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Competition and Collaboration: A Survey-based Approach to Reviewing the Role of Mathematical Olympiads in Improving Student Learning Outcomes

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Abstract: ***It is widely agreed that school learning is unable to impart all the necessary arithmetical skills to all the students equally. Gaps are often seen along the lines of class and gender divide - the higher the student in the hierarchy of wealth and the less marginalised their gender identity, the better their chances at imbibing mathematical knowledge. In order to supplement the glaring gap in learning outcomes, there is an urgent need for local support in the form of personalised teaching and assessment. Towards that end, a first-of-its-kind open mathematical olympiad (OMO) was conducted by the author in collaboration with a high-impact organisation in Delhi. The objective of the OMO was to address the gap in learning and skill between various students. In an effort to combine standardised teaching with personalised modes of delivery, the test was divided into three levels and gave the test-takers ample material (such as the test trainer) to develop their individual skills. The results of the test confirm previous hypotheses in the field and provide useful insight to classroom teachers as well as policymakers. In order to stimulate further research on supplementing learning outcomes via local modes of engagement, the test-trainer used for conducting the OMO has been attached to the end of the paper as an appendix.***

Keywords: Learning outcomes, Inequality, Gender gap, Personalised Learning

1. Introduction

Mathematics is an indispensable skill for adults. Almost all of a person's daily activities depend on how well they know their arithmetic. Whether it is calculating the time their daily commute requires or their ability to accurately record and assess their finances, all require a sound knowledge of mathematics. It was rightly said by Shakuntala Devi that without mathematics, there is nothing that one can do - "[e]verything around you is mathematics." (Trivedi, 2020, para. 4).

Yet, mathematics continues to be a dreaded subject for most school-going students. Students fear their subject owing to its perceived complexity and consequently, reduce their level of engagement with this subject (Mahapatra, 2020, para. 1). One of the causes behind such reduced engagement and unwarranted anxiety is the improper pedagogy employed in schools. Teachers themselves treat mathematics as a complex and high-stakes subject which, in turn, induces anxiety in the student. Often lack of clarity on the part of the teachers also contributes towards the dread (Mahapatra, 2020, para. 4). Reports state that an overwhelming 82% of students from grades 7 to 10 are fearful of the subject and that only 9% of students in grade 10 are even confident about their engagement with the subject (Roshni, 2021, para. 9-10). While many students worry about their performance in school and are anxious when they have to take exams, large proportions of students report feeling anxious about mathematics in particular (Ashcraft and Ridley, 2005; Hembree, 1990; Wigfield and Meece, 1988).

To the extent that mathematical knowledge depends on appropriate pedagogy, it also depends on the economic status of the student. It is already widely accepted that the higher the income, the better the services a person can afford (Caserta, 2008). Insightful and clear mathematical education is essentially a service. The better placed a student vis-a-vis the income level of their parents, the better their chances are at gaining good mathematical knowledge. This situation has undesirable implications: inequality in educational performance can come to mirror the inequality in wealth. Further, the effects of such inequality may only compound over time leaving later generations more worse off than earlier generations. In that context, it becomes essential to impart quality education to students from lower income groups so as to arrest the undesirable effects of this inequality.

The paper expounds on the impact created by an open mathematical olympiad in improving learning outcomes of children from lower-income groups. The first section of the paper details the broader role that mathematics olympiads have played in supplementing school teaching and augmenting learning outcomes. It also describes the limitations and disadvantages that have accompanied such olympiads. The second section of the paper describes the open mathematical olympiad and the results obtained. It argues that conducting such olympiads can best augment the learning outcomes of students, particularly those from lower-income groups. It concludes by giving policy suggestions for further improving learning outcomes and conducting supplementary learning activities and expanding opportunities for the same.

2. Background

Mathematical competitions began as inter-school competitions in the Austro-Hungarian Empire in the 19th century. The aim of such competitions can reasonably be inferred to have been the augmentation of a students' skills and abilities. The modern era of mathematical competitions began in 1959, when the first international mathematical Olympiad between the seven countries of the Soviet bloc took place in Romania. Each country could send a maximum of eight contestants and there were six questions with different points with a maximum possible total of forty points (International Mathematical Olympiad Foundation, n.d.). Since 1959, the number of Olympiads held has increased along with the number of students that participate in the Olympiads.

However, the Olympiads suffer from a host of associated problems. In the first place, the Olympiads have become a status symbol for parents and their children. Given the high level of prestige that surrounds the Olympiads - and the science Olympiads in particular - an entire economy of private firms and private Olympiads has emerged that preys on anxious parents (Fernandes, 2016). These companies usually conduct their exams through schools (with at least 10 students per school enrolling) and have even emerged as an alternative to the no-exam / continuous assessment model. Although the exams are voluntary, more and more children, some as young as six, have taken them (Fernandes, 2016). An entire predatory economy has thus emerged that gives priority to profit rather than learning outcomes of the students. Such conditions where learning is being outsourced give less incentive to teachers to teach properly as well. If students are going to be learning mathematical knowledge elsewhere, they might be inclined to let the private companies do all the teaching.

Secondly, there has been a gender gap as well within the Olympiads. The mathematics community has a hard time dealing with what it calls the "10 percent rule." In advanced math courses or competitions, there is only one woman for every nine men. It has gotten so bad that the math community is considering switching to an all-girls' Olympiad (Holmes, 2015). There have been discussions about whether a girl competition like IMO could increase that percentage. Its opponents argue that the European Girls' Mathematical Olympiad (EGMO) is considered a "second- class" competition (Holmes, 2015). A report by the Organization for Economic Cooperation and Development states that girls lack self-confidence in their own abilities in mathematics. Girls at every proficiency level in mathematics and science tend to report greater anxiety towards mathematics and lower levels of self-efficacy and self-concept (OECD, 2015). It is possible that girls' greater motivation to do well in school and the greater investment they make to achieve this goal are undermined by their lack of self-confidence in scientific subjects, particularly when girls are capable of achieving at the highest levels (Beilock and Carr, 2001).

