The Consequence of Thermal Diffusion on an MHD Flow with Free Convection Past an Upright Plate in Connection with Ramped Wall Temperature and Concentration

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Abstract: ***The present study attempts to investigate the influence of thermal diffusion with a special reference to ramped wall temperature and concentration in an unsteady MHD heat and mass transfer flow are studied parametrically. The MHD boundary layer equations are solved by adopting Laplace Transform technique. The impact of many relevant physical parameters is graphically illustrated and discussed.***

Keywords: MHD, Thermal diffusion, Schmdit number, Laplace Transform Technique

1. Introduction

Many natural phenomena and engineering issues are prone to MHD analysis. Geophysics encounters MHD characteristics within the interaction of conducting and magnetic fields. MHD is that the science of motion of electrically conducting fluid in presence of magnetic field. There are varied samples of application of MHD principles, as well as MHD generators, MHD pumps and MHD flow meters etc. The generator and motor could be a classical example of MHD principle. MHD principles additionally notice its application in drugs and biology. Model studies on MHD free and forced made convection with heat and mass transfer issues are applied by several authors because of their application in several branches of science and technology. a number of them are Ahmed [1], Elbashbeshy [2] and Singh and Singh [3]. Gregantopoulos et al. [4] studied two-dimensional unsteady free convection Associate in mass transfer flow of incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate. Several investigators have studied the impact of reaction in several convective heat and mass transfer flows of whom Apelblat [5] and Anderson *et.al* [6] are price mentioning.

The instability of mass-produced by a change in temperature defines the Soret Effect. Charles Soret in 1879, was the first scientist who executed the investigational study of this effect on mass transfer associated problems and that is why the effect was named after the scientist as a tribute of respect for his work. Uwanta et al. [7] examined MHD fluid flow over a vertical plate with Soret and Dufour effects. Reddy et al. [8] analyzed viscous dissipation, Soret and Dufour effects on free convection flow from a porous vertical surface. Reddy [9] examined the effects of Soret and Dufour on free convective MHD flow over a vertical porous plate provided heat generation present.

In this research, Laplace Transform technique is developed to solve the governing equations. The major goal of this work is to use the above mentioned method to explore the effects of thermal diffusion on MHD fluid flow. The basic idea of the present work is developed by considering the influence of thermal diffusion as the generalization of the work of Mahanta M and Sinha S [10].

2. Mathematical formulation of the problem

The objective of present study is to investigate the influence of thermal diffusion with a special reference to ramped wall temperature and concentration in an unsteady MHD heat and mass transfer flow. A co-ordinate system is introduced, where X-axis is considered along vertical direction of the wall and Y-axis is considered along the normal to the wall as shown **in Figure 1**.

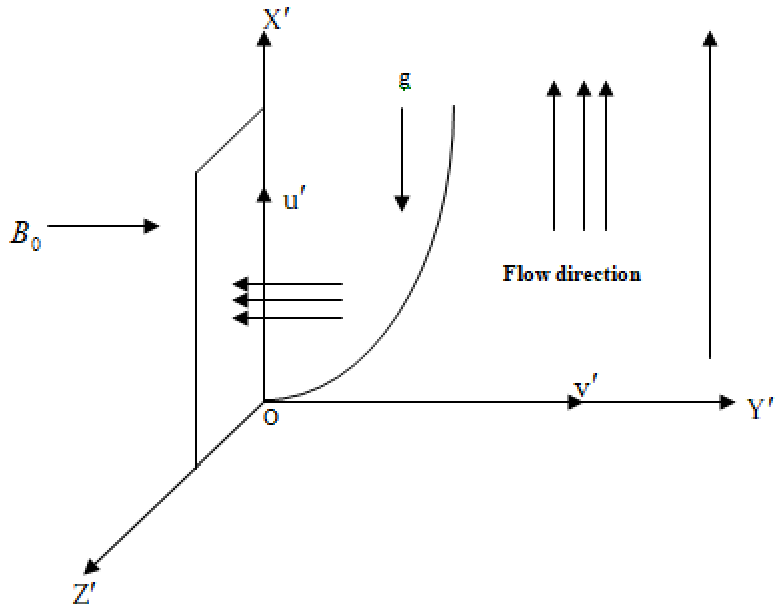

Figure 1: Physical model of the problem

Mahanta M and Sinha S [10] proposed some standard assumptions, based on which the following governing equations defining physical circumstances are evaluated.

