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Finite Frequency Controller for Vehicle Active Suspension System

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Abstract: ***This paper investigates the problem of vertical vibration control of an active vehicle suspension system. An active suspension system is a possible way to improve suspension performance. This paper illustrates the design of an output feedback H∞ controller and state feedback H∞ controllers in the entire frequency range as well as in finite frequency range for an active suspension system. The H∞ performance is used to measure ride comfort so that more general road disturbances can be considered. Human body is much sensitive to vibrations of frequency 4 - 8 Hz in the vertical direction. The main objective is to reduce the body vertical vibration in this frequency range using an active suspension system. By using Generalized Kalman - Yakubovich - Popov (KYP) lemma, the H∞ norm from the disturbance to the controlled output is reduced in the particular frequency band. A state feedback controller is designed in the framework of linear matrix inequality (LMI) optimization. In addition, the time domain constraints are guaranteed in the controller design. A quarter car model with active suspension system is considered in this work and the effectiveness of the approach is illustrated by using simulations.***

Keywords: Active suspension system, Quarter car model, H∞ controllers, Vertical vibration control, Generalized KYP lemma, Linear matrix inequality (LMI)

1. Introduction

Vehicle suspension has been a hot research topic due to their important role in vehicle performance. Performance requirements for vehicle suspension include ride comfort, road holding suspension deflection and actuator saturation. However these requirements are often conflicting and a compromise of the requirements must be reached.

Vehicle suspension basically consists of wish - bone, spring and shock absorber to transmit and also filter all forces between body and road. The spring carries the body mass and isolates the body from the road disturbances and thus contributes to ride comfort. The damper contributes to both driving safety and comfort. Its task is the damping of body and wheel oscillations. For obtaining these performances, many type of suspension systems, such as passive, semi - active and active suspension, have been developed. Among these types, active suspension system is a possible way to improve suspension performance by overcoming the conflict between ride comfort and road handling.

Unlike conventional passive suspension, in active vehicle suspension systems the external excitation is counteracted with the generation of a control force depending on the vehicle response through an actuator driven by an external energy source. The main concept is use an active suspension to reduce the vibration energy of the vehicle body induced by the road excitation, while keeping the vehicle stability within an acceptable limit. A lot of efforts have been made to develop models for suspension systems and to define design specifications that reflect the main objectives. Many active suspension control approaches are proposed, based on various control techniques such as linear quadratic Gaussian control [5], adaptive and nonlinear control [6], active fault - tolerant control [7], fuzzy logic and neural network control, μ control [8], velocity feedback control [9], sliding mode control [10] and H∞ control. In particular, active suspension systems using H_∞ control have been intensively discussed in the context of robustness and disturbance attenuation, and they have been well recognized to be an effective way to manage the tradeoff between conflicting performance requirements.

For the active suspension systems, the main task is to design the controller which can stabilize the vertical motion of the car body and isolating the force transmitted to the passengers. Most of the reported approaches are aiming to improve ride comfort and are considered in the entire frequency domain. However, active suspension system may just belong to certain frequency band, and ride comfort is known to be frequency sensitive. From the ISO2361, the human body is much sensitive to vibrations of 4 – 8 Hz in the vertical direction. Hence the development of H_∞ control in finite frequency domain is significant for active suspension system.

Frequency gridding can be used to grid the frequency axis. In this case infinitely many frequency domain inequalities (FDI) are approximated by a finite number of FDIs at selected frequency points. This approach has a practical significance especially when the system is well damped and the frequency response is expected to be smooth. But it lacks a rigorous performance guarantee in the design process.

In this paper, a finite frequency output feedback H_∞ controller is designed by incorporating weighting functions. In the H_∞ frame work, weighting functions allow for frequency informations to be incorporated into the analysis. But the additional weights increase the system complexity and the process of selecting appropriate weights is time - consuming. Another approach that avoids both weighing functions and frequency gridding is to generalize the fundamental machinery, the Kalman - Yakubovich - Popov (KYP) lemma approach. The standard KYP lemma establishes the equivalence between a frequency domain inequality for a transfer function and a linear matrix inequality (LMI) associated with its state space realization.

However the standard KYP lemma is applicable for the infinite frequency range. A very significant development made by Iwasaki and Hara is the generalized KYP lemma [11]. It establishes the equivalence between a frequency domain property and an LMI over a finite frequency range allowing designers to impose performance requirements over finite or infinite frequency ranges. The generalized KYP lemma is very useful for the analysis and synthesis problems in practical applications.