Lastly, as already pointed above, the Olympiads are greatly inaccessible. Almost all the editions of the International Mathematical Olympiad have taken place in Europe (International Mathematical Olympiad Foundation, n.d.). As such, there are huge monetary costs associated with reaching even the venue of the Olympiads, never mind the skillset required to solve the problems that eventually appear in the Olympiad. The end result is that only the students with the resources to cover the costs, or with the capabilities to secure funding, are able to go to the International Mathematical Olympiads and learn useful skills to supplement their learning.

Keeping the aforementioned in mind, the students who lose out in this competitive process often belong to lower- income groups and tend to be girls, largely in the Indian context, where the OMO was conducted. What is required to build the gap are grassroots efforts that deliver quality learning to students from these groups. It is in effort to bridge such a gap that the Open Mathematical Olympiad was designed and conducted.

3. Discussion

The Open Math Olympiad (OMO) was an alternative mathematical test designed to encourage academic enrichment and achievement in lower income schools in India. It was a paper-based test - keeping in mind the income status of the test-takers - and was developed in collaboration with experienced educators to provide an accessible yet challenging opportunity for talented young minds to improve their mathematical potential, without being constrained by the formatting of traditional curricula and exams. The necessity for the OMO was felt due to the inaccessible nature of good mathematical pedagogy and the International Mathematical Olympiads. The OMO partnered with the Prajna foundation which is an organisation that works towards poverty upliftment of the children living in the slums of Delhi. In order to generate impact effectively, Prajna operates six community centres across New Delhi (Prajna Foundation, n.d.). As such, the OMO was conducted in partnership with Prajan to utilise its ability to deliver large-scale social impact.

There are three test levels within the OMO that prospective test-takers can apply for depending on their grade and ability. While the levels correspond to some prior level of mathematical knowledge, the test is not bound by an academic curriculum and is instead designed to test intelligence and aptitude. It focuses on encouraging the application of knowledge whereby students can derive their own techniques and methods to solve problems. This approach was preferred as opposed to the rather arcane approach of conforming to standardised test formulas which only encourage rote-learning. The questions were designed using community centred topics and relatable word problems. The three levels of the test were as follows (see Appendix-1 for further details):

1) Level 1: This level was also called 'The Shakuntala Devi' as it covered topics in basic multiplication, division, shapes, matrices. It was meant for students from grades six to eight.
2) Level 2: This level was also called 'The Brahmagupta'. It covered topics in geometry and algebra, and was meant for students from grades nine to ten.
3) Level 3: This level was also called 'The Bhaskara'. It covered more advanced topics like pre-calculus and basic calculus and it was meant for students from grades eleven to twelve.

The OMO test was preceded by a test trainer. The test trainer was intended to be a resource that enables students to practise problem solving and to familiarise themselves with the format of the olympiad prior to the test day. The test trainer was a crucial component in bridging the gap between the opportunities and learning afforded to students from a lower-income group and to those from a higher-income group. Components of the test trainer and parts of the actual test were designed keeping in mind the first-hand experience of the author of teaching the test trainer to the test-takers in Delhi. It was also conducted keeping in mind previous research on the strong correlation between gender and confidence in mathematical knowledge. The introduction of the test trainer was meant to ensure that all students feel comfortable and confident enough to tackle the main exam - especially those who may have never had the chance of participating in international olympiads before. Each level had its own corresponding test trainer.

The first edition of the OMO was held with seventeen test-takers across level one and two - which included seven girls and ten boys between the ages of ten to sixteen. Students were provided with the stationery required for taking the test, the question paper, an answer sheet, and a rough paper to work on. Based on the administration of the OMO, a number of trends could be observed among the test takers: firstly, in the level one paper, students faced difficulty in questions regarding units that pertained to mensuration problems. Therefore, whenever there are surface area/volume related problems being taught in classroom settings, an emphasis needs to be placed on the units of measurement in addition to formulaic applications for certain shapes. Secondly, for the level 2 paper, the students found it taxing to navigate layered problems. This meant that less developed verbal aptitude was an impediment to students being able to attempt a wide range of problems. The average score of students was 9.6 out of 20, and 9.2 out of 25 for Level 1 and Level 2, respectively. The corresponding percentage for these averages were 48%, for Level 1, and 36.8%, for Level 2. Thus, it can be inferred that students are marginally more comfortable with mathematics in younger classes with fundamental concepts, but there proves to be a gap in the skills acquired over time and instructions as more concepts get added on to previous ones. Across levels one and two, girls performed better than boys. This further indicates that when girls are given proper training and attention, there is no reason for them to be less confident about their abilities in mathematics.

4. Conclusion

Thus, the first edition of the Open Math Olympiad proved to be insightful in a number of ways. It showcased the different problems that lie in imparting quality education to students belonging to lower-income groups and to those who belong to the gendered minority in educative opportunity. In covering the concepts and administering the test, key areas of focus were identified that can now be worked upon by future administrators. Areas of knowledge such as units, mensuration, and formulas need to be emphasised upon by anybody who undertakes the task of imparting knowledge in a classroom setting. It should be further noted that these areas are fundamental and a broader policy- based effort is needed to bolster learning at the early stages of a student's school life.

The three levels administered by the Open Math Olympiad also further the credibility to these insights since it served the purpose of relevantly dividing the sample of test-takers. The strong performance of girls across levels one and two supports the previous research hypotheses that given enough training and knowledge, there is no reason why girls should underperform on tests as compared to boys. Such a finding has importance for international competitions like the International Mathematical Olympiad and can be used to rectify the gendered and class-based imbalances present there. As expected, it is not a matter of innate disposition between genders – although there may be innate disposition between individuals where some people may be born with more of a mathematical mind – but a matter of socially created mental blocks. Further, the entire enterprise of conducting local olympiads must be sustained owing to how it adequately supplements the learning outcomes of children. The research by OMO also suggests that following a pre-test test trainer approach may be helpful in boosting outcomes.