1) **Momentum Equation:**

$$\frac{\partial u}{\partial t'} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta^*(T - T_\infty) - \frac{\upsilon}{K^*}u - \frac{\sigma B_0^2 u}{\rho} \quad (1)$$

2) **Energy Equation:**

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (2)$$

3) **Concentration Equation:**

$$\frac{\partial C}{\partial t'} = D\frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} \quad (3)$$

Initial and boundary conditions for velocity, temperature and concentration fields are:

$$y \ge 0: \quad u = 0,\ T = T_\infty,\ C = C_\infty \quad for\ t' \le 0 \quad (4.1)$$

$$y = 0: \quad u = U_0, \quad for\ t' > 0$$

$$T = T_\infty + (T_w - T_\infty)\frac{t'}{t_0},$$

$$C = C_\infty + (C_w - C_\infty) \quad for \quad 0 < t' \le t_0$$

$$T = T_w,\ C = C_w \quad for\ t' > t_0 \quad (4.2)$$

$$y \to \infty: \ u \to 0, T \to T_\infty,\ C = C_\infty \quad for\ t' > 0 \quad (4.3)$$

Radiative heat flux is

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda \quad (5)$$

Where K_{λ_0} is the absorption co-efficient, λ is wave length, $e_{\lambda h}$ denotes Planck's function. Subscript 0 means that all physical quantities have been found out at temperature T_∞. Applying the equation (5) in equation (2), the equation (2) is transformed to as follows:

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{4}{\rho C_p}(T - T_\infty)\, I \quad (6)$$

$$Where,\ I = \int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\lambda T}\right)_0 d\lambda$$

To normalize dimensional governing equations, introduced following non-dimensional

Variables and parameters:

$$\eta = \frac{y}{U_0 t_0},\ t = \frac{t'}{t_0},\ u_1 = \frac{u}{U_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$M = \frac{\sigma B_0^2 t_0}{\rho},\ Gr = \frac{g\beta^* \upsilon (T - T_\infty)}{U_0^3},\ Gm = \frac{g\beta' \upsilon (C - C_\infty)}{U_0^3},\ \Pr = \frac{\upsilon \rho C_p}{k},$$

$$K=\frac{K^{*}U_0^2}{\upsilon^2},\ t_0=\frac{\upsilon}{U_0^2},\ Sc=\frac{\upsilon}{D},\ Sr=\frac{D_T\left(T_w-T_\infty\right)}{\upsilon\left(C_w-C_\infty\right)} \quad (7)$$

Using equation (7) in the equations (1), (3) and (6), these equations are transformed to non- dimensional form as:

$$\frac{\partial u_1}{\partial t}=\frac{\partial^2 u_1}{\partial \eta^2}+Gr\,\theta+Gm\,\phi-\frac{u_1}{K}-M\,u_1 \quad (8)$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2}-Ra\,\theta,$$

Where, (9)

$$Ra=\frac{4\upsilon I}{\rho Cp U_0^2},\ I=\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \theta}{\partial \eta^2}+Sr\frac{\partial^2 \theta}{\partial \eta^2} \quad (10)$$

By applying equation (7) in the boundary conditions defined by equations (4.1), (4.2) and (4.3), the following equations of non-dimensional boundary conditions are obtained:

$$\eta=0:\ u_1=0,\ \theta=0,\ \phi=0,\ for\ t\le 0 \quad (11.1)$$

$$\eta=0:\ u_1=1,\ for\ t>0$$

$$\theta=t,\ \phi=t,\ for\ 0<t\le 1,$$

$$\theta=1,\ \phi=1,\ \text{for } t>1, \quad (11.2)$$

$$\eta\to\infty:\ u_1\to 0,\ \theta\to 0,\ \phi\to 0\ for\ t>0 \quad (11.3)$$

Method of Solution

Applying Laplace Transform technique in the equation (8) to (10), solutions are written in following way:

$$\frac{d^2\bar{u}_1}{d\eta^2}-(s+M_1)\bar{u}_1=-Gr\,\bar{\theta}-Gm\,\bar{\phi}\ ,\ Where\ M_1=\frac{1+MK}{K}$$

