An Inventory Model under Influence of Money Inflation, Time Dependent Holding Cost and Multivariate Demand

Teekam Chand Meena[1], Anil Kumar Sharma[2], Subash Yadav[3]

[1]Department of Mathematics, Government College, Rajgarh, Alwar, Rajasthan- 301408, India

[2]Principal Government Girls College Tapukara, Alwar, Rajasthan-301001 India

[3]Asistant Director College Education Department Rajasthan, Jaipur-302017 India

Email: *teekamchand5790[at]gmail.com,*
sharma_ak002[at]yahoo.com
yadavsubh67[at]gmail.com

Abstract: ***In this manuscript, we have tried to develop an inventory model in which demand is deterministic and dependent on multiple variables like price and stock dependent. Due to decline in purchasing power of money, the concept of inflation is also considered. Deterioration rate is constant. Holding cost is time dependent. Shortages are allowed and partially backlogged.***

Keywords: Multivariate demand, money inflation, partial backlogging.

1. Introduction

The standard EOQ models assumed a constant and known demand rate over an infinite planning horizon. But the demand rate is deterministic and can be multivariate, i.e., for different goods, it may be time dependent, stock dependent and price dependent. So these factors should also be considered for developing various inventory models.

It has been observed in supermarkets that the demand rate is usually influenced by the amount of the stock-level, that is, the demand rate may go up or down with the on-hand stock-level. As pointed out by Levin et al [11], the presence of inventory has a motivational effect on the people around it and attracts the people to buy more.

A good number of authors have studied problems connected with inventories of stock-level dependent demand rate. Gupta and Vrat [9] discussed the models in which demand rate has been assumed to depend upon the order quantity. Datta and Pal [7] extended the model to one in which the demand rate depends upon inventory level down to a certain stock level and then becomes constant. Meena and Sharma [17, 18] developed an inventory model with multivariate demand and also developed model with demand is a function of selling price and quadratic time holding cost .Yadav R. et [16] developed an economic production quantity model with selling price dependent demand under inflation. Smita Y. et [15] developed an inventory model for deteriorating items with selling price dependent demand and variable deterioration under inflation with holding cost is time dependent.

Selling price of an item is one of the decisive factors for selecting the item for use. It is well known that lesser the price of an item, greater is the demand of that item whereas higher selling price has the reverse effects. Whitin [13] developed a deterministic inventory model by incorporating the effect of selling price on demand. Abad [1], Arcelus and Srinivasan [2] followed the suit and developed inventory model considering dependence on pricing and lot size.

In classical inventory models, many researchers ignored deterioration in their models and assumed the demand rates to be constant. But decay or deterioration is a very realistic feature for most of the physical goods. Generally, physical goods are subject to deterioration over time in stock. This encouraged the scientists to include these factors into consideration while developing new inventory models. Deterioration is defined as decay, spoilage, loss of utility of products etc. The process of deterioration is observed in volatile liquids, beverages, medicines, blood components, sweets, fruits and vegetables etc., that result in decrease of usefulness of the original one. Furthermore, when the shortages occur, it is assumed to be either completely backlogged or completely lost. But practically some customers are willing to wait for backorders and some would turn to buy from other sellers. Park [12], Hollier and Mak [10]; Chang and Dye [5] developed inventory models with partial backorders. Goyal and giri [8] developed production inventory model with shortages which were partially backlogged.

Inflation is a concept closely related to time. Inflation is generally associated with rapidly rising prices which causes or are caused by a decline in the purchasing power of money which varies or rather depends upon time. In recent years, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. The investigation of inflation that took off with Buzacott [4], Datta and Pal [6], and Bose et al [3] further extended the concept of inflation.

Recently, Wu et al [14] developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. We have developed an inventory model with multivariate demand i.e., demand is dependent on price and stock level. The concept of time value of money on demand rate is also considered. Numerical examples are presented to demonstrate the developed model and to illustrate the procedure.