Unlike the conventional H_∞ controllers, we present a design of H_∞ controller over a finite frequency range. Initially with the help of weighting functions, an output feedback H_∞ controller is designed. In addition to this an H_∞ controller for finite frequency range by using the generalized KYP lemma is designed. The time - domain constraints (road holding, suspension deflection and actuator saturation) are guaranteed in the controller design [1]. By using the generalized KYP lemma, the frequency domain inequalities are transformed into linear matrix inequalities, and to design a state feedback control law based on matrix inequalities such that the resulting closed loop system is asymptotically stable with a prescribed level of disturbance attenuation in certain frequency domain.

The rest of this paper is organized as follows: The problem to be solved is formulated mathematically in Section II, and the controller designs using output feedback and state feedback are presented in Section III. Section IV illustrates the usefulness and advantage of the proposed methodology through simulations. Finally, Section V concludes the paper.

2. Problem Formulation

A vehicle active suspension system controller is to be designed such that the body vertical acceleration is reduced. For analysis, we choose a quarter - car model as shown in Fig.1. Here m_s is the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents the mass of vehicle's components that are not supported by the spring (wheel assembly); c_s and k_s are damping and stiffness of the passive suspension system, respectively; c_t and k_t stand for the damping and compressibility of the pneumatic tire, respectively; z_s and z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; u represents the control force. In this brief the effect of actuator dynamics is neglected and is modeled as an ideal force generator.

The ideal dynamic equations of the sprung and unsprung masses are given by [1]

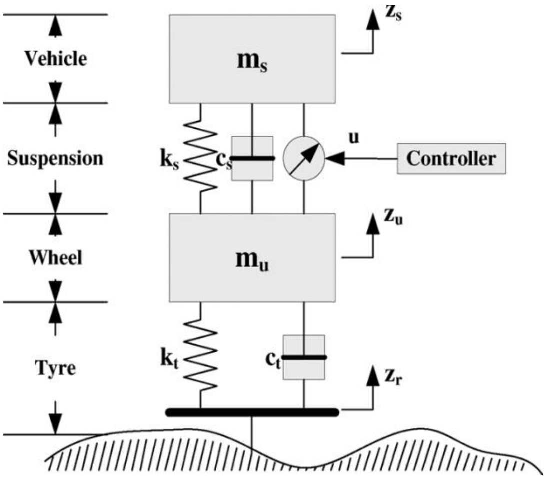

Figure 1: Quarter - car model with an active suspension

$$\begin{aligned} &m_s\ddot{z}_s(t)+c_s[\dot{z}_s(t)-z_u(t)]+k_s[z_s(t)-z_u(t)]=u(t)\\ &m_u\ddot{z}_u(t)+c_s[\dot{z}_u(t)-\dot{z}_s(t)]+k_s[z_u(t)-z_s(t)]\\ &+k_t[z_u(t)-z_r(t)]+c_t[\dot{z}_u(t)-\dot{z}_r(t)]=-u(t) \end{aligned} \quad (1)$$

The state variables are chosen as:

$$x_1(t)=z_s(t)-z_u(t), \qquad x_2(t)=z_u(t)-z_r(t)$$
$$x_3(t)=\dot{z}_s(t), \qquad x_4(t)=\dot{z}_u(t)$$

where x_1 (t) denotes the suspension deflection, x_2 (t) is the tire deflection, x_3 (t) is the sprung mass speed and x_4 (t) denotes the unsprung mass speed. The disturbance input is defined as w (t) = $\dot{z}_r(t)$. Then, by defining $x(t)=[x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T$ and the dynamic equation in (1) can be rewritten in the following state space form:

$$\dot{x}(t)=Ax(t)+B_1w(t)+B_2u(t) \quad (2)$$

where

$$A=\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-k_s}{m_s} & 0 & \frac{-c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & \frac{-k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix}, B_1=\begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, B_2=\begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}$$

The main objective for vehicle suspension systems is the improvement of ride comfort and is closely related to the body acceleration in frequency band 4 - 8 Hz. For this, it is important to keep the transfer function from the disturbance input w (t) to car body acceleration $\ddot{z}_s(t)$ as small as possible over the frequency band 4 - 8 Hz [1].

In order to make sure the car safety, we should ensure the firm uninterrupted contact of wheels to road, and the dynamic tire load should be kept small, that is $k_t(z_u(t)-z_r(t))<(m_s+m_u)g$.

Because of the mechanical structure, the suspension stroke should not exceed the allowable maximum, that is

$|z_s(t)-z_u(t)|<z_{max}$, where z_{max} is the maximum suspension deflection. Another hard constraint imposed on active suspensions is from the limited power of the actuator, that is $u(t)\le u_{max}$.