(12)

$$\frac{d^2\bar{\theta}}{d\eta^2}-\Pr\bar{\theta}\left(s+M_1\right)=0$$

(13)

$$\frac{d^2\bar{\phi}}{d\eta^2}-Sc\ s\ \bar{\phi}=-Sc\,Sr\frac{d^2\bar{\theta}}{d\eta^2} \quad (14)$$

The boundary condition equations (11.1) to (11.3) are also transformed to equation no. (15) by using Laplace Transform technique as:

$$\bar{u}_1=\frac{1}{s},\ \bar{\theta}=\frac{1}{s^2}\left(1-e^{-s}\right),\ \bar{\phi}=\frac{1}{s^2}\left(1-e^{-s}\right)\ at\ \eta=0$$

$$\bar{u}_1\to 0,\ \bar{\theta}\to 0,\ \bar{\phi}\to 0\ \ as\ \ \eta\to\infty \quad (15)$$

Applying Laplace transform technique in the equations (12) to (14) considering the boundary conditions (15), the solutions of the problem are obtained as:

$$u_1(\eta,t)=A_1+X_1A_{35}+X_2A_{36}$$

$$\theta(\eta,t)=\psi_1-\psi_2$$

$$\phi(\eta,t)=\psi_3-\psi_4-L\psi_5-M(\psi_6-\psi_7)+M(\psi_8-\psi_9)$$
$$+L(\psi_1-\psi_2)+M(\psi_{10}-\psi_{11})-M(\psi_{12}-\psi_{13})$$

Skin-friction co-efficient

Skin-friction co-efficient is denoted by τ

$$\tau=\left[\frac{\partial u_1}{\partial \eta}\right]_{\eta=0}=A_{21}+X_{21}A_{55}+X_{22}A_{56}$$

$$\tau=\psi_1+p_1\psi_{19}+p_2\psi_{33}$$

Nusselt Number

Co-efficient of rate of heat transfer in terms of Nusselt number (Nu) is given by :

$$Nu=\left[\frac{\partial \theta}{\partial \eta}\right]_{\eta=0}=\xi_1-\xi_2$$

Sherwood number

The co-efficient of rate of mass transfer in terms of Sherwood number is as follows:

$$Sh=\left[\frac{\partial \phi}{\partial \eta}\right]_{\eta=0}=\xi_3-\xi_4-L\,\xi_5-M\,\xi_6+M\,\xi_7$$
$$+L(\xi_1-\xi_2)+M\,\xi_{10}-M\,\xi_{11}$$

The constants and the functions are not shown here due to shake of brevity.

3. Results and discussions

Mathematical computations from the investigative solutions for the representative velocity field, temperature field, concentration field, skin friction, the rate of heat transfer in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number are made to get the substantial approaching in to the problem. Various graphs of the fluid flow distribution have been carried out against different physical parameters viz Soret number (Sr), Magnetic parameter (M) and Schmdit number (Sc) involved in the problem.

An effort has been made to illustrate the behavior of velocity distribution versus η under the influence of magnetic parameter M and Soret number Sr as shown in **figures 2-3**. From figure 2, it is observed that magnetic parameter tends to reduce the fluid velocity. From this observable fact it is clear that high magnetic intensity obligated the fluid motion to slow down. i.e the fluid motion is resisted by the strength of the applied magnetic field. The accelerated behavior of the velocity field against thermal diffusion is presented in figure 3. In this figure, it is found that the high temperature gradient made the fluid velocity to rise.

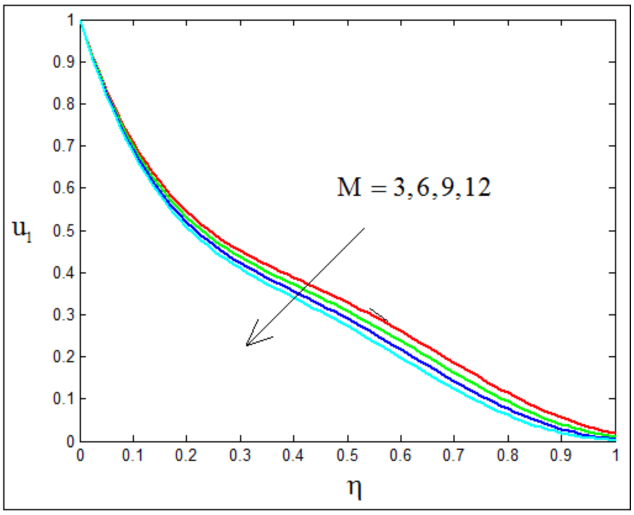

Figure 2: Velocity u versus η for K=0.04, Ra=2, Sr=2, Pr=0.71, Gr=25, Gm=25, Sc=0.60, t=0.5

