# Analyzing the Relationship Between Buoyant Force, Weight Force in Air and Liquid on Sinking Objects 

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#### Abstract

This article presents a detailed analysis of the relationship between buoyant force, weight force in air, and weight force in liquid on sinking objects. The study revisits Archimedes law and explores its application in understanding the behavior of sinking objects in a dynamic state. It is found that in a dynamic state, there was an effect on object's weight in the liquid with the equation $\boldsymbol{w}_{B}^{\prime}=\boldsymbol{w}_{\boldsymbol{B}}-\boldsymbol{F}_{\boldsymbol{A}}$


Keywords: Buoyant Force, Weight Force, Sinking Objects, Archimedes Law, Dynamic State

## 1. Introduction

One of the events or phenomena studied in physics by physicists is the object's condition of sinking, drifting, and floating when it is immersed in the fluids. Archimedes (287212 BC), a physicist from Syracuse, Greece, examined this event, suggesting that an object immersed in a fluid experiences an upward buoyant force $F_{A}$ equal to the weight of the fluid displaced $w_{f}^{\prime}$ or $F_{A}=w_{f}^{\prime}$ which is known as Archimedes' law.

Based on Archimedes' law, if we examine an object immersed in a liquid, it is found that the upward buoyant force $F_{A}$ exerted on an object submerged in the fluid, either partially or completely submerged, is equal to the weight of the fluid displaced $w_{f}^{\prime}$ by the object and works upwards towards the center of mass of the fluid being displaced which is called Archimedes' law which is formulated by $F_{A}=w_{f}^{\prime}=\rho_{f} . V_{f}^{\prime} . g$ with $\rho_{f}$ as the density of the liquid and $V_{f}^{\prime}$ as the volume of the fluid displaced.

The application of Archimedes' principle by physicists can explain the floating, drifting and sinking of an object in a liquid. In studying the events of solids, hereinafter referred to as objects that sink and float in liquids, there are still differences in the studies carried out by several physicists in some of their research results.

In the event that objects sink in liquid byLima et al. (2014)they state that Archimedes' principle does not apply to the objects that sink in liquid at the bottom of a container that does not touch the liquid at the bottom. However,Mohazzab, (2017) from the results of his research found that Archimedes' principle still applies to objects sinking in liquid at the bottom of the container by using a balance.Mohazzab, (2017)does not examine sinking objects in a dynamic state.

The purpose of this article is to provide a comprehensive analysis of the relationship between buoyant force, weight force in air, and weight force in liquid on sinking objects, with a focus on understanding the application of Archimedes law in a dynamic state.

## Archimedes' Law and Archimedes' Principle

Archimedes (287-212 BC) an Ancient Greek physicist discovered ways and formulas for calculating the volume of objects that have non-standard shapes. His discovery occurred while bathing in a tub where the water rose when his body was submerged in the water and shouted "Eureka". The cause of the liquid rising or spilling from the container is the upward force or buoyant force $F_{A}$ from the liquid against the weight of the immersed object $w_{B}$.

The relationship between the magnitude of the upward force $F_{A}$ and the weight of the liquid displaced, by Archimedes it is stated that an object (solid) that is completely or partially immersed in a fluid (liquid or gas) will get an upward force $F_{A}$ which is equal to the weight of the displaced fluid $w_{f l}^{\prime}$ which is commonly called Archimedes' law (Tipler, 1996; Halliday and Resnick, 1978; Giancoli; 2001).

The magnitude of the upward force $F_{A}$ with the weight of the fluid displaced satisfies the equation:

$$
\begin{equation*}
F_{A}=w_{f l}^{\prime}=m_{f l .}^{\prime} \cdot g=\rho_{f l} \cdot V_{f l}^{\prime} \cdot g \tag{1}
\end{equation*}
$$

with $m_{f l}^{\prime}$ as the mass of the fluid displaced, g as the gravitational field strength, and $V_{f l}^{\prime}$ as the volume of the displaced fluid (Tipler, 1996; Halliday and Resnick, 1978; Giancoli; 2001).

If a solid object is immersed in a fluid in the form of a liquid, then Archimedes' principle applies, which states that the magnitude of the upward force $F_{A}$ exerted on an object immersed in a liquid, whether fully or partially immersed, is equal to the weight of the liquid displaced by the object $w^{\prime} f$ and work in an upward direction (Tipler, 2001; Halliday and Resnick, 2006; Giancoli; 1996).

