

Understanding Financial Basics: An Introduction to Interest, Time Value of Money, and their Applications for Undergraduate Non-Math Majors

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Abstract: *This text is mainly focused on undergraduate students who have not taken mathematics as their major subject. So, it describes the context from the very scratch. It begins with the concept of interest on deposits or investments, which is very familiar even for a beginner. It explains time value of money, simple and compound interest which are the basic and simple concepts known to all with some of its applications.*

Keywords: Rate of interest, Time value of money, simple and compound interests, annuities, force of interest

1. Introduction

This text is mainly focused undergraduate students who have not taken mathematics as their major subject. So, it describes the context from the very scratch. It begins with the concept interest which is very familiar even for a beginner. It explains time value of money, simple and compound interest which is the basic and simple concept known for all with some of the problems. From the known to unknown method, the text describes accumulated function, amount function, force of interest Stoodley's formula and equation of value and yield on a transaction with suitable examples.

The meaning of interest

A clear and concrete understanding of the concept of interest is required to analyse financial transactions. In the most common context interest can be defined as an amount charged to a borrower for the use of the lender's money over a period of time. For example, if you have borrowed Rs.100 and you promised to pay back Rs.105 after one year. Then the lender in this case is making a profit of Rs.5/-, which is the fee for borrowing his money.

Looking at this from the lender's perspective the money, the lender is investing is changing value with time, due to the interest being added. For that reason, interest is sometimes referred to as the **time value of money**.

The subject matter of mathematics of finance is basically concerned with evaluation of growth, in a deposited amount over a period of time due to the interest that money earns on it.

The amount deposited is termed as Capital or Principal amount and we denote it by P. Rate of interest is expressed in terms of percentage per annum. The unit in which time of investment is measured is called measurement period. The most common measurement period is one year but may be longer or shorter.

Rate of interest

Interest rate may be defined as the amount which is paid in one unit of time period for one unit of capital invested at the beginning of the period. We denote it by 'i'.

Simple interest and Compound interest

Simple interest

When interest is calculated on the principal amount only, it is called simple interest. Example: Suppose simple interest 18% per annum on deposits made for a period of 5 years. An initial deposit of Rs.100 will earn Rs.18 only at the end of each year for five years. And consequently, at the end of five years it will grow to $Rs.100 + Rs.18 \times 5 = Rs.190$. This value is named as amount value or accumulated value and we denote it by 'A'.

We denote the principal unit by P, the period of investment t and the annual interest rate 'i', accumulated value or amount value, A is calculated by $A = P + P i t$
 $= P (1 + it)$

Problems:

- 1) Ben deposited Rs.1000 into a savings account. One year later the account has accumulated to Rs.1050.
 - a) What is the principal in this investment?
 - b) What is the interest earned?
 - c) What is the annual interest rate?

Solution:

- a) Rs.1000
- b) Rs.50
- c) $i = (A/P) \times 100$
 $= (50/1000) \times 100$
 $= 5$

- 2) You invest Rs.3200 in a savings account in January 1, 2004. On 31st December 2004, the account has accumulated to Rs.3244.08. What is the annual interest rate?

Solution: Interest amount = 3294.08 – 3200
=94.08
Rate=(94.08/3200) x100
= 2.94%

- 3) Raju borrowed Rs.12000 from a bank, the loan is to be repaid in full in one year time with a payment due of Rs. 12780.
- What is the interest amount paid on the loan.
 - Annual interest rate.
- Solution: a) interest rate paid on the loan =Rs.12780-Rs.12000=Rs.780
Annual interest rate=(interest amount/Principal) x100
c) (780/12000) x100
= 6.5%
- 4) The current interest rate quoted by a bank on its savings account is 9% per year. You open an account with Rs.1000/-.Assuming there are no transactions on the account such as depositing or withdrawing during the full year. What will be the amount value at the end of the year in the account.

Solution: $A=P(1+it)$
= 1000[1+(9/100) x1] =1000(1.09)
= 1090 Rupees.