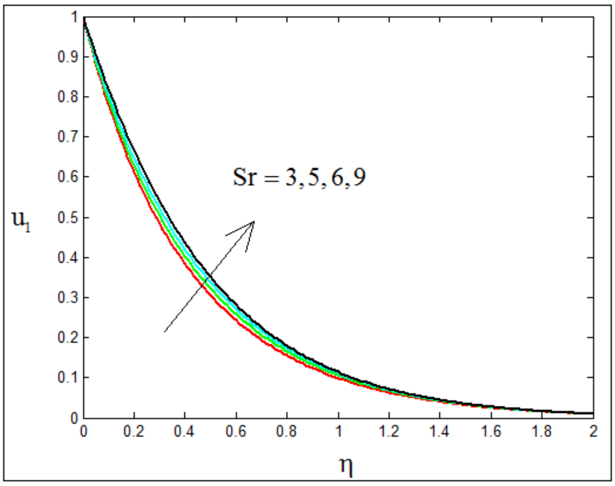

Figure 3: Velocity u versus η for K=0.04, Ra=2, Pr=0.71, Gr=25, Gm=25, M=5, Sc=0.60, t=0.5

The effects of Schmidt number and thermal diffusion on fluid concentration is demonstrated in **figures 4-5.** Figure 4 presents the variation of the species concentration under the influence of Schmdit number. The figure predicts that the species concentration decreases to zero in the infinite direction. This phenomenon physically states that for enlarging the mass diffusivity of the flow, the species concentration gets mounted up. Figure 5 shows that the concentration level of the fluid drops down due to thermal diffusion. i.e the diffusion due to temperature difference made the species to minimize.

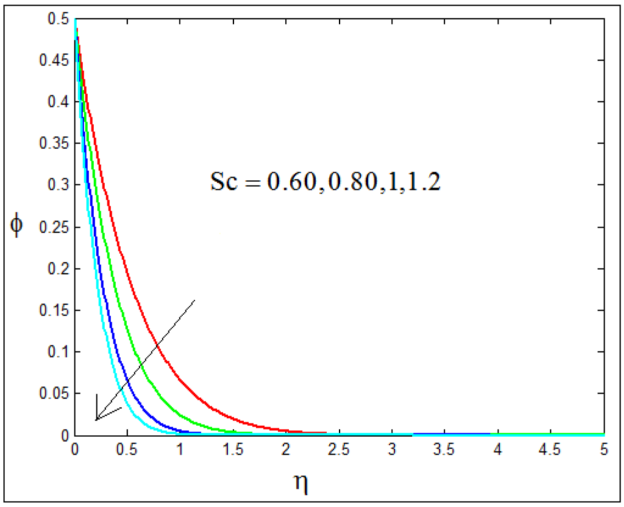

Figure 4: Concentration ϕ versus η for Sr=2, t=0.5

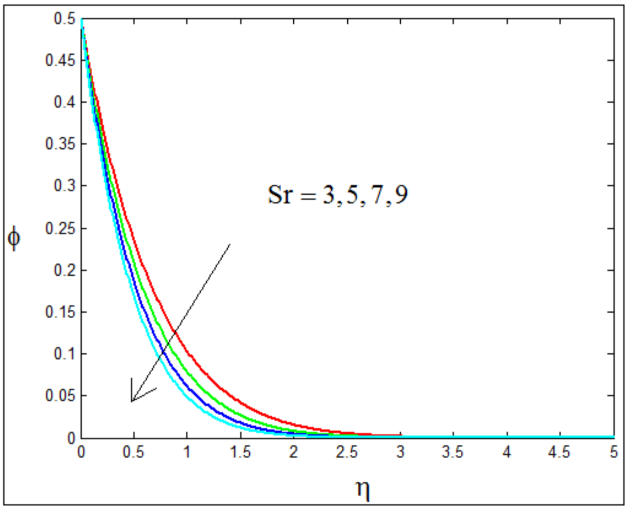

Figure 5: Concentration ϕ versus η for Sc=0.60, t=0.5

In the **figures 6-7,** the co-efficient of Skin-friction τ against t under the action of magnetic parameter and Soret number is depicted. It is explained from these two figures that high magnetic intensity tends to maximize the co-efficient of Skin-friction while the viscous drag from the plate to the fluid gets reduced on account of thermal diffusion.