According to the above conditions, we choose the H_∞ norm as performance measure and the body acceleration $\ddot{z}_s(t)$ as performance output, and suspension stroke and relative dynamic tire load as controlled outputs

$$z_1(t)=\ddot{z}_s(t),\quad z_2(t)=\begin{bmatrix}\frac{z_s(t)-z_u(t)}{z_{max}} & \frac{k_t(z_u(t)-z_r(t))}{(m_s+m_u)g}\end{bmatrix}^T \tag{3}$$

Therefore, the vehicle suspension control system can be described by

$$\begin{aligned}\dot{x}(t)&=Ax(t)+B_1w(t)+B_2u(t)\\ z_1(t)&=C_1x(t)+D_{12}u(t)\\ z_2(t)&=C_2x(t)\end{aligned} \tag{4}$$

Where A, B_1 and B_2 are defined earlier, and

$$C_1=\begin{bmatrix}-\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s}\end{bmatrix}\qquad D_{12}=\frac{1}{m_s}$$

$$C_2=\begin{bmatrix}\frac{1}{z_{max}} & 0 & 0 & 0\\ 0 & \frac{k_t}{(m_s+m_u)g} & 0 & 0\end{bmatrix}$$

Consider $G(j\omega)$ as the transfer function from the disturbance inputs w to the controlled output $z_1(t)$. The finite frequency H_∞ control problem is to design a controller such that the closed - loop system guarantees that the transfer function from $w(t)$ to car body acceleration $\ddot{z}_s(t)$ is as small as possible over 4 to 8 Hz.

In addition, from the safety and mechanical structure point of view, the constraints

$$|u(t)|\le u_{max},\ |\{z_2(t)\}_i|\le 1,\ i=1,2 \tag{5}$$

need to be guaranteed.

3. Controller Design

3.1 Output Feedback Controller Design

For the active suspension system given in (4), an output feedback H_∞ control problem is formulated to find an internally stabilizing controller to minimize the H_∞ norm of the closed loop transfer function from the disturbance input to controlled output $z_1(t)$. That is

$$\sup_{\omega}\|G(j\omega)\|_\infty<\gamma \tag{6}$$

where $\gamma>0$ is a prescribed scalar.

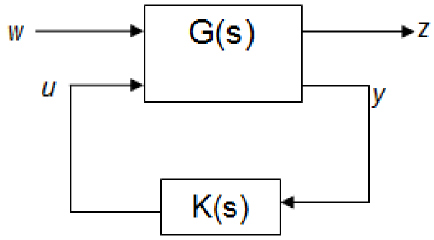

Figure 2: Standard system configuration

Standard feedback configuration using an H_∞ controller is shown in Fig.2. The signals w, u, z and y are vector - valued signals; $w(t)$ contains all exogenous inputs, such as disturbances; $u(t)$ is the control signal generated by the controller $K(s)$; $z(t)$ is the controlled output and $y(t)$ is the measured output used by the controller to generate $u(t)$. It is supposed, that transfer matrices G and K are real rational and proper with K constrained to provide internal stability. Suppose that the realizations G and K are stabilizable and detectable, then a standard suboptimal H_∞ control problem is to find all admissible controllers K (s) such that $\|G(j\omega)\|_\infty<\gamma$.

The output feedback control law is given by

$$u=K(s)y(t) \tag{7}$$

where K (s) is the feedback controller to be designed. Let

$$K(s)=\begin{bmatrix}A_K & B_K\\ C_K & D_K\end{bmatrix}$$

By combining (7) with (4), the closed - loop system is obtained by

$$\begin{aligned}\dot{x}(t)&=\bar{A}x(t)+\bar{B}w(t)\\ z_1(t)&=\bar{C}x(t)+\bar{D}w(t)\\ z_2(t)&=C_2x(t)\end{aligned} \tag{8}$$

where

$$\begin{bmatrix}\bar{A} & \bar{B}\\ \bar{C} & \bar{D}\end{bmatrix}=\begin{bmatrix}A+B_2D_KC_2 & B_2C_K & B_1\\ B_KC_2 & A_K & 0\\ C_1+D_{12}D_KC_2 & D_{12}C_K & 0\end{bmatrix}$$

Using weighted performance specification has certain advantages in system design. First of all, some components of a vector signal are usually more important than others. Secondly, each component of the signal may not be measured in the same matrix. Also we might be interested in rejecting errors in a certain frequency range, hence some frequency dependent weights must be chosen. So the weighting functions allow for frequency information to be incorporated into the design. The weighting functions reflect the frequencies at which different input - output pairs of the transfer function matrix T_{zw} are sought to be minimized. The performance objectives are achieved by minimizing the weighted transfer function.