- 4) At what rate will Rs.500 accumulate to Rs.615 in 2.5 years? Solution: Interest received =615-500=115
 $P \times i \times t = 115$ $P=500$, $t=2.5$
 $i = 115 / (2.5 \times 500) = 9.2\%$
- 5) In how many years will 500 accumulate to 630, if the annual interest rate is 7.8%. $A = P (1 + it)$
 $A = P + Pit$
 $A = 630$, $P = 500$
 $i = 7.8\%$
 $A = P + Pit$
 $A - P = Pit$
 $t = (A - P) / PI$
= (630 – 500) / (500x7.8/100)
= 3.33 years.
- 6) Mr. X borrows Rs.200 for one year and agrees to repay the lender Rs.220 at the end of the year. Find the interest rate?

Solution: $P=200$, $A=220$, $t=1$ year
Interest received =220-200=20 $P \times i \times t = 20$
 $I = (20/200) \times 100$
= 10%

- 7) If Rs.64 amounts to Rs.78.4 in 3 years. What will Rs.86 amount in 4 years at the same rate of interest? Solution:
Interest received = 78.4-64=14.4

$P \times i \times t = 14.4$
= (14.4/3) x100
= 7.5
 $A = P(1+it)$
= 111.8

- 8) A sum of money at simple rate of interest doubles in 8 years. Find the rate of interest? Solution: $A = 2P$, $t=8$,
 $A = P(1+it)$
 $2P = P (1 + i \times 8)$ $2P/P = 1 + 8 i$ $1 = 8 i$
 $i = 1/8 \times 100$
= 12.5%

Compound Interest

Compounding is the process of adding accumulated interest back to the principal, so that interest is earned on interest from that moment on. We can consider simple interest as a series of back-to-back simple interest contracts. The interest earned in each period is added to the principal amount of the previous period to become the principal amount for the next period. We calculate the compound interest using the following formula:

$$A = P(1+i)^t$$

Problems

- 1) Suppose we invest Rs.100 at a compound interest rate of 10% per annum and want to find out the amount to which this initial investment of Rs.100 will grow at the end of 5 years.
 $P=100$, $i=10$, $t=5$ $A=P(1+i)^t$
=100(1+10/100)⁵
= 100(1.1)⁵
= 100x1.610
= 161.05
- 2) You borrow Rs.10000 for 3 years at 5% annual interest compounded annually. What is the amount value at the end of 3 years? $P=10000$, $i=5$, $t=3$
 $A=P(1+i)^t$
=10000(1+5/100)³
= 100(1.05)⁵
= 100x1.1576
= 11576.25
- 3) Using compound interest formula what principal does X need to invest at 15% compounding annually so that he ends up with Rs.10,000 at the end of 5 years?

Solution: $A=10000$, $i=15$, $t=5$ $A=P(1+i)^t$
10000 = P (1+15/100)⁵
= P (1.15)⁵
= P x 2.0113
 $\Delta P = 10000/2.0113$
= 4971.908

- 4) How long would it take for an investment of Rs.15000 to increase to Rs.45000, if the annual compound interest rate is 2%?

Solution: $A=Rs.45000$, $P=Rs.15000$, $i=2\%$, $t=?$ $A=P(1+i)^t$
45000 = 15000(1+2/100)^t
45000/15000 = (1.02)^t
3 = (1.02)^t $\log 3 = \log (1.02)^t$ $\log 3 = t \times \log (1.02)$
0.4771 = t x 0.0086

$$t = 0.4771/0.0086$$

$$= 55.47$$

5) You have Rs.10,000 to invest now and are being offered Rs.22000 after 10 years as the return from the investment. The market rate is 10% compound interest. Ignoring the complications such as the effect of taxation the reliability of the company offering the contract etc. Do you accept the investment.

$$\text{Solution} = 10,000 (1 + 0.10)^{10}$$

$$= 10000(1+10/100)^{10}$$

$$= 10000(1.1)^{10}$$

$$= 10000 \times 2.5937$$

$$= 25937.423$$

As the accumulated amount that we get from the business using this interest rate is much higher than the expected amount, we do not accept the offer of investment.

