

# Advanced and Easy Expression of Bit Error Rate Calculation for Multiple DF Relay using Rayleigh Fading Channel

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**Abstract:** In recent times, cooperative communication has been considered as the most developing stage in wireless communication because of its enabling technology for efficient spectrum use and its low power consumption. In this paper, we derive a novel and simple expression of cooperative communication which can overcome the unwanted effects of wireless channel on the message received at the destination which is nearly the same as the theoretical value. In cooperative communication, the three nodes (source, relay, and destination) system, where information transfers between the source and the destination through the selected relay is available in node. The node which is selected should maximize cooperation benefits for the whole system. By applying the technique which is discussed in the paper, we can reduce the error or save the transmit power by which, in turn, we can improve the performance of the network. This paper focuses on the performance of BER using DF relay over the Rayleigh fading channel by using the number of signals received at destination with assistance from the MATLAB tool.

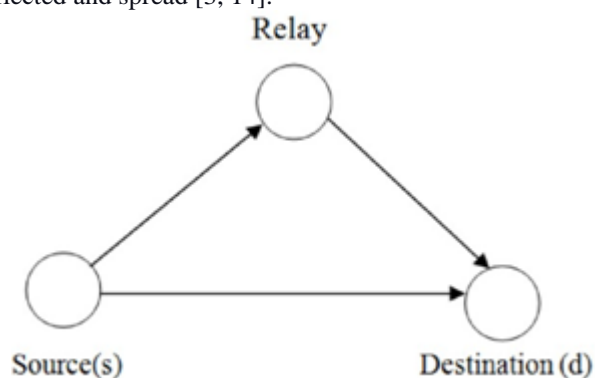
**Keywords:** Bit error rate (BER), Cooperative communication, DF Relay, fading channel Wireless network

## 1. Introduction

In the ambience of mobile communication, the channel is time-variant and it is this gently diversifying trait of the channel that leads to an occurrence known as fading. Fading channels comprise active amplitude variation in the signal that is received. However, if they are not required for them, this will lead to a serious deterioration of performance [1-3]. In recent years, very large data rate techniques have achieved significant concern in the communication system; the fundamental concept of orthogonal frequency division multiplexing is the arrangement of the spectrum at hand into multiple orthogonal sub channels encounters almost flat fading [4-5]. The signal which is transmitted fades when it flows from the transmitter to the receiver, as a consequence of the fading nature of the wireless channel [6-7]. Cooperative communication is utilized to lower the fading effect of the wireless channel by transmitting images of the orthogonal signal by means of multiple relays [8-9]. On account of this consequence in cooperative diversity wireless network, the cooperative diversity communication is an analogous compound of the direct communication line of sight as well as the communication within the relay as depicted in Fig. 1 [10-11].

In today's digital communication systems, it is possible to ascertain the authenticity of the entire radio system from the Bit error rate, including "bit in" to "bit out" [12-13]. With noise-free and strong signals, this number is very small. It is more advantageous when we desire to keep an adequate signal-to-noise ratio in case of the occurrence of defective transmission via electronic systems. In a manner similar to several wireless communication networks, the medium of transmission suffers two key challenges in the WiMAX communication mechanism; these challenges are AWGN noise, Rayleigh and Rician fading. The Rayleigh channel describes when LOS paths do not lie in between the transmitter and the receiver but solely has a diversionary

path. Thus, the resultant signal that is received at the receiver end turns out to be the aggregate of all the waves which are reflected and spread [3, 14].



**Figure 1:** Single Relay Cooperative Communication

The most extensively utilized configuration for multipath fast fading is the Rayleigh distribution, the PDF of which is presented as follows:

$$f_{ray}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad r \geq 0 \quad (1)$$

Rayleigh technology has been contracted in order to strengthen the realization of the wireless network by ensuring significant reinforcement of performance in link security, spectral power, energy productivity and the rate of production of error [2-3, 12]. In a Rayleigh, or fading, the path of the signal is not discarded in the straightforward perception of the usual conviction character of white noise but, in Gaussians noise, it is a causal method that is investigated in the same way.