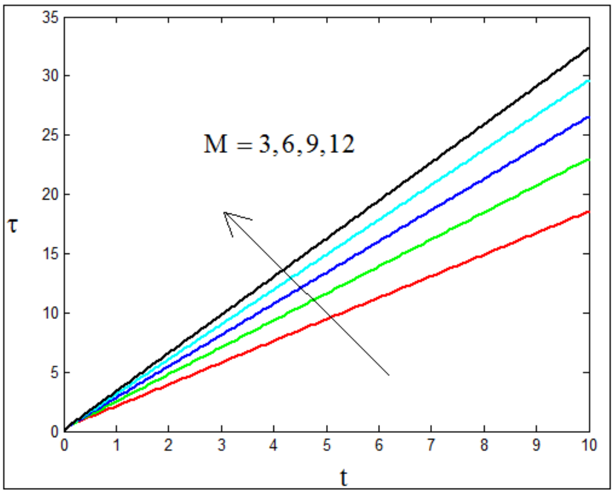

Figure 6: Skin friction τ versus t for K=0.04, Ra=2, Sr=2, Pr=0.71, Gr=25, Gm=25, Sc=0.60

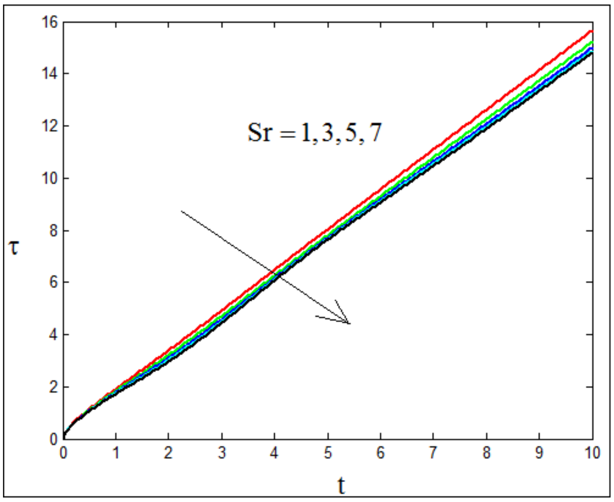

Figure 7: Skin friction τ versus t for K=0.04, Ra=2, Sr=2, Pr=0.71, Gr=25, Gm=25, Sc=0.60

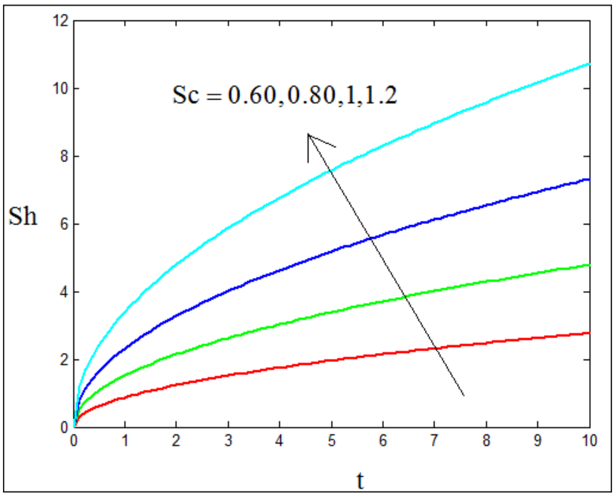

Figure 8: Sherwood number Sh versus t for Sr=2

The co-efficient of rate of mass transfer in terms of Sherwood number from the plate to the fluid under the effect of Schmdit number and thermal diffusion have been illustrated in **figure 8-9.** Figure 8 demonstrates that high mass diffusivity has made an increase in Sherwood number. It suggests that the Sherwood number accelerates for any increasing value of Sc, i.e. mass flux from the plate to the fluid gets accelerated under the influence of mass diffusivity. Consequence of thermal diffusion on Sherwood number is established in figure 9. From this figure, it is evident that thermal diffusion made the mass flux to accelerate.