Kinds of interest rate

Interest rates are expressed in three ways according to the number of time interest rate is paid, during the period for which interest rate is given.

- 1) Effective interest rate
- 2) Nominal interest rate
- 3) Force of interest.

Accumulation factor or Growth factor

If a principal amount P is invested at interest rate i compounded annually, then the accumulated amount is $A = P(1+i)^t$. The factor $(1+i)^t$ is the **accumulation factor** of each amount invested.

Accumulation function and Amount function

Assume that the change or growth in the fund is due to interest only, that is no deposit or withdrawals occur during the period of investment. If it is the length of time measured in years for which the principal has been invested then the amount of money at that time will be denoted by A(t). This is called **amount function**. A (0) is just the principal P. Now in order to compare various amount functions we use $\alpha(t) = A(t)/A(0)$. This is called **accumulated function**. A(t) is the accumulated value of an original investment A (0).

Problem: Suppose $A(t) = \alpha t^2 + 10\beta$. If X is invested at time 0 accumulates to Rs.500 at time 4 and to Rs.1000 at time 10. Find the amount of the original investment X.

$$\text{Solution: } A(t) = \alpha t^2 + 10\beta$$

$$\text{At } t=0, A(t) = A(0) = \alpha \times 0 + 10\beta = X$$

$$X = 10\beta \dots\dots\dots (1)$$

$$\text{At } t=4, A(t) = A(4) = \alpha 4^2 + 10\beta = 500$$

$$= 16\alpha + 10\beta = 500 \dots\dots\dots (2)$$

$$\text{At } t=10, A(t) = A(10) = \alpha 10^2 + 10\beta = 1000$$

$$= 100\alpha + 10\beta = 1000$$

$$\text{From, Eqn (1), } 100\alpha + X = 1000$$

$$\text{From, Eqn (2), } 16\alpha + X = 500$$

$$\text{Solving, } 1600\alpha + 100X = 50000 \dots\dots\dots (4)$$

$$1600\alpha + 16X = 16000 \dots\dots\dots (5)$$

$$(4)-(5) \Rightarrow 84X = 34000$$

$$X = 34000/84$$

$$= 404.76$$

In general Rs.k is deposited at t years, then the accumulated value of Rs.k at s, (t>s) is $k \alpha(t)/\alpha(s)$, $\alpha(t)/\alpha(s)$ is the growth factor or accumulation factor. The nth period of time is defined to be the period of time between t=n-1 and t=n.

If t=4 and t=6, then nth period of time =6-4 =2.

Interest earned during nth period of time

$$I_n = A(n) - A(n-1)$$

The amount of interest earned on an original investment on Rs.K between time ‘s’ and ‘t’,

$$I_{(s,t)} = A(t) - A(s)$$

Problems

1) Consider the function $A(t) = t^2 + 2t + 1$. Find I_n in terms of n.

$$\text{Solution: } A(t) = t^2 + 2t + 1$$

$$I_n = A(n) - A(n-1) \quad A(n) = n^2 + 2n + 1$$

$$A(n-1) = (n-1)^2 + 2(n-1) + 1$$

$$= n^2 - 2n + 1 + 2n - 2 + 1$$

$$= n^2$$

$$= n^2 + 2n + 1 - n^2$$

$$= 2n + 1$$

2) Show that $A(n) - A(0) = I_1 + I_2 + I_3 + \dots + I_n$. Interpret the result. Solution:

$$A(n) - A(0) = [A(1) - A(0)] + [A(2) - A(1)] + [A(3) - A(2)]$$

$$+ \dots + [A(n) - A(n-1)]$$

$$= I_1 + I_2 + I_3 + \dots + I_n$$

$I_1 + I_2 + I_3 + \dots + I_n$ is the interest ‘n’ earned on the capital 0. The capital A (0) is the interest earned over the concatenation of n periods is the sum of the interest earned in each of the periods separately.