Appendix-1: The Test-Trainer for the Open Mathematical Olympiad

The trainer pack is specific to each level and contains 5 problem sets on the following concepts with answers and explanations. The first page outlines the topics covered under each test and then subsequently.

Level 1: The Shakuntala Devi

1) Multiplication and division
2) Unitary Method
3) Mensuration -
    a) Circle
    b) Rectangle
    c) Triangle
4) Surface Areas and Volumes

Level 2: The Brahmagupta

1) Linear Equations in Two Variables
    a) Word problems
2) Distance - Time Problems
3) Triangles
4) Surface Areas and Volumes:
    a) Parallelograms
    b) Circles
    c) Triangles
    d) Cylinders
5) Properties of Triangles

Level 3: The Bhaskara

1) Trigonometry
2) Complex Numbers and Quadratic Equations
3) Linear Inequalities
4) Limits and Derivatives
5) Probability
6) Relations and Functions
7) Continuity and Differentiability

Level 1: The Shakuntala Devi

Multiplication and Division—

1) Rohan piled the class-IV Maths books into 8 stacks of 6 each. How many books did he put on the shelf?
a) 8 + 6
b) 8 – 6
c) 8 ÷ 6
d) 8 x 6
Answer: (d)

There are 8 stacks of 6 books each which means 6 books 8 times, which is 48.

2) He piled the class-III English reader into 9 stacks of 5 each. How many books did he put on the shelf?
a) 45
b) 54
c) 14
d) 25

Answer: (a)
There are 9 stacks of 5 books each which means 5 books 9 times, which is 45.

3) He organised 63 class-V Social Studies books in 9 neat stacks. How many books per stack is that?
a) 63
b) 7
c) 9
d) 11

Answer: (b)
The 63 books were divided into 9 piles. 63 divided by 9 is 7, thus there are 7 books in each of the 9 piles.

4) Reshma has 49 toffees which she wants to eat over the week. How should she divide the toffees so that she eats the same number of toffees everyday?
a) 8 toffees every day
b) 5 toffees every day
c) 7 toffees every day
d) 49 toffees every day

Answer : (c)
Since there are 7 days in a week and Reshma wants to eat the same number of toffees each day, she must divide the 49 toffees equally over the days of the week. 49 divided by 7 is 7, therefore she must eat 7 toffees every day.

5) Kamla went to the market with 20 rupees. She spent half of her money at the cycle repair shop. On her way home she spent half of the remaining money on chocolate. How much money does Kamla have left?
a) 20 rupees
b) 10 rupees
c) 5 rupees
d) No rupees

Answer: (c)
Rs 20 halved once is Rs 10. Rs 10 halved again is Rs 5. Thus Kamla has 5 rupees left.

The Unitary Method—

1) If the cost of 24 oranges is Rs 72, then what is the cost of 120 oranges?
a) Rs 18
b) Rs 360
c) Rs 172
d) Rs 500 Answer: (b)

24 oranges = Rs 72 thus
1 orange= Rs 72 ÷ 24 = Rs 3, therefore
120 oranges = Rs 3 x 120 = ***Rs 360***

2) If the cost of 15 eggs is Rs 75, then find out the cost of 4 dozen eggs.
a) Rs 240
b) Rs 300
c) Rs 150
d) Rs 185 Answer: (a)

15 eggs = Rs 75 thus
1 egg = Rs 75 ÷ 15 = Rs 5 and
4 dozen eggs = 4 x 12 eggs = 48 eggs therefore
4 dozen eggs = Rs 48 x 5 = ***Rs 240***

3) A worker makes a toy every 2 hours. If they work for 80 hours, then how many toys will they make?
a) 40 toys
b) 54 toys
c) 45 toys
d) 39 toys

Answer: (a)
2 hrs =>1 toy
1 hrs =>0.5 toy
80 hrs =>80 x 0.5 toys = ***40 toys***

4) 12 people can do a piece of work in 24 days. How many days are needed to complete the work if 8 people are engaged in the same work?
a) 52 days
b) 28 days
c) 48 days
d) 36 days

Answer: (d)
12 people can do the work in 24 days, then
1 person will do it in 24 × 12 days, i.e. 288 days, therefore 8 people will do it in 288 ÷ 12 days, i.e. ***36 days.***

5) If 45 m of a uniform rod weighs 171 kg, then what will be the weight of 12 m of the same rod?
a) 49 kgs
b) 42.5 kgs
c) 45.6 kgs
d) 55 kgs Answer: (c)

45 m weighs 171 kgs, then
1 m weighs 171 ÷ 45, i.e. 3.8 kgs thus
12 m weighs 12 x 3.8, i.e. ***45.6 kgs.***

Mensuration—

1) What is the area of a triangle with base: b, and height: h ?

a) b x h
b) b ÷ h
c) 0.5 x b x h
d) 2.0 x b x h

Answer: (c)

2) How do you calculate the area of a square of side: a ?

a) 2a
b) 4a
c) a ÷ a
d) a x a

Answer: (d)

3) The area of a parallelogram with base: b, and height: h, is:

a) b x h
b) ½ x b x h
c) b + h
d) None of the above

Answer: (a)

4) Area of a circle with radius 'r' is:

a) πr^2
b) ½ πr^2
c) $2\pi r^2$
d) $4\pi r^2$

Answer: (a)

5) If there are 10 millimetres in a centimetre, how many square millimetres are there in a square centimetre?

a) 10 sq mm
b) 100 sq mm
c) 20 sq mm
d) None of the above

Answer: (b)
1 cm = 10 mm, thus
$(1\ cm)^2 = (10\ mm)^2$, therefore 1 sq cm = ***100 sq mm***

Surface Areas and Volumes—

1) If the radius of a cylinder is 4 cm and its height is 10 cm, then the total surface area of a cylinder is: [Take π=22/7]

a) 440 sq cm
b) 352 sq cm
c) 400 sq cm
d) 412 sq cm

Answer: (b)
Total Surface Area of a Cylinder = 2πr(r + h) Here, r = 4 cm, and h = 10 cm