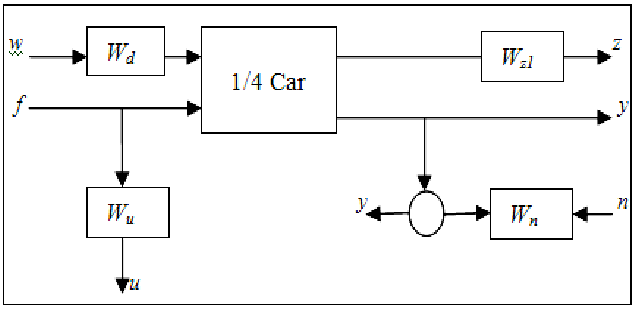

Figure 3: Quarter - car model with weighting functions

A block diagram of the H_∞ control design interconnection for the active suspension problem is shown in Fig.3. The weight W_n serves to model sensor noise in the measurement channel and is chosen as equal to 0.01 for this problem. The weight W_d is chosen to reflect the frequency content of the disturbance *w*. Here it used to scale the magnitude of road disturbances and assume the maximum road disturbance is 7cm/s and hence choose W_d =0.07.

The weighting matrix W_{z1} is used to reflect the requirements on the shape of certain closed loop transfer functions. Here it is used to keep the car body vertical acceleration small over the desired frequency range. The weight magnitude rolls off at low frequency and flattens out at a small at high frequency. Similarly, W_u is used to reflect some restrictions on the control or actuator force. Here the magnitude and frequency content of the control force are limited by the weighting function W_u.

The augmented plant model is obtained by using *sysic* command from Robust Control Toolbox in Matlab. Then the H_∞ controller is synthesized for the weighted plant by using Matlab commands, which minimizes $\|T_{zw}\|_\infty$ under the constraint that *K* (s) internally stabilizes the plant.

3.2 State Feedback Controller Design

In this section, the problem formulated as shown in (4) is solved by using state feedback approach. The main advantage of using state feedback controller is that, we can control the unstable modes of a system by using states which are uncontrollable from the output. In H_∞ state feedback controller, pole placement can be done so as to minimize the H_∞ norm of the system. For the system shown in (4) the following assumptions are made:

1) (A, B_2) is stabilizable;
2) There is a matrix D⊥ such that [D_{12} D⊥] is unitary;
3) $\begin{bmatrix} A - j\omega L & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω. Then a standard suboptimal H_∞ control problem can be formulated to find all admissible controllers K (s) such that $\|G(j\omega)\|_\infty < \gamma$. It is assumed that all the state variables can be measured, and we consider a state feedback case with

$$u(t) = Kx(t) \tag{9}$$

where *K* is state feedback gain to be designed. By combining (9) with (4), the closed - loop system is given by

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + \bar{B}w(t) \\ z_1(t) &= \bar{C}x(t) + \bar{D}w(t) \\ z_2(t) &= C_2 x(t) \end{aligned} \tag{10}$$

where

$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} = \begin{bmatrix} A + B_2 K & B_1 \\ C_1 + D_{12} K & 0 \end{bmatrix}$$

In this section we have to design a controller which can ensure the transfer function from the disturbance input *w* (*t*) to the controlled output z_1 (*t*) as small as possible over the entire frequency range, while the time domain constraints (5) are guaranteed.

The lemmas used in the paper are given below:

Lemma I (KYP Lemma) [4]: Consider the linear system $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$. Given symmetric matrix ∏, and scalar $\omega \in \Re$, then the following statements are equivalent.
(1) The frequency domain inequality

$$\begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix}^T \prod \begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix} < 0 \tag{11}$$

(2) There exist symmetric matrix *P* > 0 such that the LMI

$$\begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix} + \prod < 0 \tag{12}$$

The above lemma gives the design process of the state feedback controller over the entire frequency range.

Let *γ* and *ρ* be given. A state feedback controller in the form of (9) exists, such that the closed loop system in (10) is asymptotically stable with *w* (*t*) =0, and satisfies $\|G(j\omega)\|_\infty < \gamma$ for all nonzero $\omega \in L_2[0, \infty]$, while the constraints in (5) are guaranteed, if there exist symmetric matrices $\bar{P}_1 > 0$, and $\bar{K}$ satisfying

$$\begin{bmatrix} [\bar{P}_1 A^T + \bar{K}^T B_2{}^T]_S & B_1 & \bar{P}_1 C_1{}^T + \bar{K}^T D_{12}{}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{13}$$