3) Consider the amount function $A(t) = t^2 + 2t + 3$
 a) Find the corresponding accumulation function
 b) Find interest in terms of n

$$\text{Solution: a) } \alpha(t) = A(t)/A(0)$$

$$A(t) = t^2 + 2t + 3$$

$$A(0) = 0^2 + 2 \times 0 + 3 = 3$$

$$\alpha(t) = (t^2 + 2t + 3)/3$$

$$\text{b) } I_n = A(n) - A(n-1)$$

$$A(n) = n^2 + 2n + 3$$

$$A(n-1)$$

$$= (n-1)^2 + 2(n-1) + 3$$

$$= n^2 - 2n + 1 + 2n - 2 + 3$$

$$= n^2 + 2$$

$$= n^2 + 2n + 3 - (n^2 + 2)$$

$$= 2n + 1$$

- 3) It is known that the accumulation function $\alpha(t)$ is of the form $\alpha(t) = b(1.1)^t + ct^2$ where b and c are constants to be determined. If Rs. 100 is invested at time $(t=0)$ accumulates to Rs.170 at time $t=3$. Find the accumulated value at time $t = 12$ of Rs. 100 invested at time $t = 1$

$$\alpha(t) = b(1.1)^t + ct^2$$

$$\alpha(0) = b(1.1)^0 + c(0)^2 = 1$$

$$\alpha(0) = 1$$

$$\Rightarrow b = 1$$

$$A(t) = \alpha(t) \times A(0)$$

$$A(3) = \alpha(3) \times A(0)$$

$$= \alpha(3) \times 100$$

Now,

$$170 = 100[1(1.1)^3 + c(3)^2]$$

$$170 = 100[1.331 + 9c]$$

$$1.7 = 1.331 + 9c$$

$$C = (1.7 - 1.331)/9$$

$$= 0.041$$

$$\alpha(1) = 1(1.1)^1 + 0.041(1)^2 = 1.141$$

$$\alpha(12) = 1(1.1)^{12} + 0.041(12)^2 = 9.042$$

Accumulated value of Rs.100 invested from time $t=1$ to 12 is,

$$100 \times \alpha(12) / \alpha(1) = 100 \times 9.042 / 1.141$$

$$= 792.46$$

- 5) It is known that $\alpha(t)$ is of the form $at^2 + b$. If Rs.100 invested at $t=0$ accumulates to Rs.172 at time 3. Find the accumulated value at time 10 of Rs.100 invested at $t=5$.

Solution: $\alpha(t) = at^2 + b$,

$$\alpha(0) = a(0)^2 + b = 1 \Rightarrow b = 1$$

$$A(t) = \alpha(t) \times A(0)$$

$$A(3) = \alpha(3) \times A(0)$$

$$172 = 100 \times [ax^3 + 1]$$

$$1.72 = 9a + 1$$

$$0.72 = 9a \Rightarrow a = 0.08$$

$$\alpha(10) = 0.08(10)^2 + 1$$

$$= 8 + 1$$

$$= 9$$

$$\alpha(5) = a(5)^2 + b$$

$$= 0.08 \times 25 + 1$$

$$= 2 + 1$$

$$= 3$$

$$100 \times \alpha(10) / \alpha(5) = 100 \times 9 / 3$$

$$= 300$$

Accumulated value = Rs.300.

- 6) Rs.100 is deposited at time $t=0$ into an account whose accumulated function $\alpha(t) = 1 + 0.03\sqrt{t}$.

- a) Find the amount of interest at time 4, i.e., between $t=0$ and 4.
b) Find the amount of interest generated between $t=1$ and 4

Solution:

$$a) A(0) = 100$$

$$\alpha(t) = 1 + 0.03\sqrt{t}$$

$$A(t) = \alpha(t) \times A(0)$$

$$\text{At } t=4$$

$$A(4) = (1 + 0.03\sqrt{4}) \times 100$$

$$= 1.06 \times 100$$

$$= 106$$

b) Amount of interest generated between $t=0$ and 4 = $A(4) - A(0)$

$$= 106 - 100 = 6$$

At time $t=1$

$$A(1) = (1 + 0.03\sqrt{1}) \times 100$$

$$= 1.03 \times 100$$

$$= 103$$

Amount of interest generated between $t=1$ and 4 = $A(4) - A(1)$

$$= 106 - 103$$

$$= 3 \text{ rupees}$$

- 7) Suppose that the accumulation function for an account $\alpha(t) = 1 + 0.5t$. You invest Rs.500 in this account today find 'i' if the account value after 12 years for now is Rs.1250.