Assumptions and Notations:

1) The demand rate D(t) at time 't' is
$$D(t) = \{\mu - \nu s + \xi q(t)\}$$
Where μ, ν and ξ are positive constants.
2) The deteriorating rate is constant $(0 < \theta \le 1)$ and is proportional to on hand inventory.
3) The replenishment occurs instantaneously at an infinite rate.
4) Lead time is zero.
5) Shortages are allowed and partially backlogged.
6) During time t_1, the inventory has no shortage, $t_1^{'}$ is the length of time in which product has no deterioration.
7) Q is the order quantity per cycle and T is the length of order cycle.
8) There is no repair or replenishment of deteriorated units.
9) C_0 denotes the ordering cost per order, C_h is the inventory holding cost coefficient per unit time, C_d is deteriorating cost per unit, C_s is the shortage cost for backlogged items and π is the unit cost of lost sales. All the cost parameters are positive constants.
10) $q_1(t)$ denotes the inventory level at time t $(0 \le t \le t_1^{'})$, in which the product has no deterioration, $q_2(t)$ is the inventory level at time t $(t_1^{'} \le t \le t_1)$, in which the product has deterioration. $q_3(t)$ denotes the inventory level at time t $(t_1 \le t \le T)$, in which the product has shortage.
11) 'r' is a constant representing the difference between the discount rate and inflation rate.
12) The model has been developed for a finite planning horizon.

2. Mathematical Model

The differential equations describing the instantaneous states of q(t) in the interval (0, T) are given by

$$\frac{dq_1(t)}{dt} = -(\mu - \nu s + \xi q_1(t)); \quad \ldots(1)$$

$$\frac{dq_2(t)}{dt} + \theta q_2(t) = -(\mu - \nu s + \xi q_2(t)); \quad (t_1^{'} \le t \le t_1), \quad (2)$$

$$\frac{dq_3(t)}{dt} = -\mu e^{-\epsilon(T-t)}; \quad (t_1 \le t \le T), \quad \ldots \quad (3)$$

The solutions of the above differential equations after applying the boundary conditions $q_1(0) = q_m$; $q_2(t_1) = 0$; $q_3(t_1) = 0$ are

$$q_1(t) = e^{-\xi t} q_m - \frac{(\mu-\nu s)}{\xi}\left[1 - e^{-\xi t}\right]; \quad (4)$$

$$q_2(t) = \frac{(\mu-\nu s)}{(\theta+\xi)}\left[e^{(\theta+\xi)(t_1-t)} - 1\right]; \quad (t_1^{'} \le t \le t_1), \quad (5)$$

$$q_3(t) = \frac{(\mu-\nu s)}{\epsilon}\left[e^{-\epsilon(T-t_1)} - e^{-\epsilon(T-t)}\right]; \quad (t_1 \le t \le T), \quad (6)$$

Taking into consideration the continuity of q(t) at t= t_1, it follows that $q_1(t_1^{'}) = q_2(t_1^{'})$ which implies that the maximum inventory level for each cycle is

$$q_m = \frac{(\mu-\nu s)}{(\theta+\xi)}\left[e^{(\theta+\xi)(t_1-t_1^{'})} - 1\right]e^{\xi t_1^{'}} + \frac{(\mu-\nu s)}{\xi}\left[e^{\xi t_1^{'}} - 1\right] \ldots(7)$$

Substituting equation (7) in (4), we get

$q_1(t) = \frac{(\mu-\nu s)}{\xi}\left[e^{\xi(t_1^{'}-t)} - 1\right] + \frac{(\mu-\nu s)}{(\theta+\xi)}\left[e^{(\theta+\xi)(t_1-t_1^{'})} -\right.$
1eξ(t1′−t)

…(8)

The maximum amount of demand backlogged per cycle is given by

$$q_b = -q_3(T) = \frac{(\mu-\nu s)}{\epsilon}\left[1 - e^{-\epsilon(T-t_1)}\right] \quad \ldots(9)$$

Using equation (7) and (9), we have order quantity Q, as

$$Q = q_m + q_b$$

$$Q = \frac{(\mu-\nu s)}{(\theta+\xi)}\left[e^{(\theta+\xi)(t_1-t_1^{'})} - 1\right]e^{\xi t_1^{'}} + \frac{(\mu-\nu s)}{\xi}\left[e^{\xi t_1^{'}} - 1\right] + \frac{(\mu-\nu s)}{\epsilon}\left[1 - e^{-\epsilon(T-t_1)}\right]$$

…(10)

The total cost per cycle consists of the following cost components:

1. Since the order for replenishing the stock is placed at the beginning of each cycle, the value of ordering cost per cycle is

$$OC = C_o \quad \ldots (11)$$

2. Inventory holding cost per cycle is given by

$$IHC = C_h \int_0^{t_1^{'}} (a+bt) q_1(t) e^{-rt} dt + C_h \int_{t_1^{'}}^{t_1} (a+bt) q_2(t) e^{-rt} dt$$

$IHC =$
$C_h a \frac{(\mu-\nu s)}{(\theta+\xi)}\left[\frac{(\theta+\xi)}{\xi r}\left(e^{-rt_1^{'}} - 1\right) + \frac{1}{(\theta+\xi+r)}\left(e^{(\theta+\xi)\left(t_1-t_1^{'}\right)-rt_1^{'}} -\right.\right.$
e−rt1)+1re−rt1−e−rt1′+1(ξ+r)(eξt1′−e−rt1′)θξ+eθ+ ξt1−t1′+Cℎb(μ−νs)(θ+ξ)1(θ+ξ+r)−t1e−rt1+t1′eθ+ξt1 −t1′−rt1′+1(θ+ξ+r)2eθ+ξt1−t1′−rt1′−e−rt1+1r2e−rt1 −1+1(ξ+r)−t1′eθ+ξt1−t1′−rt1′−□□t1′e−rt1′+1(ξ+r)2− □□e−rt1′+θξeξt1′−eθ+ξt1−t1′−rt1′+eθ+ξt1−t1′+ξt1′+ 1rt1e−rt1+θξt1′e−rt1′+θξr2e−rt1′1

…(12)

3. Deterioration cost per cycle is given by

$$DC = C_d \int_{t_1^{'}}^{t_1} q_2(t) e^{-rt} dt$$

$=C_d \theta \frac{(\mu-\nu s)}{(\theta+\xi)}\left[\frac{1}{(\theta+\xi+r)}\left(e^{(\theta+\xi)(t_1-t_1^{'})-rt_1^{'}}\right) + \frac{1}{r}\left(e^{-rt_1} -\right.\right.$
e−rt1′

… (13)

4. Shortage cost per cycle due to backlog is given by

$$SC = C_s \int_{t_1}^{T} [-q_3(t)] e^{-rt} dt$$

$$SC = \frac{(\mu - \nu s)}{\epsilon} C_s \left[\left(\frac{(e^{-rT} - e^{(\epsilon - r)t_1 - \epsilon T})}{(\epsilon - r)} \right) + \left(\frac{e^{-(\epsilon + r)T + \epsilon t_1} - e^{(\epsilon - r)t_1 - \epsilon T}}{r} \right) \right]$$

... (14)

5. Opportunity cost per cycle due to lost sales is given by

$$LS = \pi \int_{t_1}^{T} (\mu - \nu s)(1 - \alpha e^{-\epsilon(T-t)}) e^{-rt} dt$$

$$LS = \pi(\mu - \nu s) \left[\frac{(e^{-rt_1} - e^{-rT})}{r} - \alpha e^{-\epsilon T} \left(\frac{e^{(\epsilon - r)T} - e^{(\epsilon - r)t_1}}{(\epsilon - r)} \right) \right] \quad ...$$

(15)

6. Purchase cost per cycle is given by

$$PC = pq_m + pe^{-rT} q_b$$

$$PC = p \left[\frac{(\mu - \nu s)}{(\theta + \xi)} (e^{(\theta + \xi)(t_1 - t_1')} - 1) e^{\xi t_1'} + \frac{(\mu - \nu s)}{\xi} [e^{\xi t_1'} - 1] + \right.$$

μ−νse−rTϵ1−e−ϵ(T−t1) ...(16)

Therefore, total inventory cost per unit time is given by

$$TC(t_1, T) = \frac{1}{T}[OC + IHC + DC + SC + LS + PC] \quad ... (17)$$

Solution Procedure:

Now the necessary condition for the total cost per unit time to be minimum is

$$\frac{\partial TC}{\partial t_1}(t_1, T) = 0 \quad \text{and} \quad \frac{\partial TC}{\partial T}(t_1, T) = 0 \quad ... (18)$$

provided they satisfy the sufficient conditions

$$\left. \frac{\partial^2 TC(t_1, T)}{\partial t_1^2} \right|_{(t_1^*, T^*)} > 0, \quad \left. \frac{\partial^2 TC(t_1, T)}{\partial T^2} \right|_{(t_1^*, T^*)} > 0$$