Considering *V* (*t*) =x^T (*t*) $P_1 x$ (*t*) as the energy function, and we have

$$x(t)^T P_1 x(t) \le V(0) + \eta w_{\max} = \rho \tag{14}$$

Then consider

$$\max_{t \ge 0} |u(t)|^2 = \max_{t \ge 0} \left\| x(t)^T K^T K x(t \right\|_2 \le \rho . \lambda_{\max} (P_1^{-1/2} K^T K P_1^{-1/2})$$

$$\max_{t \ge 0} \left| \{z_2(t)\}_i \right|^2 = \max_{t \ge 0} \left\| x(t)^T \{C_2\}_i^T \{C_2\}_i x(t) \right\|_2, \; i = 1, 2.$$

$$\le \rho.\lambda_{\max}(P_1^{-1/2}\{C_2\}_i^T\{C_2\}_i P_1^{-1/2}),\, i=1,2$$

where λ_{max} (.) represents the maximum eigenvalue. Then the constraints in (5) hold if

$$\rho P_1^{-1/2} K^T K P_1^{-1/2} < u_{\max}^2 I$$

$$\rho P_1^{-1/2}\{C_2\}_i^T\{C_2\}_i P_1^{-1/2} < I,\; i=1,\,2 \tag{15}$$

By Schur complement inequality (15) can be written as

$$\begin{bmatrix} -I & \sqrt{\rho}\bar{K} \\ * & -u_{\max}^2 \bar{P}_1 \end{bmatrix} \le 0 \tag{16}$$

$$\begin{bmatrix} -I & \sqrt{\rho}\{C_2\}_i \\ * & -\bar{P}_1 \end{bmatrix} < 0 \tag{17}$$

If the inequalities (13), (16) and (17) have a set of feasible solution, the controller gain K is given by

$$K = \bar{K}.\bar{P}_1^{-1}$$

3.3 Finite Frequency Controller Design

In this section the design of a controller which can ensure the transfer function from the disturbance input w (t) to the controlled output z_1 (t) as small as possible over the frequency band 4 - 8 Hz, while the time domain constraints (5) are guaranteed is discussed. In the following, we will investigate how to design a desired controller for the suspension system and we use the following lemma

Lemma II (Generalized KYP Lemma [11]: Consider the linear system $(\bar{A},\bar{B},\bar{C},\bar{D})$. Given symmetric matrix ∏, and scalars $\omega_1,\omega_2 \in \Re$, then the following statements are equivalent.

(1) The finite frequency inequality

$$\begin{bmatrix} (j\omega I - A)^{-1}B \\ I \end{bmatrix}^T \prod \begin{bmatrix} (j\omega I - A)^{-1}B \\ I \end{bmatrix} < 0,\, \omega_1 \le \omega \le \omega_2. \tag{18}$$

(2) There exist symmetric matrixes P and Q satisfying Q>0 and

$$\begin{bmatrix} \Gamma[P,Q.\bar{C},\bar{D}] & [\bar{C} \quad \bar{D}]^T \\ * & -I \end{bmatrix} < 0 \tag{19}$$

where

$$\Gamma[P,Q,\bar{C},\bar{D}] = \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q & P + j\omega_c Q \\ P - j\omega_c Q & -\omega_1\omega_2 Q \end{bmatrix} \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} 0 & \bar{C}^T\Pi_{12} \\ * & [\bar{D}^T\Pi_{12} + \Pi_{22}] \end{bmatrix} \tag{20}$$

where ω_c= (ω_1+ω_2) / 2 and $\prod_{12}$, $\prod_{22}$ are the upper right and lower right block matrices of ∏.

By using GKYP lemma, Projection Lemma [15] and Reciprocal Projection Lemma [15] the finite frequency H_∞ control problem is formulated to minimize the H_∞ norm from the disturbance inputs to the controlled output over the fixed frequency band $\omega_1 \le \omega \le \omega_2$. We can use the following theorems [3] as well, for the design of the controller.

Theorem 1: Let γ, η and ρ be given. A state feedback controller in the form of (9) exists, such that the closed loop system in (10) is asymptotically stable with w (t) =0, and satisfies $\|G(j\omega)\|_\infty^{\omega_1\le\omega\le\omega_2} < \gamma$ for all nonzero $\omega \in L_2[0,\infty]$, while the constraints in (5) are guaranteed with the disturbance energy under the bound w_{max} = (ρ - V (0)) /η, if there exist symmetric matrices P, P_1>0, Q_1>0 and general matrix F satisfying

$$\begin{bmatrix} -[F]_S & F^T\bar{A}+P_1 & F^T & F^T\bar{B} \\ * & -P_1 & 0 & 0 \\ * & * & -P_1 & 0 \\ * & * & * & -\eta I \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} -Q & P + j\omega Q - F & 0 & 0 \\ * & -\omega_1\omega_2 Q + [F^T\bar{A}]_S & F^T\bar{B} & \bar{C}^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -I & \sqrt{\rho}K \\ * & -u_{\max}^2 P_1 \end{bmatrix} \le 0 \tag{24}$$

$$\begin{bmatrix} -I & \sqrt{\rho}\{C_2\}_i \\ * & -P_1 \end{bmatrix} < 0 \tag{25}$$

where ω_c= (ω_1+ω_2) / 2 is given.