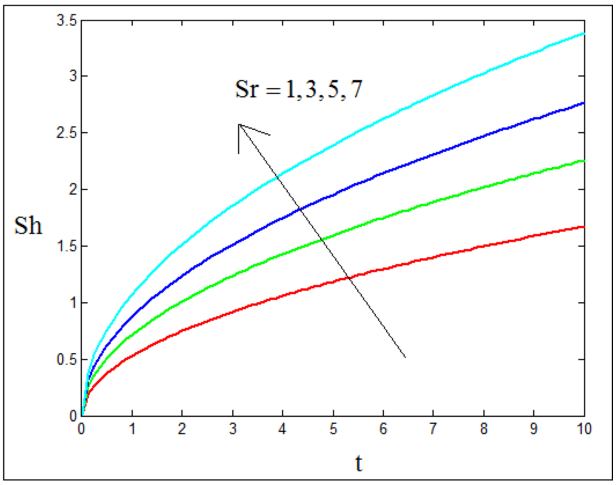

Figure 9: Sherwood number Sh versus t for Sc=0.60

<u>Comparison with Mahanta M and Sinha S [10]</u>

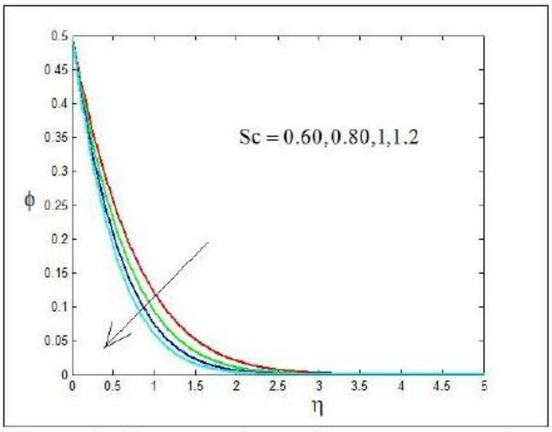

Figure 6: Concentration ϕ versus η under t=0.5

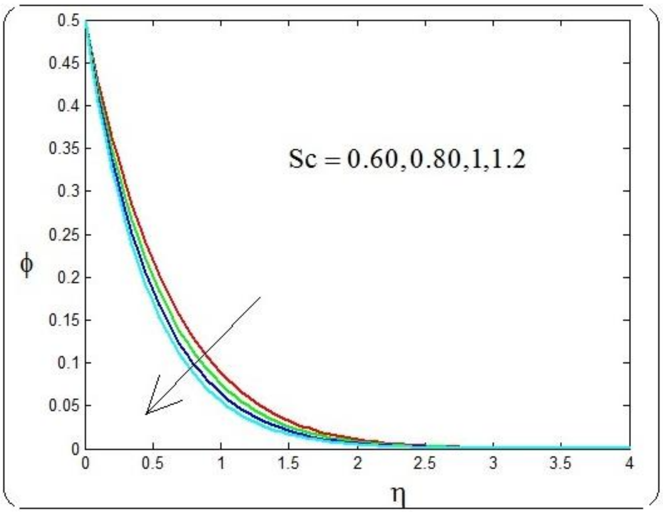

Figure 10: Concentration versus η for Sr=0, t=0.5

For comparing the results of the present work, the results of Mahanta M and Sinha S [10] are used. Comparing figure 10 with figure 6 of Mahanta M and Sinha S [10], it is observed that the same kind of behavior is occurred due to the implementation of Soret number in concentration profile. The concentration distribution is almost similar making an admirable fact with the findings investigated by Mahanta M and Sinha S [10] and the present authors.

4. Conclusion

1) The fluid motion is resisted by the strength of the applied magnetic field and the high temperature gradient made the fluid velocity to rise.
2) The enlargement of mass diffusivity of the flow mounted up the species concentration and the diffusion due to temperature difference made the species to minimize.
3) High magnetic intensity tends to maximize the co-efficient of Skin-friction while the viscous drag from the plate to the fluid gets reduced on account of thermal diffusion.
4) High mass diffusivity Sc is found to enhance the mass flux and thermal diffusion made the mass flux to accelerate.

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