Thus, TSA = 2 x 22/7 x 4(4 + 10) sq cm
= ***352 sq cm***

2) The curved surface area of a right circular cylinder of height 14 cm is 88 sq cm. The diameter of the base is: [Take π=22/7]

a) 2 cm
b) 3 cm
c) 4 cm
d) 6 cm

Answer: (a)
Given:
Curved Surface Area of the cylinder = 88 sq cm Height (h) = 14 cm
Since, for a cylinder, CSA = πdh And here, CSA = πdh = 88 sq cm

The Diameter (d) = CSA/πh
= 88/(π x 14) cm
= 88/44 cm
= ***2 cm***

3) The Curved surface area of a right circular cylinder is 4.4 sq cm. The radius of the base is 0.7 cm. The height of the cylinder will be: [Take π=22/7]

a) 2 cm
b) 3 cm
c) 1 cm
d) 1.5 cm

Answer: (c)
Given:
Curved surface area of the cylinder = 4.4 sq cm

Since, for a cylinder, CSA = 2πrh And here, CSA = 2πrh = 4.4 sq cm
The Height (h) = CSA/2πr
= 4.4/(2π x 0.7) cm
= ***1 cm***

4) The diameter of the base of a cone is 10.5 cm, and its slant height is 10 cm. The curved surface area will be:

a) 150 sq cm
b) 165 sq cm
c) 177 sq cm
d) 180 sq cm Answer: (b)

Curved Surface Area of cone = πrl Here, r = diameter/2 = 10.5/2 = 5.25 cm and, l = 10 cm

Thus, CSA = π(5.25)(10) sq cm
= ***165 sq cm***

5) If the slant height of the cone is 21 cm and the diameter of the base is 24 cm. The total surface area of the cone will be: [Take π=3]

a) 1200 sq cm
b) 1177 sq cm
c) 1222 sq cm
d) 1188 sq cm

Answer: (d)
Total Surface Area of a cone = πr(l + r) Here, r = diameter/2 = 24/2 = 12 cm and, l = 21 cm
Thus, TSA = π(12)(21 + 12) = ***1188 sq cm***

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Level 2: The Brahmagupta

Linear Equations in two variables—

1) If x = 20 and 7x - 4y = 100, then what is y?
a) 19
b) 10
c) -7
d) 0

Answer: (b)
Since x = 20,
And y = (7x - 100)/4
*y = (7*20 - 100)/4*
Thus ***y = 10***

2) A: 12x - 5y = 1, and B: 8x + 3y = 45. Which of the following pairs of x and y satisfy the equations A and B?
a) 2 and 8
b) 9 and 4
c) 1 and 0
d) 3 and 7
Answer: (d)
Given A and B as above, we shall perform A - (3B)/2, Thus we have the following equation:
*12x - 5y - (3*8x)/2 - (3*3y)/2 = 1 - (3*45)/2, i.e.*
12x - 5y - 12x - (9/2)y = 1 - (135/2), i.e.
-5y - 4.5y = 1 - 67.5, i.e.
-9.5y = -66.5
Therefore, ***y = 7***
Thus x = (1 + 5y)/12
Therefore, **x** = 36/12= **3**

3) x = 5 and y = 3 satisfy which of the following equations?
a) 2x - 9y = 12
b) 4y + 3x = -5
c) 6x = 10y
d) 4x = 2x - 3y + 7

Answer: (c)
Substituting for x and y in the above equations we obtain the following:
a. 10 - 27 = 12 i.e. -17 = 12
b. 20 + 9 = -5 i.e. 29 = -5
c. 30 = 30, and
d. 20 = 10 -9 +7 i.e. 20 = 8

We can see that of the above only (c) is true. Thus ***the given x and y satisfy (c).***

4) Do equations A: 14x + 9y = 20, and B: 18y + 28x = 25, have:
a) A unique solution
b) No solution
c) Infinitely many solutions
d) None of the above

Answer: (b)
Assuming the forms ax + by = c, and px + qy = r for equations A and B, the ratios of the coefficients for the above equations are:
a:p = 1:2
b:q = 1:2

Since these are equal and equal to the the ratio of the constants, c:r = 1:2, the given pair of equations has ***no solution.***

5) Which of these is not a linear equation in two variables:
a) x - y = z
b) 7x + 3y = 2x
c) y = 5x
d) 3x - 8y = 0

Answer: (a)
Clearly, ***equation (a)*** *is an equation in three variables (x, y and z), and is therefore not an equation in two variables.*

Word Problems—

1) Asif bought 3 kgs of Potatoes and 5 kgs of Onions for Rs 210. From the same store, Malini bought 2 kgs of Potatoes and 1 kg of Onions for Rs 70. Calculate how much Potatoes and Onions cost per kg.
a) Rs 15 and Rs 20.
b) Rs 25 and Rs 40.
c) Rs 30 and Rs 10.
d) Rs 20 and Rs 30.

Answer: (d)
Expressing the above information as linear equations, Asif: 3x + 5y = 210, and
Malini: 2x + y = 70,
where x and y represent the cost of potatoes and onions respectively.
Solving for x and y, we get ***x = 20****, and* ***y = 30****. Thus (d) accurately represents the cost of Potatoes and Onions per kg.*

2) If 0.5 litres of ghee and 2 litres of oil together weigh 2.5 kgs, and if 1.5 litre of ghee and 1 litre of oil together weigh 3.75 kgs, then calculate how much ghee and oil weigh per litre.
a) 1 kg and 2 kg
b) 2 kg and 0.75 kg
c) 3 kg and 1 kg
d) 1.6 kg and 0.5 kg

Answer: (b)
Expressing the above information as linear equations, A : 0.5x + 2y = 2.5, and
B : 1.5x + y = 3.75,
where x and y represent the densities (weight per litre) of ghee and oil respectively.
Solving for x and y, we get ***x = 2****, and* ***y = 0.75****. Thus (b) accurately represents the weight per litre of ghee and oil.*

3) If it takes a craftsman 3 hours to make 2 pots and 120 diyas, and 2.5 hours to make 3 pots and 60 diyas. How long does it take to make one pot and one diya each?
a) 30 minutes and 1 minute
b) 0.5 hours and 10 minutes

c) 20 minutes and 2 hours
d) 1 hour and 5 minutes

Answer: (a)
Expressing the above information as linear equations, A : 2x + 120y = 3, and
B : 3x + 60y = 2.5,
where x and y respectively represent the time it takes to make one pot and one diya.
Solving for x and y, we get ***x = 0.5****, and* ***y = (1/60)****. Since 0.5 hrs is 30 minutes, and (1/60) hrs is 1 minute,*
(a) accurately represents the time it takes the craftsman to make one pot and one diya.