The bit error rate (BER) is mainly influenced by noise. This process occurs randomly, as described in terms of statics. The Gaussian probability density function delineates the system's noise while the Rayleigh PDF illustrates the signal

path [1, 14]. The BER is a crucial determinant which is taken into account while estimating systems that communicate digital data across locations. The BER refers to radio data links, Ethernet, in addition to fiber optic data modes as well. The relay of data across a data link leads to the probability of the transportation of errors into the system [15]. If such is the case, it may be possible to negotiate the uprightness of the system. Consequently, there is a need to evaluate the performance of the system and BER provides an absolute means using which such an assessment can be done. The BER estimates the total functioning of an arrangement, including the transmitter, the receiver and the communication that happens between them. It is determined as the rate at which errors take place in a transmission system. Put in simpler terms,

$$\text{BER} = \frac{\text{No. of bits in error}}{\text{total no bit sent}} \quad (2)$$

From Rappaport 2002 [5], the BER expression is estimated as

$$\text{BER} = \int_0^{\infty} P_r(E/r) P(r) dr \quad (3)$$

Here,  $P_b(E/r)$  = the conditional error probability  $P(r)$  = the pdf of the SNR

There are two relaying techniques which are used in the cooperative communication system to improve the data transmission efficiency and reduce the bit error rate [16-17]. These techniques are:

## 2. Amplified and Forward Method

### Decode and forward method.

In this paper, we use decode and forward method because of the absence of magnified noise in the transmitted signal; thus, at first, the signal which is received is decoded and, thereafter, re-encoded [7-18]. The most favored way of data processing in the relay is detect and forward method; here, while the relay has the downside of wasting time, the benefit is that the received bit is corrected at the relay station [19-20]. In a previous literature review, we concluded a more complex equation and algorithm but, in this paper, we try to

extract expression for the bit error rate by using DF relay in the Rayleigh fading channel in a simple and easy way [21-23].

In contrast to much literature that we reviewed, where the analysis of the theoretical value of BER with DF was done by implementing Rayleigh fading channel, in this paper, plot analytical satisfied all theoretical values. In this paper, we used orthogonal coding and, with the help of the previous BER equation, applied the BPSK modulation technique with DF relay.

The rest of the paper is organized in the following manner: In Section 2, we describe the system model where we list the modulation technique, the relay, and through which hope signal travel. In Section 3, we perform the analysis of bit error rate on the basis of our review of various literatures, which mention the equation and try to find the easiest way of calculation and simulation of BER using MATLAB. In Section 4, we explain the result obtained by plotting graphs of the final expression that is derived and, in Section 5, we present our conclusions.

## 3. System Model

In this system, it is assumed that all relays perform TDMA and signal routed orthogonally [6-7]. The best relay selection method is shown in Fig 2. It is specially designed in orthogonal shape so that it can be used on all spectrums in antennas. If two channels transmit different frequencies, it is an orthogonal frequency [15, 21]. Since there is a single transmitter and receiver, the information received from the transmitter through different paths becomes unusable; thus, we have to pass the signal at a different frequency band or time interval so as to use the orthogonal property. The code which issues time and space extract signals is known as space time coding.

### Encoding is done in the following manner:

Let us consider that the signal  $S_1$  is transmitted by the antenna One and  $S^*1$  is the replica of the signal transmitted by antenna One;  $S_2$  is the signal transmitted by the second antenna and, its replica,  $S^*2$  is also generated by the second antenna. So, their matrix form combining signal is given as:

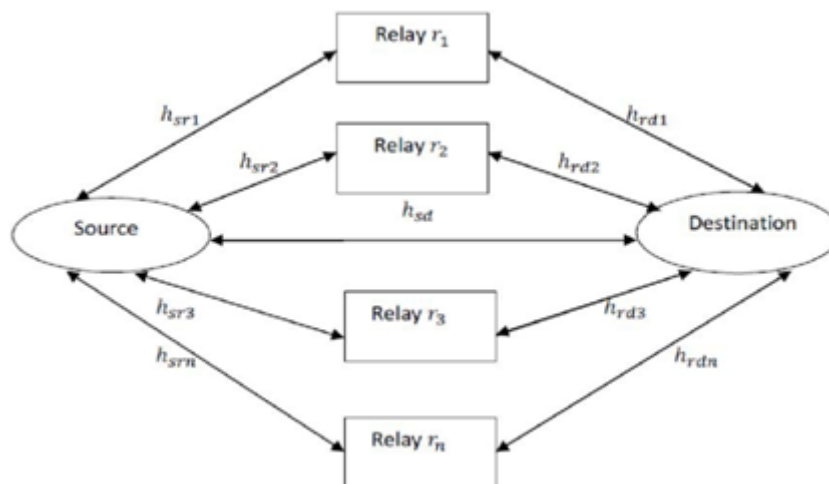


Figure 2: Multi relay cooperative Communication

$$\begin{bmatrix} S_1 & -S_2^* \\ S_2 & S_1^* \end{bmatrix} \quad (4)$$

Considering the first transmitted antenna,  $A_1$

$$A_1 = [S_1 \quad -S_2^*] \quad (5)$$

Again, for the second transmitted antenna,

$$A_2 = [S_2 \quad S_1^*] \quad (6)$$

By orthogonal property,

$$A_1 * A_2^H = 0 \quad (7)$$

First time period signal t or first time interval at t time

$$r_1 = S_1 h_1 + S_2 h_2 + n_1 \quad (8)$$

Second time interval signal at time t +  $r_2$

$$r_2 = -S_2^* h_1 + S_1^* h_2 + n_2 \quad (9)$$

**Table 1:** Space Time Coding

Time	Transmitter 1	Transmitter 2
Time T	$S_1$	$S_2$
Time T+t	$-S_2^*$	$S_1^*$

Channel estimator use to separate  $h_1$  and  $h_2$  as given in equation:

$$S_1 = h_1^* r_1 + h_2^* r_2 \quad (10)$$

$$S_2 = h_2^* r_1 - h_1^* r_2 \quad (11)$$

By putting the value of  $r_1$  and  $r_2$  we get:

$$S_1 = (|h_1|^2 + |h_2|^2) S_1 + h_1^* n_1 + h_2^* n_2 \quad (12)$$

$$S_2 = (|h_1|^2 + |h_2|^2) S_2 - h_1^* n_2 + h_2^* n_1 \quad (13)$$

$$\int_0^\infty e^{-at} t^{b-1} Q(\sqrt{ct}) dt = \left( \frac{\Gamma(b)}{a^b} \sqrt{\frac{\gamma}{1+\gamma}} \frac{(1+\gamma)^{-b} \Gamma(b+0.5)}{\sqrt{\pi} \Gamma(b+1)} \right) \times 2F_1 \left( 1, b+0.5, b+1, \frac{1}{1+\gamma} \right) \quad (15)$$

### Outage Probability Calculation

The probability density function of the Rayleigh distribution is represented below as [5]

$$p(r) = f(x) = \begin{cases} -\frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & 0 \leq r \leq \infty \\ 0 & r \leq 0 \end{cases} \quad (16)$$

In selection combiner, the outage probabilities of DF relay are given as

$$P_{out} = \left[ \prod_{i=1}^n \left\{ \min(\gamma_{sr_i}, \gamma_{rd_i}) \right\} \right]^n \quad (17)$$

$$P_{out} = \min(\gamma_{sr}, \gamma_{rd})^n \quad (18)$$

The Max term of above function is given as:

$$P_{out} = [1 - \max(\gamma_{sr}, \gamma_{rd})]^n \quad (19)$$

The outage probability of the Rayleigh fading channel is

Where  $h_1, h_2$  are the channel gain,  $r_1, r_2$  are the replica of the signal,  $n_1, n_2$  are the noise of the signal

### 4. BER Measurement

BER measurement refers to the comparison of the output to the input data stream which is sent through the system for an infinite period. Here, we assume that data transmission is a pseudorandom process [1-2]. Pseudorandom implies that we cannot create a truly random signal using deterministic (mathematical) methods. For accurate BER measurement, we use smart mathematical work [3-4].