When an object is immersed in a liquid, there are two vertical forces acting on the object (solid). The first force is the weight of the object (solid matter) $w_{B}$ which is directed downwards and the second force is the buoyant force of Archimedes $F_{A}$ which is directed upwards. Based on the comparison of the strength of these forces, we will observe
three phenomena when immersing objects into liquids, namely sinking, drifting, and floating (Abdullah, 1996).

The relationship between the concept of Buoyant Force $\vec{F}_{A}$, the weight of an object in the air $\overrightarrow{\boldsymbol{w}}_{B}$, and the weight of an object in a liquid $\overrightarrow{\boldsymbol{w}}_{B}^{\prime}$ on an object sinking in a liquid

An irregularly shaped object whose weight in the air is $w_{B}$ with a density of $\rho_{\mathrm{B}}$ hanging from a rope that has a magnitude of the tension in the rope T equal to the weight
of theobject in air $w_{B}$ or $\mathrm{T}=w_{B}$. Then it is put it in a container filled with liquid with a density of $\rho_{\mathrm{B}}$ at a height H which has a cross-sectional area A as shown in Figure 1a.

After being immersed in the liquid, the water surface will experience an increase in height by $\Delta \mathrm{H}$ with the magnitude of the tension in the rope $\mathrm{T}^{\prime}$ when the object in the liquid is equal to the weight of the object in the liquid $w_{B}^{\prime}$ which is usually called the apparent weight of the object (Mohazzabi, 2017 ) as in Figure 1b.


Figure 1: (a). Objects before being immersed in liquid, (b) Objects after being immersed in liquid (Source: Mohazzabi and James 2017)

The volume change of the displaced liquid is $V_{f}^{\prime}=A . \Delta H$ whose mass is $m_{f}^{\prime}$ so that the weight of the displaced fluid $w_{f}^{\prime}$ is:

$$
\begin{equation*}
w_{f}^{\prime}=m_{f}^{\prime} \cdot g=\rho_{f} \cdot V_{f}^{\prime} \cdot g \tag{2}
\end{equation*}
$$

The weight of the displaced fluid $\llbracket w_{f}^{\prime}$ is the same as the additional weight of the object in the liquid $\vec{w}_{B}^{\prime}$ downwards (Mohazzabi and James 2017). However, this was revised by Mohazzabi (2017) by conducting an experiment by placing a balance at the bottom of the container as shown in Figure 2a.


Figure 2: (a) Objects sink to the bottom of the container and there is a balance, (b) objects are placed on the balance (source: Mohazzabi, 2017)

If an object sinks to the bottom of a container in a liquid with the weighs $w^{\prime}{ }_{B}$, then the magnitude is equal to the magnitude of the weight designation on the balance in Figure 1.b. The weight of the object in liquid $w_{B}$ is equal to the difference in the weight of the object in air $w_{B}$ and the weight of the fluid displaced $w_{f}$ using the equation (Mohazzabi (2017)

$$
\begin{equation*}
w_{B}^{\prime}=w_{B}-w_{f}^{\prime}=w_{B}-F_{\mathrm{A}}=m_{B}^{\prime} \cdot g-r_{f} g V_{f}^{\prime} \tag{3}
\end{equation*}
$$

Therefore, the weight of an object in a liquid or the apparent weight of an object $\vec{w}_{B}^{\prime}$ in a static submerged state has the direction of the upward gravitational force in the same direction as the buoyant force $\vec{F}_{A}$.

The cause of the buoyancy force $\vec{F}_{A}$ is obtained as follows.

$$
\begin{equation*}
\vec{F}_{A}=\overrightarrow{\boldsymbol{w}}_{\boldsymbol{B}}-\overrightarrow{\boldsymbol{w}}_{\boldsymbol{B}}^{\prime} \tag{4}
\end{equation*}
$$

The result of equation (4) describes that the buoyant force of a liquid $\vec{F}_{A}$ in a system of sinking objects in a liquid at rest is caused by a reduction in the weight of the object in air $w_{B}$ with the weight of the object in liquid $w^{\prime}{ }_{B}$.

The relationship between the concept of buoyancy $\vec{F}_{A}$, the weight of an object in the air $\overrightarrow{\boldsymbol{w}}_{B}$, and the weight of an object in a liquid $\overrightarrow{\boldsymbol{w}}_{B}^{\prime}$ on an object sinking in a liquid

For example, if a sinking object in a liquid has a mass of $m_{\mathrm{B}}$, a density of $\rho_{B}$ and a weight of $w_{B}$ in the air as shown in Figure 3.a, it is placed in a non-viscous liquid with a density of $\rho_{f}$ with $\rho_{f}<\rho_{B}$ to a container as shown in Figure 3.b. The object is then released so that the object moves down as shown in Figure 3.c.