4) **At present Tara is thrice as old as her daughter. In five years' time, Tara's age will be five more than the double of her daughter's age. How old is Tara's daughter now?**
a) 5 years old
b) 10 years old
c) 8 years old
d) 15 years old

Answer: (b)
Let Tara's and her daughter's age at present be x and y respectively.
Then, at present, x = 3y —eq. (1)
After five years Tara and her daughter will be (x + 5) and (y + 5) years old, respectively.
Thus, after five years, (x + 5) = 5 + 2(y + 5) —eq. (2)
Substituting for x from eq. (1) we get,
3y + 5 = 5 + 2y + 10
Thus, ***y = 10***

Tara's daughter is therefore ***10 years old*** *at present as accurately indicated by (b)*

5) **The ratio of two numbers is 1:3. The sum of these numbers is 12. What will be the product of these two numbers?**
a) 15
b) 45
c) 27
d) 31

Answer: (c)
Let the number be x and y. Then, x/y = 1/3, i.e.
3x = y, and x + y = 12
Solving for x and y, we have x = 3 and y = 9. ***Thus the product of x and y will be 27****, as suggested in (c)*

Distance-Time Problems—

1) **The distance between Jaipur and Chandigarh is 500 kms. If a train covers this distance in 4 hours going one way, and in 5 hours going the other way. What is the approximate average speed of the train?**
a) 100 kms per hour
b) 120 kms per hour
c) 112 kms per hour
d) 97 kms per hour

Answer: (c)
Onward speed of the train = (500/4) kms per hour
= 125 kms per hour Return speed of the train = (500/5) kms per hour
= 100 kms per hour Average speed of the train = (250 + 100)/2 kms per hour
≈ ***112 kms per hour***

2) **A bus going at 1/3 of its usual speed takes half an hour extra to reach its destination. How much time does it usually take the bus to cover the same distance?**
a) 1 hour
b) Half an hour
c) 30 minutes
d) 15 minutes

Answer: (d)
*Say the usual speed of the bus is S, and the time it usually takes to cover the distance is T. The distance in question would then be D = S*T*
Now, at a third of its usual speed, the time it will take the bus to cover this distance would be,

T´ = D/(S/3)
*= 3(S*T)/S*
= 3T

Since, according to the question T´ = T + 30 min (sub. for T´)

	3T	*= T + 30 min*
Therefore,	*2T*	*= 30 min, and*
and,	***T***	***= 15 min***

3) **A car takes 4 hours to cover a certain distance when it travels at a speed of 40 km/h. What should be its approximate speed to cover the same distance in 1.5 hours?**
a) 107 km/h
b) 170 km/h
c) 117 km/h
d) 120 km/h

Answer: (a)
Say, the distance in question is D.
*Then D = 4 hours * 40 km/h i.e., D = 160 kms*

If the car covers this distance in 1.5 hrs, then its speed would be S = (160 kms)/(1.5 hrs)
i.e., S≈107 km/h

4) **Manas and Anna are standing 50 kms apart. If they both start cycling towards each other at 10 AM, and their average speeds are 10 km/h and 15 km/h respectively, at what time will they meet?**
a) 12:55 PM
b) 12:00 PM
c) 11:40 AM
d) 11:15 AM

Answer: (b)
When Manas and Anna do meet, they would have collectively traveled the entire distance of 50 kms. That is, the sum of the

distances traveled by them individually would be 50 kms.
Thus, if they meet T hours after starting to cycle,
*Manas would have traveled, M = 10 km/h * T hrs, i.e.*
*= 10T kms, and Anna would have traveled,A = 15km/h * T hrs, i.e.*
= 15T kms

Now, as established before, $M + A = 50$ *kms, i.e.*
$10T + 15T = 50$ *kms*
Thus, $T = 2$ *hrs,*

It would therefore take Manas and Anna hours to meet. Thus, starting at 10 AM, they would meet at **12 PM.**

5) It takes a train 2 minutes to pass by a woman standing at a platform. If the average speed of the train is 15km/h, how long is the train?
a) 1.2 km
b) 750 m
c) 550 m
d) 500 m

Answer: (d)
*The length of the train is simply the distance it travels in the given time at the given speed. Thus, Length of train = 15 km/h * 2 min*
*= 15 km/h * (1/30) hrs*
= ½ kms
= ***500 m***

Triangles—

1) If perimeter of a triangle is 100 cm and the length of two sides are 30 cm and 40 cm, the length of third side will be:
a) 30 cm
b) 40 cm
c) 50 cm
d) 60 cm

Answer: (a)
The perimeter of the triangle = sum of all its sides
Thus, 100 cm = 30 cm + 40 cm + length of the third side
Therefore, length of the third side = 100 - 70 = ***30 cm***

2) Triangles ABC and DEF are similar. Now if AB = 4 cm, DE = 6 cm, EF = 9 cm, and FD = 12 cm, what would be the perimeter of triangle ABC?
a) 22 cm
b) 20 cm
c) 21 cm
d) 18 cm

Answer: (d)
Since the given triangles are similar, their corresponding sides must be in the same ratio. That is, AB : DE = BC : EF = CA : FD
Since we have been given some of these sides, we get,
4 : 6 = BC : 9 = CA : 12
i.e. 2 : 3 = BC : 9 = CA : 12