Solution: $\alpha(t) = 1 + 0.5t$

$$\alpha(t) = A(t) / A(0)$$

$$1 + (0.5 \times 12) = 1250 / 500$$

$$1 + 6i = 2.5$$

$$6i = 2.5 - 1$$

$$= 1.5$$

$$i = 1.5 / 6 = 0.25$$

$$i = 0.25 \times 100$$

$$= 25\%$$

Force of interest

Before entering into force of interest we have to understand what is nominal rate of interest.

Suppose the interest rate is 12% per annum and interest is paid at the end of each quarter. This is said to be Nominal interest because the interest is paid several times during the period of one year for which the interest is given.

A rate is said to be nominal if interest is paid 'm' times. If the interest is compounded continuously the accumulated amount of function $A(t)$ is a continuous function of t . Then the nominal rate is called force of interest and is denoted by δ . Sometimes it is denoted by $i^{(a)}$.

For an investment that grows according to accumulated amount function, $A(t)$, the force of interest at time t is defined as $\delta_t = A'(t) / A(t)$, where $A'(t)$ is the derivative of $A(t)$

with respect to 't'. Using force of interest, we can measure interest at any moment of time.

Problems

- 1) You are given that $A(t)=at^2+bt+c$, for $0 \leq t \leq 2$ and $A(0) = 100$, $A(1) = 110$, $A(2) = 136$. Determine the force of interest at time $t=1/2$. Solution: $A(0) = 100$, $A(1) = 110$, $A(2) = 136$

$$A(t)=at^2+bt+c$$

$$A(0) = a \times 0^2 + b \times 0 + c = 100 = c \dots \dots \dots (1)$$

$$A(1) = a \times 1^2 + b \times 1 + 100 = 110$$

$$A + b = 110 - 100 \quad a + b = 10 \dots \dots \dots (2)$$

$$A(2) = a \times 2^2 + b \times 2 + 100 = 136$$

$$4a + 2b = 36 \dots \dots \dots (3)$$

From (2), $a = 10 - b$

$$(3) \Rightarrow 4(10 - b) + 2b = 36$$

$$40 - 2b = 36$$

$$-2b = -4$$

$$b = 2 \dots \dots \dots (4)$$

$$a + b = 10$$

$$a = 10 - 2$$

$$= 8$$

Now we got $a = 8$, $b = 2$, $c = 100$.

$$A(t) = at^2 + bt + c$$

$$= 8t^2 + 2t + 100$$

Now, $\delta_{t=1/2} = A'(t)/A(t)$

$$= [d/dt(8t^2 + 2t + 100)] / (8t^2 + 2t + 100)$$

$$= 16t + 2 / (8t^2 + 2t + 100)$$

$$= 16.1/2 + 2 / [8.1/4 + 1/2.2 + 100]$$

$$= 10/103$$

$$= 9.708\%$$

- 2) Suppose that $\alpha(t) = 0.10t^2 + 1$. The only investment made is Rs.300 at time 1. Find the accumulated value of the investment at time 10.

Solution:

$$A(1) = 300$$

$$\alpha(t) = 0.10t^2 + 1$$

$$\alpha(10) = 0.10 \times 10^2 + 1 = 11$$

$$\alpha(1) = 0.10 \times 1^2 + 1 = 1.1$$

$$A(1) = \alpha(1) \times A(0)$$

$$300 = 1.1 \times A(0)$$

$$A(0) = 300 / 1.1 = 272.72$$

$$A(10) = A(0) \times \alpha(10)$$

$$= 272.72 \times (0.10 \times 10^2 + 1)$$

$$= 3000$$

- 3) Using the constant force of interest of 4.2%. Calculate the present value of the payment Rs.1000 to be made in 8-year time. Solution:

$$P = A(1+i)^{-t}$$

$$= 1000(1+i)^{-8}$$

$$= 1000e^{-8 \times 0.042}$$

$$= 1000 \times e^{-0.336}$$

$$= 1000 \times 0.7146$$

$$= 714.6$$

- 4) A loan of Rs. 3000 is taken on June 23 rd 1997. If the force of interest is 14%, find each of the following.