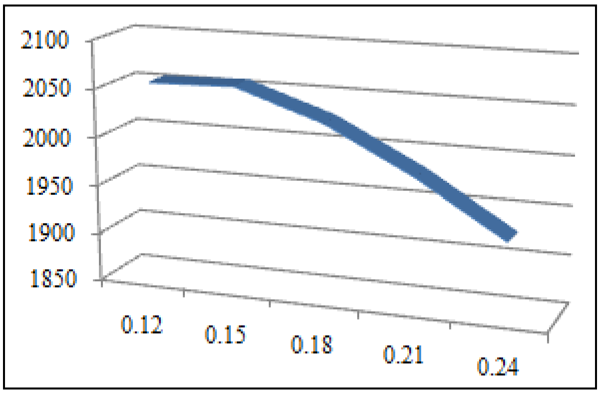
$$\left\{ \left(\frac{\partial^2 TC}{\partial t_1^2}(t_1, T) \right) \left(\frac{\partial^2 TC}{\partial T^2}(t_1, T) \right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}(t_1, T) \right)^2 \right\}_{(t_1^*, T^*)} > 0 \quad .$$

Numerical Example:

Table 1: Relation between 'r' and 'TC'
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

r	TC
0.12	2054.702
0.15	2057.263
0.18	2027.26
0.21	1981.522
0.24	1926.337

Relation between 'r' and 'TC'

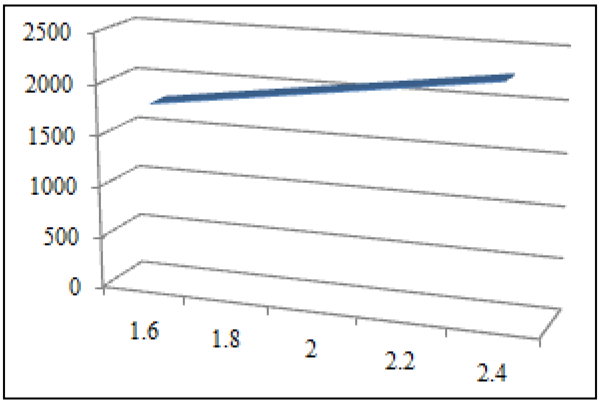


When 'r' increases, the total cost 'TC' decreases.

Table 2: Relation between 'π' and 'TC'
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

π	TC
1.6	2004.079
1.8	2016.011
2	2027.943
2.2	2039.875
2.4	2051.807

Relation between 'π' and 'TC'

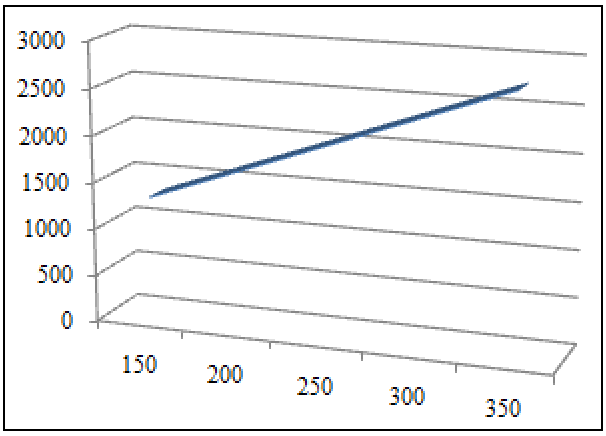


When 'π' increases, the total cost 'TC' increases.

Table 3: Relation between 'μ' and 'TC'
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

μ	TC
150	1314.82
200	1671.04
250	2027.26
300	2383.48
350	2739.701

Relation between ‘μ’ and ‘TC’

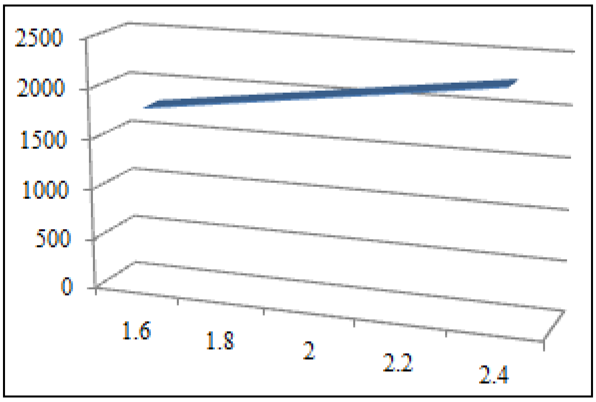


When ‘μ’ increases, the total cost ‘TC’ increases.

