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Reliability Modeling and Analysis of an Industrial System with one Main Unit with Two Subunit

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Abstract: ***The paper deals with Reliability modelling and analysis of an industrial system with one main unit with two sub unit. The system consists of one main unit and two sub unit. System will be operable when main unit and at least one subunit is in operative mode. It is assumed that only one job is taken for processing at a time. There is a single server who visits the system immediately when preventive maintenance and repair required. The unit works as new after preventive maintenance and repair. The failures of the unit are distributed exponentially while the distribution of PM and repair time are taken is arbitrary. Semi-Markov and Markov regenerating point techniques are used to calculate various reliability parameters.***

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Keywords: Main time between failure (MTBF), Preventive maintenance (PM), Regenerative point technique ,semi-Markov process

1. Introduction

It is obvious that a system is composed of a number of components and to achieve high reliability of the system; we have to use high reliable components. In many cases when it is not possible to produce such type of components, we can increase the system reliability by incorporating the redundancies of the corresponding components. Reliability models for system have widely been analyzed by a number of authors under various assumption, including Saw and Manker [4] develop a stochastic reliability model of a one-unit main system with two associate units along with sub -unit. Gopalan [2] analyzed cost benefit analysis of one server two-unit cold standby system with repair and preventive maintenance, Nakagawa and Osaki [7] studies model which describes single unit system with scheduled maintenance and variation in demand. Pathak, and Mehata [8] developed the configuration modelling of wire rod system, Singh and singh [6] analysis cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand. Gupta 2007 [1] discussed benefit analysis of distillery plant system studies some reliability models in real data of failure and repair rates in such system. Saw, Pathak and Chaturvedi [5] analysis stochastic model for analysis real industrial system modal of a RO membrane used in water purification system. Bhatt, Chitkara and Bhardwaj [3] analysis two identical unit cold standby system with single repairman has been discussed. This modal present Reliability modelling and Analysis of an industrial system with one main unit with two sub unit. Also, the involvement of preventive maintenance in the modal increases the reliability of the functioning units.

2. Materials and Method

In this study, the stochastic reliability of the system is analyzed by using semi-Markov process and regenerative point techniques expression for various reliability measures like Mean time between failure. The steady state Availability, the Busy period of the server due to repair of a failed unit at t = 0. Busy period of the server due to preventive maintenance at t = 0. Expected down time at t = 0. Expected number of visits by server at t = 0. Profit incurred to the system.

System Description

The system consists of three units namely one main unit M and two associate unit U and R. Hear the associate unit U and R dependents upon the main unit M. The main unit is employed to rotate U and R. As soon as job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repair the failed units on a priority basis. Using regenerative point technique, several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end, the expected profit is also calculated.

Assumptions

1) The system consists of one main unit and two associate units.
2) The associate units U and R starts with the help of the main unit.
3) There is a single repairman who repairs the failed units on a priority basis.
4) After repair system work in good state.
5) The repair starts immediately upon failure of units.
6) After a random period of time, the whole system goes to preventive maintenance.

Notations

P_{ij} = Transition probabilities from $S_i to S_j$

μ_i = Mean Sojourn time at time t

$x_1/x_2/x_3$ =Constant repair rate of Main unit M/Associate unit U/Associate unit R

$\alpha_1/\alpha_2/\alpha_3$ = Failure rate of Main unit M/Associate unit U/Associate unit R

$f_1/f_2/f_3 =$ Probability density function of repair time of Main unit M/Associate unit U/Associate Unit R

$\overline{F_1}/\overline{F_2}/\overline{F_3}$ =Cumulative distribution function of repair time of Main unit M/Associate unit U/Associate unit R
a(t) = Probability density function of preventive maintenance,
b(t) = Probability density function of preventive maintenance completion time,

$\overline{G}(t) =$ Cumulative distribution functions of preventive maintenance,

$\overline{H}(t) =$ Cumulative distribution functions of preventive maintenance completion time,
$ = Symbol for Laplace-Stieltjes transforms,