Figure 3: (a) a sinking object in the air with a weight $\vec{w}_{B}$, (b) a sinking object is placed in a liquid at the bottom of the vessel with a change in the height of the liquid $\Delta H$, (c) The sinkting object moves upward due to the buoyant force $\vec{F}_{A}$ and object's gravity $\vec{w}_{B}$

Based on Newton's 2nd law, if there is no external force acting on the object, then the object can move sink down with acceleration $\vec{a}$ whose downward direction is caused by the resultant force $\sum \vec{F}$ then the equation is obtained:

$$
\begin{equation*}
\vec{a}=\frac{\sum \vec{F}}{m_{B}^{\prime}} \tag{5}
\end{equation*}
$$

with $m_{\text {' }}$ as the mass of the object in the liquid or the apparent mass of the object in the liquid.

The magnitude of the resultant force $\sum \vec{F}$ exerted is caused by the difference in the magnitude of the buoyant force $\vec{F}_{A}$ by the liquid with the weight of the object in the air $\vec{w}_{B}$ or the equation is obtained as follows.

$$
\begin{equation*}
\sum \vec{F}=\vec{w}_{B}-\vec{F}_{A} . \tag{6}
\end{equation*}
$$

if equation (6) is substituted to equation (5), it can be obtained as follows.

$$
\vec{a}=\left(\vec{w}_{B}-\vec{F}_{A}\right) / m_{B}^{\prime}
$$

or

$$
\begin{equation*}
m_{B}^{\prime} \cdot \vec{a}=\vec{w}_{B}-\vec{F}_{A} \tag{7}
\end{equation*}
$$

The magnitude of the object's acceleration move down $a$ is equal to the acceleration of gravity $g$ because there is no external force exerted on the object, hence equation (5) can be written

$$
\begin{equation*}
m_{B}^{\prime} \cdot \vec{g}=\vec{w}_{B}-\vec{F}_{A} \tag{7.a}
\end{equation*}
$$

the object's weight $m_{B}^{\prime} \cdot \vec{g}$ is the object's weight in liquid $\vec{w}_{B}^{\prime}$ so that the object's weight in liquid $w^{\prime}{ }_{B}$ is directed upward in the direction of the object's weight in air $\vec{w}_{B}$ which can be formulated:

$$
\begin{equation*}
w^{\prime}{ }_{B}=m^{\prime}{ }_{\mathrm{B}} \cdot \mathrm{~g} \tag{8}
\end{equation*}
$$

thus, the equation (7.a) can be written as:

$$
\begin{equation*}
\vec{w}_{B}^{\prime}=\vec{w}_{B}-\vec{F}_{A} \tag{7.b}
\end{equation*}
$$

the weight direction a sinking object in a liquid in a dynamic state (moving) $\vec{w}_{B}^{\prime}$ points upward in the same direction to the buoyant force $\vec{F}_{A}$. Thus, the magnitude of the object's weight in the liquid $w_{B}$ is

$$
w_{B}^{\prime}=w_{B}-F_{A}
$$

According to Archimedes' law, the magnitude of the buoyant force $F_{A}$ is equal to the weight of the displaced liquid $w_{f}^{\prime}$ or $F_{A}=w_{f}^{\prime}=m_{f}^{\prime} \cdot g=\rho_{f} \cdot V_{f}^{\prime} \cdot g$, thus equation (9) can be written as follows.

$$
\begin{equation*}
w_{B}^{\prime}=w_{B}-F_{A}=w_{B}-w_{f}^{\prime}=w_{B}-\rho_{f} . V_{f}^{\prime} \cdot g \tag{10}
\end{equation*}
$$

The result of equation (10) is identical to equation (3) obtained by Mohazzabi (2017) for an object immersed in a liquid in a static state where the direction of the gravitational force (object's weight) of the object in the liquid $\vec{w}_{B}^{\prime}$ is directed upward in the direction of the buoyant force $\vec{F}_{A}$.

## 2. Conclusion

The analysis of the relationship between buoyant force $\vec{F}_{A}$, weight force in air $\vec{w}_{B}$, and weight force in liquid $\vec{w}_{B}^{\prime}$ on sinking objects reveals that in a dynamic state, there is an subtaction of the objects weight from the weight of the object in air and the weight of the object in liquid. This is caused by the resultant force between the weight of the object in air and the buoyant force. $\vec{w}_{B}^{\prime}$ which is upward in the same direction to the buoyant force $\vec{F}_{A}$ which has the magnitude $w_{B}^{\prime}=w_{B}-F_{A}$.

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