*This means, BC = (2/3)*9 and CA = (2/3)*12*
i.e. BC = 6 cm and CA = 8 cm

Therefore, the desired perimeter of ABC = AB + BC + CA
= 4 + 6 + 8
= ***18 cm***

3) What is the height of an equilateral triangle of side: a?
a) a
b) a
c) a
d) a

Answer: (b)
The height (h) of an equilateral triangle divides the base (a) into two equal parts. From the Pythagorean theorem it then simply follows that,

$h^2 = a^2 - (a/2)^2$
i.e. $h^2 = ¾ a^2$
Thus $h = \sqrt{(¾ a^2)}$
Therefore, the desired ***height (h) = a***

4) If ABC and DEF are two triangles and AB : DE = BC : FD, then the two triangles are similar if
a) ∠A = ∠F
b) ∠B = ∠D
c) ∠A = ∠D
d) ∠B = ∠E

Answer: (b)
If corresponding pairs of sides in two triangles are in the same ratio, then the triangles are similar if the angles between the pairs are equal. Thus, in this case, the given triangles are similar if ∠B = ∠D

5) The sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio:
a) 2 : 3
b) 4 : 9
c) 16 : 81
d) 81 : 16

Answer: (c)
Areas of similar triangles are always in a ratio that is the square of the ratio of their sides. Thus, in this case, the desired ratio = $(4 : 9)^2$ *=* ***16 : 81***

Surface Areas and Volumes—

1) If in a cylinder, the radius is doubled and height is halved, what would be its new curved surface area?
a) Twice as before
b) Same as before
c) Half as before
d) None of the above

Answer: (b)
Curved Surface Area of a cylinder = $2\pi rh$
Now, with $r \to 2r$*, and* $h \to ½ h$,
CSA = $2\pi rh \to 2\pi(2r)(½ h)$ *CSA =* $2\pi rh \to 2\pi rh$

The CSA thus stays ***the same as before.***

2) An equilateral triangle made of a wire is opened up and made into a circle. If the area of the initial triangle is 10 sq cm and its height is 5 cm, what will be the area of the new circle? [Take π=3]

a) 12 sq cm
b) 10 sq cm
c) 5 sq cm
d) 14 sq cm

Answer: (a)
*Given the are of the initial triangle and its height we can ascertain its base length as Base = 2 * Area / Height*
*= 2 * 10 sq cm / 5 cm*
= 4 cm

Since the initial triangle was equilateral, its perimeter would be three times this base, i.e. 12 cm. As both were fashioned of the same wire, the perimeter (circumference) of the new circle would also be 12 cm. Given this circumference the radius of the circle would be,
Radius = Circumference / 2π
= 12 / 6 = 2 cm Thus, the are of the circle would be, Area = πr^2
*= 3 * 2 * 2 sq cm*
*= **12 sq cm***

3) The radii of two cylinders of the same height are in the ratio 4 : 5. What is the ratio of their volumes?

a) 5:4
b) 4:5
c) 25:16
d) 16:25

Answer: (d)
The volume of a cylinder = $\pi r^2 h$
Thus, the volume of the first cylinder = πr1 h1 And, the ^{2}
volume of of the second cylinder = πr2 h2

Then, the ratio of these volumes = $\pi r1^2 h1^2 : \pi r2^2 h2^2$
$= r1^2 : r2^2$
$= (r1 : r2)^2$

Since, as per the question, r1: r2 = 4: 5
$(r1 : r2)^2 = 16 : 25$

*Therefore, the desired **ratio of volumes = 16: 25***

4) A parallelogram is joined at the shorter edges to form a hollow cylinder. If the area of the initial parallelogram is 12 sq cm and its height is 3 cm, what will be the volume of the resulting cylinder? [Take π=3]

a) 5 sq cm
b) 3 cubic cm
c) 4 sq cm
d) 4 cubic cm Answer (d)

A cylinder formed by morphing a parallelogram like such would have the same height as the parallelogram, and its CSA would be the same as the area of the parallelogram.

Thus, height of the cylinder (h) = 3 cm And, CSA of the cylinder (2πrh) = 12 sq cm
Then, radius of the cylinder = 12/(2πh)
= 6/(πh)

Now, the volume of the cylinder = πr2h
= πh * $36/(\pi h)^2$
= 36/(πh)
= 36/(3*3)
= 4 cubic cm

5) 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. What will be the length of the wire in metres? [Take π=22/7]

a) 90 m
b) 168 m
c) 84 m
d) 336 m

Answer: (c)
Assuming the wire to be a uniform cylinder,
Radius of the cylinder (r) = diameter/2 = 0.5 mm
Volume of the cylinder (πr2h) = volume of the silver = 66 cu cm

Thus, length of the wire = height of the cylinder (h) = $66/\pi r^2$
= 66/{(22/7)(0.25)}*
= 84 m

Properties of Triangles—

1) Which of the following does not represent possible sides of a triangle?

a) 3, 5, and 4
b) 7, 2, and 8
c) 12, 13, and 9
d) 6, 18, and 30

Answer: (d)
All triangles share the property that the sum of any two of their sides is greater than the third (the triangle inequality rule). In the above examples, (d) does not satisfy said property, since 6 + 18 = 24 < 30. Therefore, (d) does not represent possible sides of a triangle.

2) If a right triangle is also isosceles, what will be its smallest angle?

a) 30 deg
b) 45 deg
c) 90 deg
d) 60 deg

Answer: (b)
One of the angles of a right triangle always measures 90˚. Thus, since the sum of all the internal angles of a triangle is always 180˚, the other two angles of this triangle must add up to 90˚.
Now, since in an isosceles triangle at least two of the internal angles must be equal, the triangle in question can either have the measures:

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90˚, 90˚ and 0˚, OR —ΔP
90˚, 45˚, and 45˚. —ΔQ
Clearly ΔP is not a valid triangle, thus ΔQ must be the triangle in question. The smallest angle of ΔQ is 45˚, *thus (b) is the correct answer.*

3) If the ratio of the corresponding sides of two similar triangles is 2 : 5, what is the ratio of the angles opposite to these sides?
a) 2 : 5
b) 3 : 3
c) 4 : 3
d) 4 : 25