$\phi =$ Symbol for Laplace-convolution

$Q_{i,j}(t) =$ Cumulative distribution function of transition

time from $S_i to S_j$

$B_0(t)$ = Busy period of the server for repair due to failed unit at time t=0
B = Set of regenerative states

$\pi_i(t) =$ CDF of time to system failure when starting from

state $B_0 = S_I \in B$

$\mu_i(t)$ =Mean Sojourn time in the state $B_0 = S_i \in B$,

$D_i(t) =$ Repairman is busy in the repair at time t

$B_0 = S_i \in B$,

Symbols

$M_0/M_G/M_r$- Main unit 'M' under operation/good and non-operative mode/repair mode,

$R_0/R_g/R_r$ — Associate unit 'R' under operation/good and non-operative mode/repair mode

$U_0/U_g/U_r$-Associate unit 'U' under operation/good and non-operative mode/repair mode,
P.M. — System under preventive maintenance

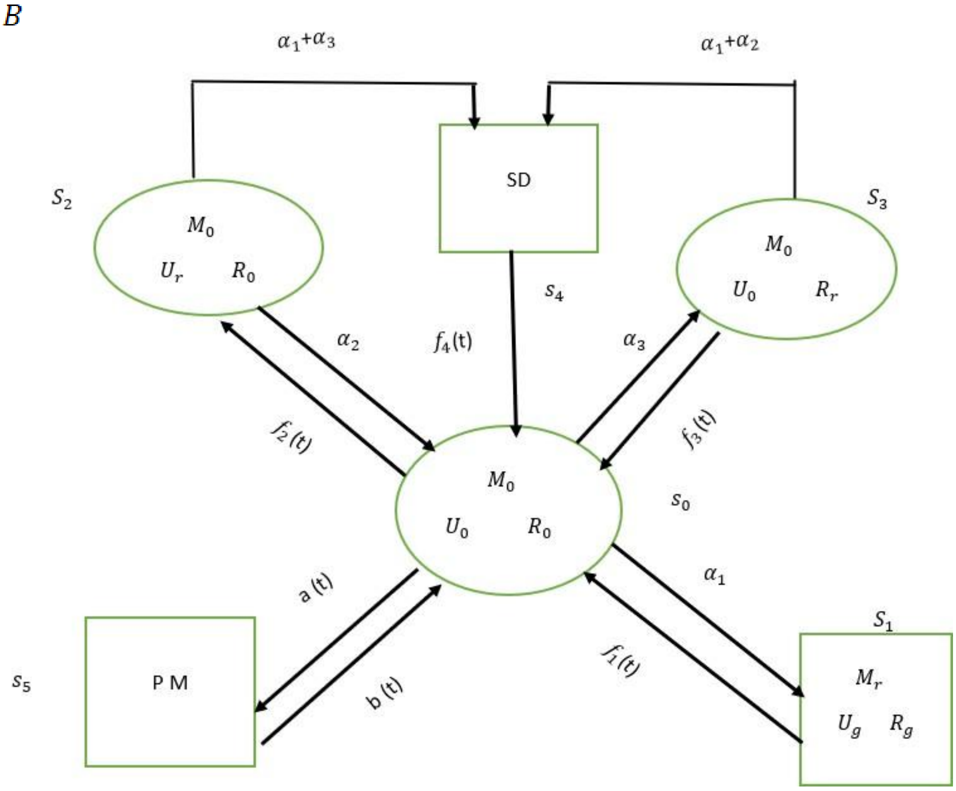
Up States

$$S_0 = (M_0, U_0, R_0), S_2 = (M_0, U_r, R_0), S_3 = (M_0, U_0, R_r)$$

Down States

$$S_1 = (M_r, U_g, R_g), S_4 = (S.D.), S_5 = (P.M.)$$

State transition diagram



Mathematical Analysis of the System
Transition Probabilities and Mean Sojourn Times
Simple probabilistic considerations yield the following non-zero transition probabilities:

$$Q_{01}(t) = \int_0^t \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3)t} \overline{G}(t)$$

$$Q_{02}(t) = \int_0^t \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3)t} \overline{G}(t) dt$$

$$Q_{03}(t) = \int_0^t \alpha_3\, e^{-(\alpha_1+\alpha_2+\alpha_3)t}\overline{G}(t)dt$$

$$Q_{05}(t) = \int_0^t a\,(t)e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt$$

$$Q_{10}(t) = \int_0^t f_1\,(t)dt$$

$$Q_{20}(t) = \int_0^t f_2\,(t)e^{-(\alpha_1+\alpha_3)t}dt$$

$$Q_{30}(t) = \int_0^t f_3\,(t)e^{-(\alpha_1+\alpha_2)t}dt$$

$$Q_{24}(t) = \int_0^t (\alpha_1+\alpha_3)\,e^{-(\alpha_1+\alpha_3)t}\overline{F_2}(t)dt$$

$$Q_{34}(t) = \int_0^t (\alpha_1+\alpha_2)\,e^{-(\alpha_1+\alpha_2)t}\overline{F_3}(t)dt$$

$$Q_{50}(t) = \int_0^t b\,(t)dt$$

$$Q_{40}(t) = \int_0^t f_4\,(t)dt$$

The nonzero element P_{ij} can be obtained as

$$P_{ij} = \lim_{Pt\to\infty} C_{ij}(t)$$

$$P_{01}(t) = \int_0^\infty \alpha_1\, e^{-(\alpha_1+\alpha_2+\alpha_3)t}\overline{G}(t)dt = \frac{\alpha_1[1-f^*(\alpha)]}{\alpha}$$

$$P_{02}(t) = \int_0^\infty \alpha_2\, e^{-(\alpha_1+\alpha_2+\alpha_3)t}\overline{G}(t)dt = \frac{\alpha_2[1-f^*(\alpha)]}{\alpha}$$

$$P_{03}(t) = \int_0^\infty \alpha_3\, e^{-(\alpha_1+\alpha_2+\alpha_3)t}\overline{G}(t)dt = \frac{\alpha_3[1-f^*(\alpha)]}{\alpha}$$

$$P_{05}(t) = \int_0^\infty a\,(t)e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = f^*(\alpha)$$

$$P_{20}(t) = \int_0^\infty e^{-(\alpha_1+\alpha_3)t}\, f_2(t)dt = f_2^*(\alpha_1+\alpha_3)$$

$$P_{30}(t) = \int_0^\infty e^{-(\alpha_1+\alpha_2)t}\, f_3(t)dt = f_3^*(\alpha_1+\alpha_2)$$

$$P_{24}(t) = \int_0^\infty (\alpha_1+\alpha_3)\, e^{-(\alpha_1+\alpha_3)t}\overline{F}_2(t)dt = 1 - f_2^*(\alpha_1+\alpha_3)$$

$$P_{34}(t) = \int_0^\infty (\alpha_1+\alpha_2)\, e^{-(\alpha_1+\alpha_2)t}\overline{F}_3(t)dt = 1 - f_3^*(\alpha_1+\alpha_2)$$

$$P_{10}(t) = \int_0^\infty f_1\,(t)dt = 1$$

$$P_{50}(t) = \int_0^\infty b\,(t)dt = 1$$

It can be easily verified that;

$$P_{01}+P_{02}+P_{03}+P_{05} = 1$$

$$P_{20}+P_{24} = 1, P_{30}+P_{34} = 1$$

The main sojourn times (μ_i) in the generative state i is defined as the time of stay in that state before transition to any other state.