Table 4: Relation between ‘p’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

p	TC
1.6	1793.908
1.8	1910.584
2	2027.26
2.2	2143.936
2.4	2260.613

Relation between ‘p’ and ‘TC’

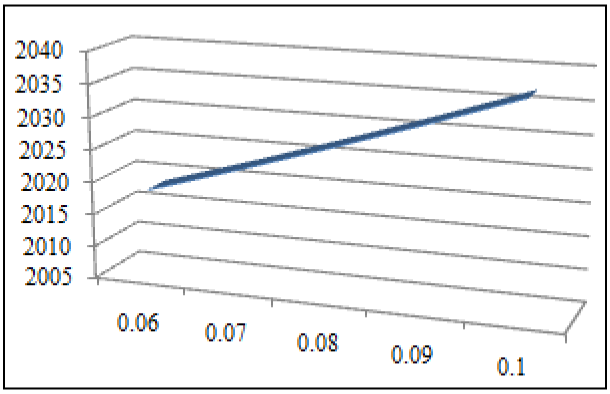


When ‘p’ increases, the total cost ‘TC’ increases.

Table 5: Relation between ‘θ’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

θ	TC
0.06	2018.463
0.07	2022.768
0.08	2027.26
0.09	2031.937
0.1	2036.794

Relation between ‘θ’ and ‘TC’

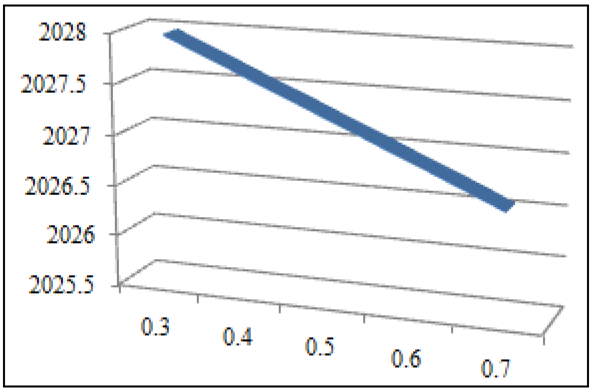


When ‘θ’ increases, the total cost ‘TC’ increases.

Table 6: Relation between ‘ν’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

ν	TC
0.3	2027.973
0.4	2027.617
0.5	2027.26
0.6	2026.904
0.7	2026.548

Relation between ‘ν’ and ‘TC’

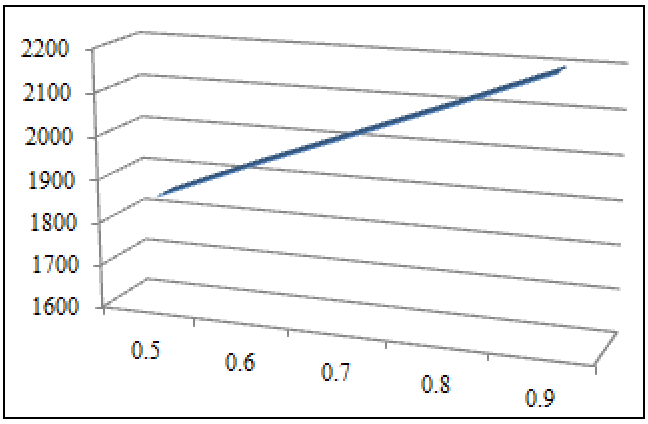


When ‘ν’ increases, the total cost ‘TC’ decreases.