The constraints given in (22) and (23) involve the forms of FBK, and then the resulting feasibility problem is nonlinear. LMI optimization technique cannot handle nonlinear problems. By congruence transformation with J_1=diag{F^{-1}, F^{-1}, F^{-1}, I}, J_2=diag{F^{-1}, F^{-1}, I, I}, J_3=diag{I, F^{-1}}, we can convert (22) - (25) to linear inequalities.

Defining

$$\bar{Q} = (F^{-1})^T Q F^{-1},\;\; \bar{P} = (F^{-1})^T P F^{-1},$$

$$\bar{P}_1 = (F^{-1})^T P_1 F^{-1},\;\; \bar{K} = K F^{-1},\;\; \bar{F} = F^{-1}$$

the following theorem is obtained.

Theorem 2: Let γ, η and ρ be given. A state feedback controller in the form of (9) exists, such that the closed loop system in (10) is asymptotically stable with w (t) =0, and satisfies $\|G(j\omega)\|_\infty^{\omega_1\le\omega\le\omega_2} < \gamma$ for all nonzero $\omega \in L_2[0,\infty]$, while the constraints in (5) are guaranteed with the disturbance energy under the bound w_{max} = (ρ - V (0)) /η, if there exist symmetric matrices $\bar{P}, \bar{P}_1 > 0, \bar{Q} > 0$ and general matrix $\bar{F}$ satisfying

$$\begin{bmatrix} -[\bar{F}]_S & A\bar{F} + B_2\bar{K} + \bar{P}_1 & \bar{F} & B_1 \\ * & -\bar{P}_1 & 0 & 0 \\ * & * & -\bar{P}_1 & 0 \\ * & * & * & -\eta I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} -\bar{Q} & \bar{P}+j\omega_c\bar{Q}-\bar{F} & 0 & 0 \\ * & -\omega_1\omega_2 Q+[A\bar{F}+B_2\bar{K}]_S & B_1 & \bar{F}^T C_1^T+\bar{K}^T D_1^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} -I & \sqrt{\rho}\bar{K} \\ * & -u_{\max 2}^2 \bar{P}_1 \end{bmatrix} \le 0 \quad (28)$$

$$\begin{bmatrix} -I & \sqrt{\rho}\{C_2\}_i \bar{F} \\ * & -\bar{P}_1 \end{bmatrix} < 0,\ i=1,2 \quad (29)$$

If the inequalities (26) - (29) have a set of feasible solutions, the controller gain K is given by

$$K=\bar{K}.\bar{F}^{-1} \quad (30)$$

4. Simulation Results

In this section, we provide an example to illustrate the effectiveness of the finite frequency H_∞ controller design method. The quarter - car model parameters are listed in Table I. The output feedback H_∞ controller is designed with weighting functions selected as shown below

$$W_n = 0.01, \qquad W_d = 0.07$$

$$W_{z1} = 8\frac{(2\pi * 0.5)}{s+2\pi * 5} \quad W_u = \frac{1}{13}\frac{(s+5)}{(s+500)}$$

Then the H_∞ controller for the weighted system is obtained by using Robust Control Toolbox in Matlab. The controller K (s) is obtained as

$$A_K = \begin{bmatrix} 0 & -0.256 & 1 & -1 & 0 & -0.0568 \\ 0 & -388.2 & 0 & 1 & 0 & 8.97*e-07 \\ -47.93 & -410 & -3.286 & 3.254 & 0.1644 & -0.1456 \\ 383.4 & -1856 & 26.24 & -26.29 & -1.315 & 3{,}298 \\ 21310 & 83320 & -365.8 & 331.3 & -79.25 & -925.9 \\ 766.8 & 473.7 & -52.29 & 52.09 & 2.63 & 35.21 \end{bmatrix}$$

$$B_K = \begin{bmatrix} 0.03341 \\ 50.55 \\ 57.63 \\ -443.3 \\ 0 \\ 6.131 \end{bmatrix}, \quad C_K = [360.8 \quad 1411 \quad -6.193 \quad 5.61 \quad 7.124 \quad -15.68]$$