$$\mu_0 = \frac{[1-f^*(\alpha)]}{\alpha}$$

$$\mu_1 = \int_0^\infty \overline{F_1}\,(t)dt$$

$$\mu_2 = \frac{[1-f_2^*(\alpha_1+\alpha_3)]}{\alpha_1+\alpha_3}$$

$$\mu_3 = \frac{[1 - f_3^*(\alpha_1 + \alpha_2)]}{\alpha_1 + \alpha_2}$$

$$\mu_4 = \int_0^\infty \overline{F_4}\,(t)dt$$

$$\mu_5 = \int_0^\infty \overline{H}\,(t)dt$$

Laplace Stieltjes transform of $C_{ij}(t)$ we get.

$$\overline{Q}_{ij}(s) = \int_0^\infty e^{-st}\, C_{ij}(t)dt = L[C_{ij}(t)] = q_{ij}^*(s)$$

$$\overline{Q}_{01}(s) = \int_0^\infty \alpha_1\, e^{-(s+\alpha)t}\overline{G}(t)dt = \frac{\alpha_1[1 - g^*(s+\alpha)]}{s+\alpha}$$

$$\overline{Q}_{02}(s) = \int_0^\infty \alpha_2\, e^{-(s+\alpha)t}\overline{G}(t)dt = \frac{\alpha_2[1 - g^*(s+\alpha)]}{s+\alpha}$$

$$\overline{Q}_{03}(s) = \int_0^\infty \alpha_3\, e^{-(s+\alpha)t}\overline{G}(t)dt = \frac{\alpha_3[1 - g^*(s+\alpha)]}{s+\alpha}$$

$$\overline{Q}_{05}(s) = \int_0^\infty e^{-(s+\alpha)t}\, a(t)dt = g^*(s+\alpha)$$

$$\overline{Q}_{10}(s) = \int_0^\infty e^{-st}\, f_1(t)dt = f_1^*(s)$$

$$\overline{Q}_{20}(s) = \int_0^\infty e^{-(s+\alpha_1+\alpha_3)t}\, f_2(t)dt = f_2^*(s+\alpha_1+\alpha_3)$$

$$\overline{Q}_{26}(s) = \int_0^\infty (\alpha_1+\alpha_3)\, e^{-(s+\alpha_1+\alpha_3)t}\overline{F_2}(t)dt = \frac{\alpha_1+\alpha_3[1 - f_2^*(s+\alpha_1+\alpha_3)]}{s+\alpha_1+\alpha_3}$$

$$\overline{Q}_{30}(s) = \int_0^\infty e^{-(s+\alpha_1+\alpha_2)t}\, f_3(t)dt = f_3^*(s+\alpha_1+\alpha_2)$$

$$\overline{Q}_{36}(s) = \int_0^\infty (\alpha_1+\alpha_2)\, e^{-(s+\alpha_1+\alpha_2)t}\overline{F_3}(t)dt = \frac{\alpha_1+\alpha_2[1 - f_3^*(s+\alpha_1+\alpha_2)]}{s+\alpha_1+\alpha_2}$$

$$\overline{Q}_{50}(s) = \int_0^\infty e^{-st}\, b(t)dt = b^*(s)$$

$$\overline{Q}_{40}(s) = \int_0^\infty e^{-st}\, f_6(t)dt = f_6^*(s)$$

We define m_{ij} as follows:

$$m_{ij} = -\left[\frac{d\overline{Q}_{ij}(s)}{ds}\right]_{s=0} = -Q_{ij}'(0)$$

It can be easily verified

$$m_{01} + m_{02} + m_{03} + m_{05} = \mu_0$$

$$m_{20} + m_{24} = \mu_2$$

$$m_{30} + m_{34} = \mu_3$$

Where $\alpha_1 + \alpha_2 + \alpha_3 = \alpha$ Main Time Between Failure

$\pi_i(t)$ is defined as the CDF of first passage time from i^{th} state to a failed state.