Table 7: Relation between ‘ξ’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

ξ	TC
0.5	1855.035
0.6	1943.697
0.7	2027.26
0.8	2111.206
0.9	2198.372

Relation between ‘ξ’ and ‘TC’

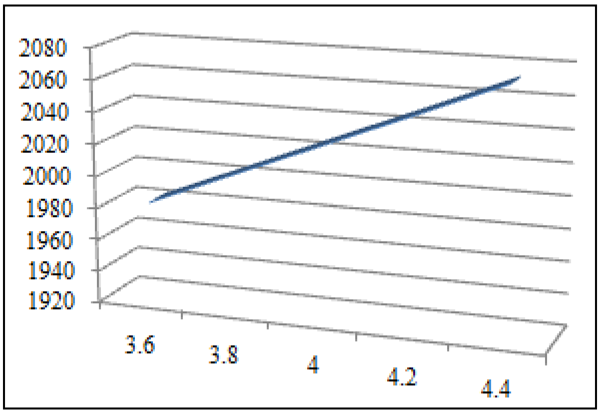


When ‘ξ’ increases, the total cost ‘TC’ increases.

Table 8: Relation between ‘a’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

a	TC
3.6	1982.255
3.8	2004.757
4	2027.26
4.2	2049.763
4.4	2072.266

Relation between ‘a’ and ‘TC’

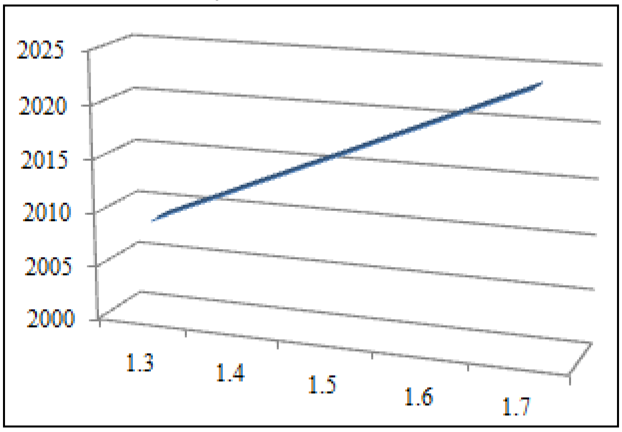


When ‘a’ increases, the total cost ‘TC’ increases.

Table 9: Relation between ‘C_d’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

C_d	TC
1.3	2009.073
1.4	2012.732
1.5	2016.391
1.6	2020.05
1.7	2023.709

Relation between ‘C_d’ and ‘TC’

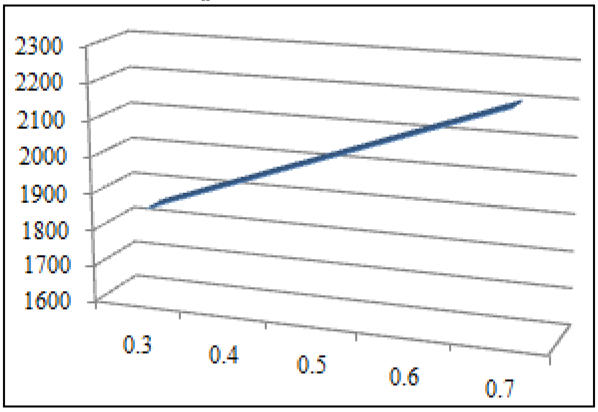


When ‘C_d’ increases, the total cost ‘TC’ increases.

Table 10: Relation between ‘C_h’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

C_h	TC
0.3	1851.918
0.4	1939.589
0.5	2027.26
0.6	2114.931
0.7	2202.602

Relation between ‘C_h’ and ‘TC’

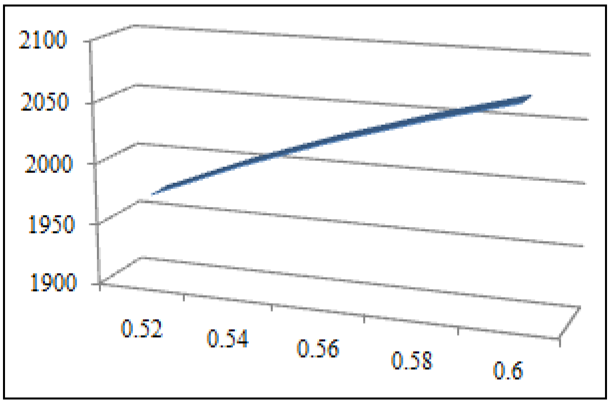


When ‘C_h’ increases, the total cost ‘TC’ increases.