With respect to the probability of error (POE), the Bit Error Rate (BER) may be explicated as,

$$P.O.E. = \frac{1}{2} ((1 - \text{erf}) \sqrt{\frac{E_b}{N_0}}) \quad (14)$$

Where, erf refers to the error function,  $E_b$  stands for the energy in one bit and  $N_0$  represents the noise power spectral density.

Here, we consider that the radio noise is Gaussian in its spectral power density. In this consideration, we add Rayleigh characteristics to the signal using multiple channels with unstable time delays to stimulate changing the path cases. The conflation of Gaussian noise and Rayleigh fading has become very significant in development of wireless communication systems. The mathematical nature of digital signal transmission also allows MATLAB simulation [5, 8, 12]. The bit error rate is the function of Q function which is given as:

represented as:

$$P_{out} = 1 - \exp\left(\frac{-\gamma_{th}}{\gamma}\right) \quad (20)$$

The cumulative function (CDF) of the mentioned function is represented as:

$$P_{out} = 1 - \exp\left(\frac{-\gamma_{th}}{\gamma}\right) \quad (21)$$

Hence, the Rayleigh fading channel's outage probability from source to relay and again from relay to destination, using the selection combiner, is represented as: [6]

$$P_{out} = \left[ 1 - \left\{ \exp\left(\frac{-\gamma_{th}}{\gamma_{sr}}\right) \right\} \times \left\{ \exp\left(\frac{-\gamma_{th}}{\gamma_{rd}}\right) \right\} \right]^n \quad (22)$$

The CDF of the function is given as:

$$CDF = \left[ 1 - \left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \right\} \left\{ \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} \right]^n \quad (23)$$

Where  $\gamma$  signal is the noise ratio (SNR) and  $\gamma_{sr}$  refers to the SNR from source to relay and  $\gamma_{rd}$  stands for the SNR from

relay to destination.

PDF represents the differential function of the CDF function and, thus, the differential of above equation is given as:

$$PDF = -n \left[ 1 - \left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \right\} \left\{ \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} \right]^{n-1} \frac{1}{\gamma_{sr}} * \frac{1}{\gamma_{rd}} \left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \right\} * \left\{ \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} \quad (24)$$

Bit Error Rate is given as

$$\int_0^\infty pdf * Q(\sqrt{cx}) dx \quad (25)$$

So, the bit error rate of cooperative communication for multiple DF relays is given as

$$\int_0^\infty -n \left[ 1 - \left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \right\} \left\{ \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} \right]^{n-1} \frac{1}{\gamma_{sr}} * \frac{1}{\gamma_{rd}} * \exp\left[-\left(\frac{\gamma}{\gamma_{sr}} + \frac{\gamma}{\gamma_{rd}}\right)\right] * Q\sqrt{cx} dx \quad (26)$$

Assume that

$$\left[ 1 - \left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \right\} \left\{ \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} \right] = X \quad (27)$$

By comparing equation (33) with equation (15) we get: a=1; b=n Thus, the bit error for the Rayleigh fading channel becomes

Thus, the above equation can be written as:

$$\left\{ \exp\left(-\frac{\gamma}{\gamma_{sr}}\right) \exp\left(-\frac{\gamma}{\gamma_{rd}}\right) \right\} = 1-X \quad (28)$$

$$-\left(\frac{\gamma}{\gamma_{sr}} + \frac{\gamma}{\gamma_{rd}}\right) = \log(1-X) \quad (29)$$

$$\frac{n\Gamma(n)}{\gamma_{sr} * \gamma_{rd}} \left( \sqrt{\frac{\gamma}{1+\gamma}} \frac{(1+\gamma)^{-n} \Gamma(n+0.5)}{2\sqrt{\pi} \Gamma(n+1)} \right) \times 2F_1\left(1, n+0.5, n+1, \frac{1}{1+\gamma}\right) \quad (34)$$

Where n is the number of relays.