$$D_K = [0]$$

Table I: Quarter - Car Model Parameters

m_s	m_u	k_s	k_t	c_s	c_t
320kg	40kg	18kN/m	200kN/m	1kNs/m	10Ns/m

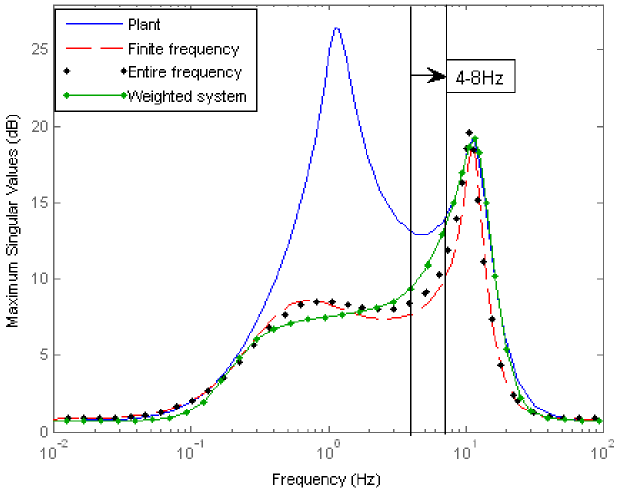

Figure 4: Frequency response of body vertical acceleration

The closed - loop performance of the suspension system is analyzed using this controller and this controller is denoted as Controller I.

Then the H_∞ state feedback controller over the entire frequency range is designed based on the assumption that all the state variables can be measured. Solve the matrix inequalities (13), (16) and (17) using the LMI Control Toolbox in Matlab with zero initial conditions for matrices $\bar{P}_1 > 0$ and $\bar{K}$ with the optimized parameter γ > 0. Constraints are given using the command *lmiterm* and finally the controller is synthesized using the command *feasp*. Then we get the feasible values of the LMI variables and the controller gain $K_E = \bar{K}.\bar{P}_1^{-1}$ is obtained as

$$K_E = 10^4 \times [1.29 \quad 0.536 \quad -0.093 \quad -0.04]$$

and we denote this controller as Controller II for brevity.

A state feedback H_∞ controller in the finite frequency domain for the system (4) is designed based on the assumption that all the state variables can be measured. Solve the matrix inequalities (26) - (29) using the LMI Toolbox in Matlab with zero initial conditions for matrices $\bar{P}, \bar{P}_1 > 0$ and $\bar{Q} > 0$ with the optimized parameter $\gamma > 0$ and ω_1=4 Hz, ω_2=8 Hz, ρ=0.9, η=10, 000, z_{max}=100 mm, u_{max}=2500 By solving the convex optimization problem formulated in the above section, the minimum guaranteed closed - loop H_∞ performance obtained is γ=2.46. Then an admissible control gain matrix is

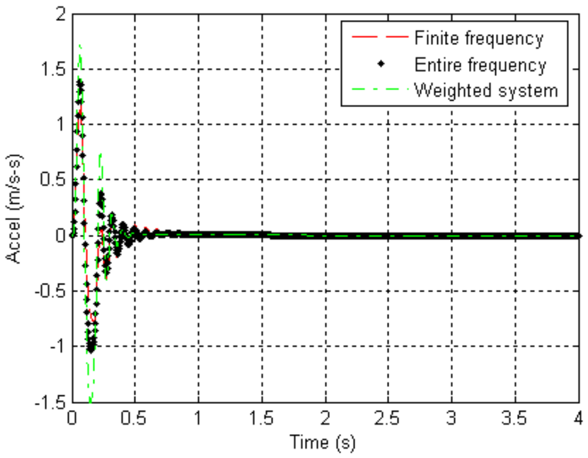

Figure 5: Time - domain response of body vertical acceleration

$$K_F = 10^4 \times [0.905 \quad -0.543 \quad -0.205 \quad -0.6114]$$

we denote this finite frequency controller as Controller III.

In the following, we will illustrate the performance of the closed - loop suspension system. By the simulation, the frequency response of the open - loop system, the closed - loop system with Controller I, Controller II, and Controller III are compared in Fig.4. The dashed, dotted, and dash - dotted lines are the frequency responses of the suspension system with finite frequency controller, entire frequency controller, and output feedback controller respectively, and the solid line is the response of the passive system. From Fig.4, we can see that the finite frequency controller yields the least value of H_∞ norm over the frequency range 4 - 8 Hz, compared with the others, which clearly shows that improved ride comfort has been achieved.