$$\pi_0(t) = Q_{01}(t) + Q_{02}\$\pi_2(t) + Q_{03}(t)\$\pi_3(t) + Q_{05}(t)$$

$$\pi_2(t) = Q_{20}\$\pi_0(t) + Q_{26}(t)$$

$$\pi_3(t) = Q_{30}\$\pi_0(t) + Q_{36}(t)$$

Taking Laplace- Stieltjes Transforms (L.S.T.) on both side and solving we get, The MTBF when the system starts from the state S_0 is given by.

$$E(T) = -\left[\frac{d\overline{\pi}_0(s)}{ds}\right]_{s=0} = \frac{D(0) - N(0)}{D(0)}$$

$$= \frac{\mu_2 P_{02} + \mu_3 P_{03} - \mu_0}{1 - P_{02}P_{20} - P_{03}P_{30}}$$

Availability Analysis

Using probabilistic argument, we have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)\phi A_1(t) + q_{02}(t)\phi A_2(t) + q_{03}(t)\phi A_3(t) + q_{05}(t)\phi A_5(t)$$

$$A_1(t) = q_{10}(t)\phi A_0(t)$$

$$A_2(t) = M_2(t) + q_{20}(t)\phi A_0(t) + q_{24}(t)\phi A_4(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)\phi A_0(t) + q_{34}(t)\phi A_4(t)$$

$$A_4(t) = q_{40(t)}\phi A_0(t)$$

$$A_5(t) = q_{50}(t)\phi A_0(t)$$
Where

$$M_0(t) = e^{-(\alpha_1+\alpha_2+\alpha_3)t}\overline{G}(t)$$

$$M_2(t) = e^{-(\alpha_1+\alpha_3)t}\overline{F_2}(t)$$

$$M_3(t) = e^{-(\alpha_1+\alpha_2)t}\overline{F_3}(t)$$
Taking Laplace-Stieltjes Transforms (L.S.T.) of above equation and solving the steady -state availability is given by.

$$A_0^* = \lim_{s\to 0} sA_0^*(s) = \frac{N_1(0)}{D(0)}$$

Where

$$N_1(0) = \mu_2 P_{02} + \mu_3 P_{03} - \mu_0$$

$$D(0) = 1 - P_{02}P_{20} - P_{03}P_{30}$$

Busy Period Analysis
Using probabilistic argument, we have the following recursive relation for

$$B_0(t) = q_{01}(t)\phi B_1(t) + q_{02}(t)\phi B_2(t) + q_{03}(t)\phi B_3(t) + q_{05}(t)\phi B_5(t)$$

$$B_1(t) = W_1(t) + q_{10}(t)\phi B_0(t)$$

$$B_2(t) = W_2(t) + q_{20}(t)\phi B_0(t) + q_{24}(t)\phi B_4(t)$$

$$B_3(t) = B_3(t) + q_{30}(t)\phi B_0(t) + q_{34}(t)\phi B_4(t)$$

$$B_4(t) = q_{40(t)}\phi B_0(t)$$

$$B_5(t) = q_{50}(t)\phi B_0(t)$$
Where

$$W_1(t) = \overline{F_1}(t), W_2(t) = \overline{F_2}(t), W_3(t) = \overline{F_3}(t),$$

Taking Laplace transform of above equation and solving for

$$B_0(s)$$

$$B_0(s) = \frac{N_2(s)}{D_2(s)}$$
Where

$$N_2(s) = \mu_2 P_{02} + \mu_3 P_{03} - \mu_0$$

$$D_2(s) = 1 - P_{02}P_{20} - P_{03}P_{30}$$

Profit Analysis
G(t) = Expected total revenue earned by the system in (0, t]
- Expected repair cost of the failed units
- Expected repair cost of the repairman in preventive maintenance
- Expected repair cost of the Repairman in shut down

$$G(t) = C_0 A_0 - C_1 B_0 - C_2 B'_0 - C_3 B''_0$$

Where
C_0 be the per unit time revenue by the system
$C_1, C_2 and C_3$ be the per unit time for which the system is under simple repair, preventive maintenance and shut down repair, respectively.

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