidal fuzzy number[6]
2	n= 2	m= 8	Octagonal fuzzy number[1]
3	n= 3	m = 12	Dodecagonal fuzzy number[8]
4	n= 4	m = 16	Hexadecagonal fuzzy number[4]
5	n= 5	m = 20	Icosagonal fuzzy number[9]
6	n= 6	m = 24	Icosikaitetragonal fuzzy number[15]
7	n= 7	m = 28	Icosikaioctagonal fuzzy number[7]
…	…	…	… … …
	For n ∈ N	m = 4n	Generalized 4thmultiple polygonal fuzzy number (purposed)

6. Conclusion

The aim of this paper is to introduce the Robust's ranking function of Generlized4thmultiple polygon fuzzy number with the help of alpha cut. The purposed ranking function of Generlized4thmultiple polygon fuzzy number is used for converting different type of fuzzy optimization problems into crisp problem. The complexity in solving this type of problems has reduced to easy computation.

7. Future Scope

This ranking function can be used for optimization problems as future research studies.

References

[1] B. Amutha and G. Uthra, Defuzzification of Symmetric Octagonal Intuitionistic Fuzzy Number. Advances and Applications in Mathematical Sciences, **20**(9) (2021), 1719-1728.

[2] A. Felix and A. Victor Devadoss, A New Dodecagonal Fuzzy Number under uncertain Linguistic Environment, International Journal of Mathematics and its Application, **3**(1) (2017), 89- 97.

[3] R. Jain, Decision making in the presence of fuzzy variables. IEEE Transactions on systems, Man and Cybernetics, **6** (1976), 698-703.

[4] U. Jayalakshmi and K. A. Mohan, New Method for Solving Fuzzy Transportation Problem using Hexadecagonal Fuzzy number. International Journal of Innovative Research in Technology, **5**(9) (2019), 195-201.

[5] Kumar D. and Singh J., "Arithmetic Operations for Generalized 4thMultiple Polygonal Fuzzy Numbers", ISOR Journal of Mathematics, vol.- **18**(4), pp 12-19 (2022).

[6] M. Ramesh Kumar and S. Subramanian, Solution of Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers using Robust Ranking Methodology, International Journal of Pure and Applied Mathematics, **119**(16) (2018), 3763-3775.

[7] S. Maria Jesu Raja and V. Raju, Elucidating Fuzzy Assignment Problem Employing Icosikaioctagonal Fuzzy Number, Journal of Xi'an University of Architecture & Technology,**12**(6) (2020), 1681-1688.

[8] T. S. Pavithra and C. Jenita, A New Approach to Solve Fuzzy Assignment Problem Using Dodecagonal Fuzzy Numbers, International Journal of Scientific Research in Science, Engineering and Technology, **3**(6) (2017), 320- 323.

[9] V. Raju and M. paruvatha vathana, Fuzzy Critical Path Method with Icosagonal Fuzzy Numabers using Ranking Method, A Journal of composition theory, **12**(9) (2019), 62-69.

[10] V. Raju and R. Jayagopal, An Approach on Icosikaioctagonal Fuzzy Number-Traditional operations on Icosikaioctagonal fuzzy number. A Journal of Composition Theory, **12**(10) (2019), 727-734.

[11] V. Raju and R. Jayagopal, A Rudimentary Operations on Octagonal Fuzzy Numbers. International Journal of Research in Advent Technology, **6**(6) (2018), 1320-1323.

[12] V. Raju and R. Jayagopal, An Arithmetic Operations of Icosagonal fuzzy number using Alpha cut. International Journal of Pure and Applied Mathematics, **120**(8) (2018), 137-145.

[13] V. Raju and R. Jayagopal, A new Operation on Icosikaiteteragonal fuzzy number. Journal of Combinatorial Mathematics and Combinatorial Computing, **112** (2020), 127-136.

[14] S. Rezvani, Multiplication Operation on Trapezoidal Fuzzy Numbers. Journal of Physical Sciences, **15** (2011), 17-26.

[15] S. Sasikumar and V. Raju, Study on Fuzzy Game Problem in Icosikaitetragonal Fuzzy Number, Annals of R. S. C. B., **25**(6) (2021) 7793-17801.

[16] A. Sahaya Sudha and R. Gokilamani, An Arithmetic Operation on Hexadecagonal Fuzzy Number. International Journal of Fuzzy Logic System, **7**(1) (2017), 17-26.

[17] S. Singh, D. S. Hooda and S. C. Malik, On Soft and Fuzzy Soft Relations With Their Applications. Jnanabha, **50**(1) (2020), 102- 114.

[18] L.A. Zadeh, Fuzzy sets. Information and Control, **8**(3) (1965), 338-353.