Now, equation (26) can be written as:

$$\int_0^\infty -\frac{n}{\gamma_{sr} * \gamma_{rd}} X^{n-1} e^{\log(1-X)} * Q\sqrt{cX} dX \quad (30)$$

As we know that

$$\log(1-X) = -X + \frac{X^2}{2} - \frac{X^3}{3} + \frac{X^4}{4} \dots \dots \dots \quad (31)$$

Upon placing the value of (31) in (30), we get:

$$\int_0^\infty -\frac{n}{\gamma_{sr} * \gamma_{rd}} X^{n-1} e^{-X + \frac{X^2}{2} - \frac{X^3}{3} + \frac{X^4}{4} \dots} * Q\sqrt{cX} dX \quad (32)$$

On neglecting the higher term we get:

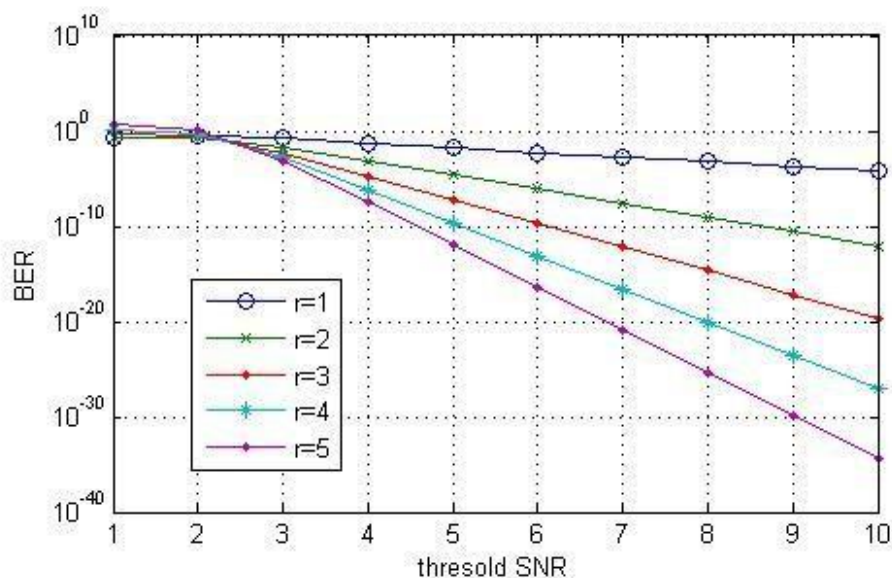
$$\int_0^\infty -\frac{n}{\gamma_{sr} * \gamma_{rd}} X^{n-1} e^{-X} * Q\sqrt{cX} dX \quad (33)$$

## 5. Simulation and Result

In Fig. 3 we simulated the Rayleigh fading channel's bit error rate for 1x1Tx and Rx system, where the transmit antenna of the carrier is deployed to transmit BPSK data symbol over the Rayleigh fading channel. Our conclusion is that the two hops, that is, the source to relay and relay to destination via multiple DF relay protocol.

Table 2: Simulation Parameter for Bit Error Rate

Parameter	Value
Symbols	BPSK
No. of hop	2
T <sub>x</sub> and R <sub>x</sub>	1x1
No. of relay	5
Type of relay	DF
SNR	3
Threshold SNR	1:10



**Figure 3:** Bit Error Rate Performance for Mobile Communication with DF Relay

With reference to Fig. 3, for relay 1, if the thresholds SNRs are 4db and 5db, the bit error rate is 0.05499 and 0.01754, respectively and, for relay 2, if thresholds SNRs are 4db and 5db, the bit error rate is 0.0008166 and 2.629e-005, respectively.

## 6. Conclusion

In this paper, we evaluate BER performance for mobile communication with DF relay by employing the BPSK transmission scheme that has been assessed with random data. The result shows a good bond between the simulated and theoretical plots (as studies in previous literature) for BPSK over Rayleigh fading. As the number of relays is stepped-up, the BER reduces, implying that if the maximum number of relays is four, the error is nearly removed. The benefits of this method are that if any image, data or text is sent through it, they may be more visible and remain unharmed.

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