a) The value of the loan on June 23 rd 2002

b) The value of i Solution:

We have, $\delta = 14\%$, $P = 3000$, $t = 5$ years

$$A = P(1+i)^t$$

$$= 3000(1+i)^5$$

$$= 3000 \times e^{\delta t}$$

$$= 3000 \times e^{0.14 \times 5}$$

$$= 3000 \times 2.014$$

$$= 6042$$

b) Value of i

$$(1+i)^t = e^{t\delta}$$

$$(1+i) = e^{\delta}$$

$$i = e^{\delta} - 1$$

$$i = e^{0.14} - 1$$

$$= 1.15 - 1$$

$$= 0.15$$

Value of i = 15%

Stoodley's Formula for Force of Interest

$P + S / (1 + ixe^{st}) = \delta(t)$, Where P and S are constants.

Basic compound interest relations

Relationship between s, i, v and d

S – Sum of the value or accumulated value

V -initial amount value

i-interest rate

d-duration of time

$$S = v(1+i)^d$$

The equation of value and yield on a transaction

All financial decision must take into account the basic ideas that money has time value. In a financial transaction each payment should have an attached date, i.e., mathematics of finance deals with the dated values.

Example: At a simple interest rate 12% of Rs.100 due in one year is considered to be equivalent to Rs.112 in 2 years, since Rs.100 would accumulate to Rs.112 in the 2nd year.

In the same way, $A(1+i)^{-1} = 100(1+0.12)^{-1} = 89.29$ would be considered an equivalent sum at present.

Definition of equation of value (dated value)

Rs. X due on a given date is equivalent at a given simple interest rate 'i' to Rs.Y due T years later if, $Y = X(1+it)$

$$X = Y(1+it)^{-1}$$

i.e., when we move money forward, we accumulate by the multiplication factor $(1+it)$.

When we move money backward, we discount by the multiplication factor $(1+i)^{-1}$.

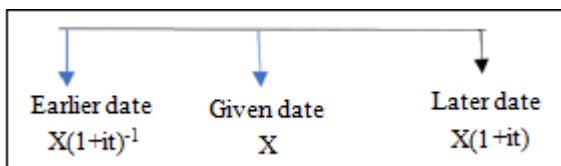
The sum of a set of dated values due on different dates has no meaning. We have to replace all the dated values by equivalent dated values due on same date. The sum of equivalent values is called the dated value of the set.

We say that two set of payments are equivalent at a given simple interest rate, if the dated values on a common date on 2 set of payments are equal is called an equation of value or an equation of equivalents.

The date used is called the focal date or the comparison date.

Use the following steps for solving problems.

Step 1: Make a good time diagram showing the dated values of one set of payments on one side of the time line and the dated values of the second set of payments on the other side.



Step 2: Select a focal date and bring all the dated values to this focal date using the specified interest rate.

Step 3: Set up an equation of value at the focal date.

Step 4: Solve the equation of value using the appropriate methods of algebra.

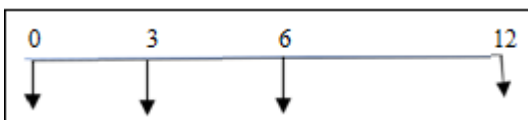
Problems

- 1) An obligation of Rs.1500 is due in 6 months with interest rate at 11%. At 15% simple interest find the value of the obligation,
 - a) at the end of 3 months, b) at the end of 12 months.

Solution:

The value of obligation in 6 months is $=1500[1+(11/100 \times 6/12)]$
 $= 1582.5$

Let X be the value of obligation at the end of 3 months and Y be the value of obligation at the end of 6 months.