Answer: (b)
*For similar triangles, the measures of all the corresponding angles are the same. Thus, the desired ratio will always be 1 : 1 (**in this case 3 : 3**). Therefore, option (b) is the correct answer.*

4) The shorter angle in an isosceles triangle measures half as much as the sum of the bigger angles. If the shorter side is 5cm, what will be the parameter of the triangle?
a) 15 cm
b) 10 cm
c) 5 cm
d) 25 cm

Answer: (a)
Since the triangle in question is isosceles, and the shorter angle (call A) seems to be the odd one out, the bigger angles (call B and C) must be the same. That is, B = C
Now, it is provided that, A = ½(B + C)
A = ½(2B) A = B

Since A = 5 cm, B = C = 5 cm
Therefore, the perimeter of the triangle = A + B + C
*= **15 cm***

5) ΔP and ΔQ are similar. Both of these triangles have a side that measures 5 cm. Are ΔP and ΔQ congruent?
a) Yes
b) No
c) Can't say.
d) None of the above

Answer: (c)
The information provided is not sufficient to reach a conclusion about the congruence of ΔP and ΔQ. One cannot say.

Level 3: The Bhaskara

Trigonometry—

1) In Δ ABC, right-angled at B, AB = 24 cm, and BC = 7 cm. The value of tan(C) is:
a) 12/7
b) 20/7
c) 7/24
d) 24/7

Answer: (d)
Given, AB = 24 cm and BC = 7 cm
*tan C = side opposite to C ÷ side adjacent to C therefore, tan C = **24/7***

2) (sin 30° + cos 60°) - (sin 60° + cos 30°) is equal to:
a) 0
b) 1 + 2√3
c) 1 -√3
d) 1 + √3

Answer: (c)
sin30°=½, sin60°= √3 / 2, cos30°= √3 / 2 andcos60°=½
Substituting for these values, we get:
(½+½)-(√3/2+√3/2) = 1 –[(2√3)/2]
= 1 –√3

3) The value of tan 60°/cot 30° is equal to:
a) 0
b) 1
c) 2
d) 3

Answer: (b)
tan60°= √3 andcot30°= √3 Hence,tan60°/cot30°= √3/√3 = ***1***

4) 1 – $\cos^2$A is equal to:
a) $\sin^2$ A
b) $\tan^2$ A
c) 1 - $\sin^2$ A
d) Sec^2A

Answer: (a)
We know from trigonometric identities that,
$\sin^2 A + \cos^2 A = 1$
*Therefore, $1 - \cos^2 A$ = **$\sin^2 A$***

5) Sin (90° - A) and cos A are:
a) Different
b) The Same
c) Not related
d) None of the above

Answer: (b)
From trigonometric identities, we know that, sin (90° - A) = cos A
*Therefore the above values are **the same.***

Complex Numbers and Quadratic Equations

1) The value of √(-144) is
a) 144i
b) ±12i
c) 12i
d) -12i

Answer: (c)
√(-144) = √(-1*144)
= √-1*√144
*= i * 12*
*= **12i***

2) A: $x^2 = 1$, and B: $x^3 = -1$. How many solutions do equations A and B each have?

a) 1 and 2
b) 1 and 0
c) 1 and 3
d) 2 and 3

Answer: (d)
Since all quadratic equations have two, and cubic equations have three solutions, the answer is (d): ***2 and 3.***

3) The square of a complex number is:

a) Always complex
b) Sometimes complex
c) Always real
d) Never real

Answer: (b)
A complex number is a number of the form:
z =a+bi, where b≠0 and **i** =√-1
then $z^2 = (a + bi)^2$
$= a^2 + b^2 i^2 + 2abi$
$= a^2 - b^2 + 2abi$
Thus depending on whether a = 0 or not, z^2 *can be either real or complex. Thus the square of a complex number is only* ***sometimes complex.***

4) Consider E: $x^3 + 4x^2 + 5x = 0$. Which of the following is a possible value of x?

a) -1
b) i - 2
c) 2 - i
d) -3 - i

Answer: (b)
The above polynomial factors as: $x(x^2 + 4x + 5)$
Thus, one of its roots is zero, while, using the quadratic formula: the other two roots =
= -2 ± i
= -2 - i and **-2 + i**
Thus, **i - 2** is a possible value for x.

5) Evaluate the expression, where is a complex cube root of unity. What do you get?

a) -1
b) 0
c) 1
d) i

Answer: (a)
The sum of the cube roots of unity is always zero: i.e.
Thus,
So, (since)

Linear Inequalities

1) If |x| < 5 then the value of x lies in the interval:

a) (-5, 5)
b) (-∞, -5)
c) (∞, 5)
d) (-5, ∞)

Answer: (a)
Given, |x| < 5
Then, x < 5 and -x < 5
i.e. x < 5 and x > -5 Thus, 5 > x > -5
Therefore, $x \in (-5, 5)$

2) The solution of the -12 < (4 - 3x)/(-5) < 2 is

a) 56/3 < x < 14/3
b) -56/3 < x < -14/3
c) 56/3 < x < -14/3
d) -56/3 < x < 14/3

Answer: (d)
Given, -12 < (4 - 3x)/(-5) and (4 - 3x)/(-5) < 2
Then. 12 > (4 - 3x)/5 and (4 - 3x)/5 > -2
60 > 4 - 3x and 4 - 3x > -10
i.e. 56 > -3x and -3x > -14
-56/3 < x and -x > -14/3
Thus, x > -56/3 and x < 14/3 Therefore, ***-56/3 < x < 14/3***

3) The solution of |2/(x – 4)| > 1 where x ≠ 4 is:

a) (2, 6)
b) (2, 4) ∪ (4,6)
c) (2, 4) ∪ (4,∞)
d) (-∞, 4) ∪ (4, 6)

Answer: (b)
Given, |2/(x – 4)| > 1
Then, 2/(x – 4) > 1 and 2/(x - 4) < -1 2 > x - 4 and 2 < 4 - x
i.e. 6 > x and x < 2 Thus, 6 > x > 2
Therefore, x ∈(2,6),butsincex≠4,x∈(2,4)∪(4,6)