Time - domain performance characteristics are critical to the success of the active suspension systems. Time response plots of the three H_∞ controllers are shown in following figures. The dash - dotted, dotted and dashed lines correspond to the suspension system with, H_∞ Controller I, Controller II and Controller III respectively. All responses correspond to the road disturbance $w(t)$:

$$w(t) = \begin{cases} A\sin(2\pi f t), & \text{if } 0 \le t \le T \\ 0, & \text{if } t > T \end{cases} \tag{29}$$

where A = 0.5m, f = 5 Hz, and T=0.2 s. The time - domain response of body vertical acceleration for the active suspension system is shown in Fig.4 and 5. We can clearly see that the vertical body acceleration is less for the system with finite frequency controller. Fig.5 shows the ratio $x_1(t)/z_{max}$, the relation dynamic tire load $k_t.\ x_2(t) / z_{max}$, and the force of the actuator. From the figure, it is found that the actuator force is less than the maximum bound u_{max}, which means the time domain constraints are guaranteed by the Controller III.

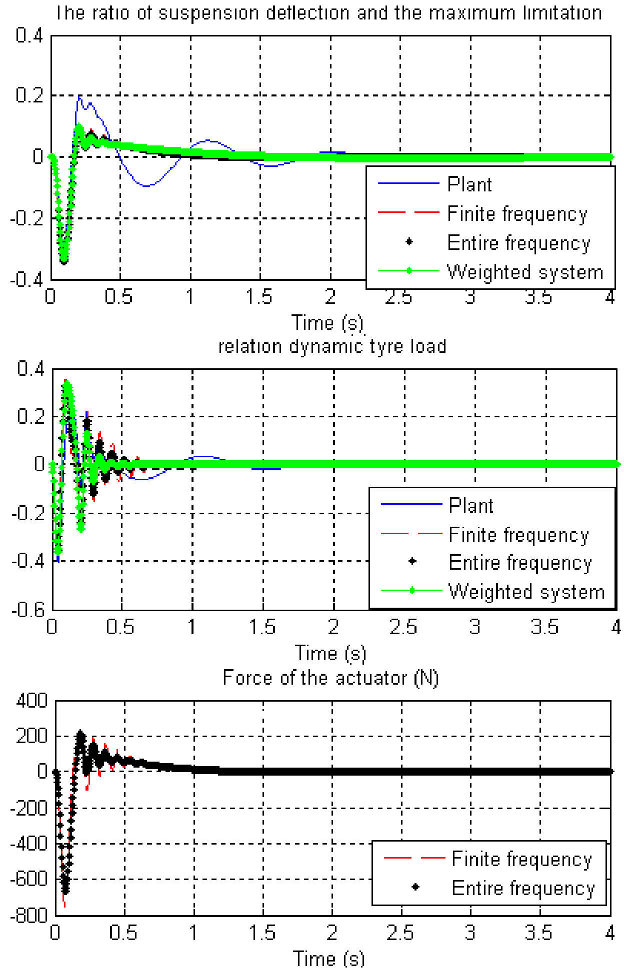

Figure 6: Time - domain response of constraints for active suspension system.

From Fig.5 it is found that the response of the finite frequency controller is better than the response of the system with weighting functions. The body vertical acceleration is also gets reduced in finite frequency system. The time domain characteristics such as dynamic tire load and suspension deflection are quite satisfactory with Controller I. The closed - loop system with this controller also gives satisfactory result and the method is effective. But, the choice of weighting function is quite time consuming, especially when the designer has to shoot for a good tradeoff between the complexity of weights and the accuracy in capturing desired specifications.

From Fig.6, we found that larger actuator forces are needed for finite frequency controller than the entire frequency controller. The reason is that the finite frequency control requires more force to match the finite frequency features. All other time domain constraints are satisfied by both systems.

A state feedback H_∞ control based on generalized KYP lemma is a more reliable and convenient method to deal with problems in the finite frequency domain. This method avoids the usage of weighing functions. The improvement in ride comfort has been achieved with the finite frequency controller.

5. Conclusions

This paper investigates the problem of finite frequency H_∞ control with time domain constraints for active vehicle suspension systems using weighting function method and in the frame work of LMI optimization. By using both the methods we can reduce the body vertical acceleration in the 4 - 8 Hz domain. Using Generalized KYP lemma, the ride comfort has been improved by minimizing the H_∞ norm in specific frequency band, and the time domain constraints have been guaranteed. The effectiveness of this approach has been shown by the analysis and simulation of a given quarter car model. The finite frequency H_∞ controller design can be done by considering the actuator dynamics as a future work.

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