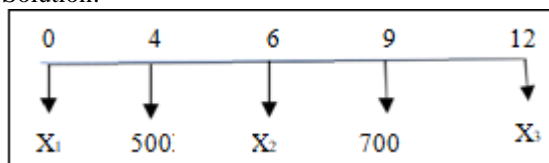
a) The value of obligation at the end of 3 months
 $X=1582.5(1+0.15 \times 3/12)^{-1}$
 $=1525.30$

b) The value of obligation at the end of 12 months
 $Y=1582.5(1+0.15 \times 6/12)$
 $= 1701.1875$

- 2) Mr. Gill owes Rs.500 due in 4 months and Rs.700 due in 9 months. What single payment a) now) in 6 months c) in

1 year will liquidate these obligations if money is worth 11%.

Solution:



$$X_1 = 500(1+0.11 \times 4/12)^{-1} + 700(1+0.11 \times 9/12)^{-1}$$

$$= 482.16 + 646.65$$

$$= 1128.81$$

$$X_2 = 500(1+0.11 \times 2/12) + 700(1+0.11 \times 3/12)^{-1}$$

$$= 509.13 + 681.2652$$

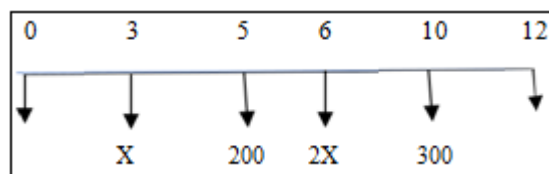
$$= 1190.39$$

$$X_3 = 500(1+0.11 \times 8/12) + 700(1+0.11 \times 3/12)^{-1}$$

$$= 536.65 + 719.25$$

$$= 1255.9$$

- 3) Mrs. Adam has two options available in repaying a loan. She can pay Rs. 200 at the end of 5 months and Rs.300 at the end of 10 months. Or she can pay rupees X at the end of three months and rupees 2X at the end of six months. If the options are equivalent and money is worth 12% find X using the focal date a) fund at the end of six months b) the end of three months.



Dated value of Option 1 is

$$X(1+i) + X(1+i)^{-1} = 200(1+12/100 \times 1/12) + 300(1+12/100 \times 4/12)^{-1}$$

$$= 201.92 + 288.57$$

$$= 490.49$$

Dated value of option 2 is

$$X(1+i) + 2X = X(1+12/100 \times 3/12) + 2X$$

$$= 1.03X + 2X$$

$$= 3.03X$$

Since the options are equivalent $3.03 X = 490.46$
 $X = 490.6/3.03 = 161.86$

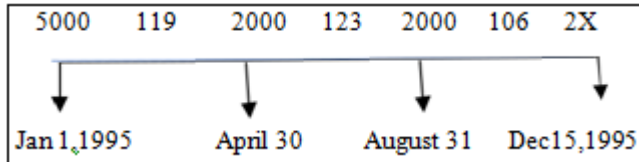
b) Dated value of Option 1 = $X(1+i) + X(1+i)^{-1}$
 $= 200(1+12/100 \times 2/12) + 300(1+12/100 \times 7/12)$
 $= 196.27 + 280.47$
 $= 476.74$

Dated value of option 2 is $X + 2X(1+i)^{-1} = X + 2X/1.03$
 $= (1.03X + 2X)/1.03$
 $= 3.03X/1.03$
 $= 2.94 X$

Since the options are equivalent, $476.74 = 2.94X$
 $X = 162.156$

- 4) Ramu borrowed rupees 5000 on Jan 1, 1995. He paid rupees 2000 on April 30, 1995 and rupees 2000 on August 31, 1995. The final payment was made on December 15, 1995. Find the size of the final payment if the rate of interest was 7% and the focal date was a) December 15, 1995 and b) January 1, 1995.