Table 11: Relation between ‘ε’ and ‘TC’
t_1'=0.0833, C_s=2.50, ξ =0.70, p=2, μ =250, θ=0.08, π=2, C_o=250, ε=0.56, ν=7, s=130, C_h=0.50, C_d=1.50

ε	TC
0.52	1972.434
0.54	2002.037
0.56	2027.943
0.58	2050.72
0.6	2070.828

Relation between 'ε' and 'TC'



When 'ε' increases, the total cost 'TC' increases.

3. Conclusion

In this paper, we have proposed inventory models for perishable items having stock and price dependent demand and constant deterioration. Obviously, the demand is a decreasing function of time. The extent to which inflation has affected the business world is clearly elucidated. Our research implies that the effect of inflation and time value of money on the present value of total cost is more significant and highlights that total cost decreases as the inflation rate increases.

From the graph, it can be seen that total cost has a increase in its value with the increase in purchase cost. It is also observed total cost decreases when the backordering rate increases. But when the unit cost of lost sales increases, total cost also increases. Also, when any of the costs such as shortage cost, ordering cost, holding cost etc. increases then the total cost of the system also increases. Thus, this model incorporates some realistic features that are likely being associated with some kinds of inventory.

References

[1] Abad P. L. (1996): Optimal pricing and lot-sizing under conditions of perish ability and partial backlogging, *Management Sciences, 42, 1093-1104.*

[2] Arcelus F. J. And Srinivasan G. (1993): Delay in payments for extra ordinary purchases, *Journal of Operations Research Society, 44, 785-795.*

[3] Bose S., Goswami A. and Chaudhari K. S. (1995): An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting, *Journal of Operations Research Society, 46, 771-782.*

[4] Buzacott L. A. (1975): Economic order quantities with inflation, *Operations Research, 26, 553-558.*

[5] Chang H. J. and Dye C. Y. (1999): An EOQ model for deteriorating items with time varying demand and partial backlogging, *Journal of Operations Research Society, 50, 1176-1182.*

[6] Datta T. K. and Pal A. K. (1991): Effects of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages, *European Journal of Operations Research, 52, 326-333.*

[7] Datta T. K. and Pal A. K. (1998): Demand promotion by upgradation under stock dependent demand situation-a model, *International Journal of Production and Economy, 55, 31-38.*

[8] Goyal S. K. and Giri B. C.(2003): The production inventory problem of a product with time varying demand, production and deterioration rates, , *European Journal of Operations Research, 147, 549-557.*

[9] Gupta R. and Vrat P. (1986): Inventory model with multi-items under constraint systems for stock dependent consumption rate, *Operations Research, 24, 41-42.*

[10] Hollier R. H. and Mak K. L. (1983): Inventory replenishment policies for deteriorating items in a declining market, *International Journal of Production Research, 21, 813-826.*

[11] Levin R. I., Mc Laughlin C. P., Lamone R. P. and Kottas J. F. (1972): Production operations management: contemporary policy for managing operating systems, *McGraw Hill, New York, 373.*

[12] Park K. S. (1982): Inventory models with partial backorders, *International Journal of System Sciences, 13, 1313-1317.*

[13] Whitin T. M. (1955): Inventory control and price theory, *Management Sciences, 2, 61-68.*

[14] Wu K. S., Ouyang L. Y., C. T. (2006): An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging, *International Journal of Production and Economy, 101, 369-384.*

[15] Smita Y., Sharma A. K.and Kumar N. (2015) : An inventory model for deteriorating items with selling price dependent demand and variable deterioration under inflation with time dependent holding cost, International Journal of Scientific & Engineering Research, 6, 1241-1244.

[16] Yadav R., Sharma A.K.and Yadav S. (2022): An economic production quantity model with selling price dependent demand under inflation and variable production rate for deteriorating items, Jñānābha, 52(1), 113-117.

[17] Meena T.C. and Sharma A .K. (2022) : Integrated vendor-buyer co-operative model for deteriorating Items with multivariate demand and salvage value, IOSR Journal of Mathematics, 18(6), 01-11.

[18] Meena T.C. and Sharma A.K. (2022) : A deterministic inventory model for non instantaneous deteriorating items with allowed backorders and quadratic time varying holding cost, International Journal of Science and Research, 11(11), 1117-1123.