Solution



a) Equation of value on Dec 15, 1995
 Dated value of payment = Dated value of debits
 $2000(1+7\% \times 229/365) + 2000(1+7\% \times 106/365) + X$
 $= 5000(1+7\% \times 348/365)$
 $4126.6 + X = 5332.5$
 $X = 5332.5 - 4126.6$
 $= 1205.9$

b) Dated value of payment = dated value of debits

$2000(1+7\% \times 119/365)^{-1} + 2000(1+7\% \times 242/365)^{-1}$
 $+ X(1+7\% \times 348/365)^{-1} = 5000$
 $1955.4 + 1912.04 + X/1.0667 = 5000$
 $3867.44 + X/1.0667 = 5000$
 $X/1.0667 = 1132.56$
 $X = 1208.1017$

Equation of value -Compound interest rate

Rs.X due on a given date is equivalent at a given compound interest rate 'i' to Rs.Y due on 't' periods later, then $Y = X(1+i)^t$ and $X = Y(1+i)^{-t}$.

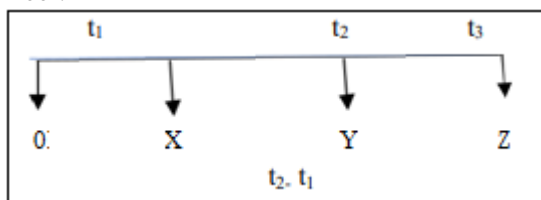
Two important properties of equivalents at compound interest rate.

- At a given compound interest rate, if X is equivalent to Y and Y is equivalent to Z, then Z is equivalent to X (transitivity)
- If two set of payments are equivalent on one focal date, then they are equivalent on any focal date.

Problems

- Prove that a given compound interest rate if $X=Y$ and $Y=Z$, then prove that $Z=X$.

Proof:



If X is equivalent to Y, then $Y = X(1+i)^{t^2-t^1}$

If Y is equivalent to Z, then $Z = Y(1+i)^{t^3-t^2} = X(1+i)^{t^2-t^1} X(1+i)^{t^3-t^2}$

$= X(1+i)^{t^2-t^1+t^3-t^2}$

$= X(1+i)^{t^3-t^1}$

$Z = X(1+i)^{t^3-t^1}$

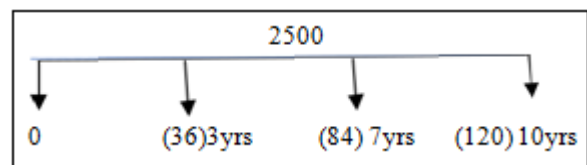
Therefore $X=Z$

- An obligation of Rs.2500 false due at the end of 7 years. If money is worth $j_{12} = 10\%$. Find an equivalent debit at the end of a) 3 years b) 10 years.

($j_{12} = 10\%$ means that nominal rate of 10% compounded 12 times per year)

Solution:

$j_{12} = 10\%$
 $i = (10/12) \times (1/100)$
 $= 0.0083$



a) $X = 2500(1+i)^{48}$
 $2500(1.0083)^{48}$
 3717.48

b) $Z = 3717.48(1+i)^{84} = 3717.48(1.0083)^{84} = 7443.703$

- A consumer buys goods worth Rs.1500 paying Rs. 500 down and rupees 500 at the end of six months. If the store charges interest at $J_{12} = 18\%$ on the unpaid balance, what final payment will be necessary at the end of first year?

Solution:

Focal date at the end of 12 months.
 $500(1+18/1200)^6 + X = 1000(1+18/1200)^{12}$
 $546.72 + X = 1195.618$
 $X = 1195.618 - 546.72$
 $= 648.90$

- A man stipulates in his will that rupees 50,000 from his estate is to be placed in a fund from which his three daughters are to receive the same amount when aged 21. When the man dies the girls are aged 19, 15 and 13. How much will each receive if the fund earns interest at $J_2 = 12\%$.

Solution: If each daughter gets X each then,
 $50000 = X(1+12/200)^{-4} + X(1+12/200)^{-12} + X(1+12/200)^{-16}$
 $50,000 = 1.683X$
 $X = 50000/1.683 = 29708.85$

2. Conclusion

Dovetailed different parts of the referred books and online contents to bring a simple and concrete way to understand the concept clearly.

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