4) If (|x| – 1)/(|x| – 2) ≥ 0, x ∈ R, and x ≠ ±2 then the interval of x is

a) (-∞, -2) ∪ [-1, 1]
b) [-1, 1] ∪ (2, ∞)
c) (-∞, -2) ∪ (2, ∞)
d) (-∞, -2) ∪ [-1, 1] ∪ (2,∞)

Answer: (d)
Given,(|x|-1)/(|x|-2)≥0 Then, |x|-1≥0 (sincex≠±2)
i.e. |x|≥1
Therefore x ∈[-1, 1]
Further,
In the case that, (|x| - 1)/(|x| - 2) > 0
either (i)|x| - 1 > 0 and |x| - 2 > 0 (both +ve)
i.e. |x| > 1 and |x| > 2 therefore x ∈(-∞, -2) ∪ (2, ∞)
or (ii) |x| - 1 < 0 and |x| - 2 < 0 (both -ve)
i.e. |x| < 1 and |x| < 2 therefore x ∈(-1, 1)
Thus, ultimately, x∈(-∞, -2) ∪ [-1, 1] ∪ (2, ∞)

5) Solve: f(x) = {(x – 1)(2 – x)}/(x – 3) ≥ 0, where x ≠ 3

a) (-∞, 1) ∪ (2, ∞)
b) (-∞, 1) ∪ (2, 3)
c) (-∞, 1] ∪ [2,3)
d) None of these

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Answer: (c)
Given, $\{(x-1)(2-x)\}/(x-3) \geq 0$
i.e. $\{(x-1)(x-2)\}/(x-3) \leq 0$

A: *{(x - 1)(x - 2)}/(x - 3) = 0*
i.e. $(x-1)(x-2)=0$ (since $x \neq 3$)
i.e. x = 1 OR x = 2

i.e. $x \in \{1\} \cup \{2\}$

Therefore $x \in \{1,2\}$

B: *{(x - 1)(x - 2)}/(x - 3) < 0*
(i) *{(x - 1)(x - 2)} < 0 AND x - 3 > 0*
i.e. ***(a)*** *x - 1 < 0 AND x - 2 > 0 AND x - 3 > 0*
x < 1 AND x > 2 AND x > 3

$x \in (-\infty,1) \cap x \in (2,\infty) \cap x \in (3,\infty)$ $x \in \{\} \cap x \in (3,\infty)$

Thus, $x \in \{\}$
or ***(b)*** *x - 1 > 0 AND x - 2 < 0 AND x - 3 > 0*
x > 1 AND x < 2 AND x > 3

$x \in (1,\infty) \cap x \in (-\infty,2) \cap x \in (3,\infty)$ $x \in (1,2) \cap x \in (3,\infty)$

Thus, $x \in \{\}$

from ***(a)*** *and* ***(b)***, ***(i):*** $x \in \{\}$

(ii) *{(x - 1)(x - 2)} > 0 AND x - 3 < 0*
i.e. ***(a)*** *x - 1 < 0 AND x - 2 < 0 AND x - 3 < 0*
x < 1 AND x < 2 AND x < 3

$x \in (-\infty,1) \cap x \in (-\infty,2) \cap x \in (-\infty,3)$ $x \in (-\infty,1) \cap x \in (-\infty,3)$

Thus, $x \in (-\infty,1)$
or ***(b)*** *x - 1 > 0 AND x - 2 > 0 AND x - 3 < 0*
x > 1 AND x > 2 AND x < 3

$x \in (1,\infty) \cap x \in (2,\infty) \cap x \in (-\infty,3)$ $x \in (2,\infty) \cap x \in (-\infty,3)$

Thus, $x \in (2,3)$

from ***(a)*** *and* ***(b)***, ***(i):*** $x \in (-\infty,1) \cup (2,3)$

from ***(i)*** *and* ***(ii)***, ***B:*** $x \in \{\} \cup (-\infty,1) \cup (2,3)$

Therefore, $x \in (-\infty,1) \cup (2,3)$

from ***A*** *and* ***B:*** $x \in \{1,2\} \cup (-\infty,1) \cup (2,3)$

Ultimately, $x \in (-\infty,1] \cup [2,3)$

Limits and Derivatives—

1) **Evaluate:**
a) 0
b) 1
c) -1
d) 2

Answer: (b)
Substituting for x, Therefore,

2) **Evaluate:**
a) x
b) -1
c) 0
d) 1

Answer: (c)
Using the logarithmic series: Thus,

3) **Evaluate:**
a) 0
b) inf
c) 1
d) -2

Answer: (a)
Upon rationalising the numerator we get, Thus,

4) **Evaluate:**
a) i
b) -i
c) 0
d) 1

Answer: (d)
We know that if such that exists, then, In this case, let f(x) = cos(x) - 1
And, g(x) = sin(x)
Then, Therefore,

5) **Evaluate:**
a) 1
b) 0
c) log x
d) -1

Answer: (a)
Dividing the numerator and denominator by x, we get Now, by using the formula: since

6) **What would be the derivative of |x| w.r.t. x?**
a) 1
b) ±1
c) 1 ÷ |x|
d) |x| ÷ x

Answer: (d)
Say $f(x) = |x|$ *Then,* $f^2(x) = x^2$
Differentiating both sides w.r.t. x $2f(x) * f'(x) = 2x$
$f'(x) = x \div f(x)$
$= x \div |x|$
$= \boldsymbol{|x| \div x}$

7) **If the derivative of e^x is e^x, what would be the derivative of x^e ?**

a) x^e
b) e^x
c) ex^{e-1}
d) ex

Answer: (c)
There is nothing much to the question really, it simply tries to see if the reader remembers that the Euler's number (e) is after all a constant, and thus x^e *would be differentiated similar to* x^a, *i.e.* $\boldsymbol{ex^{e-1}}$

Probability

1) **If two fair dice are rolled, the probability of getting sum greater than 3 will be:**
a) 11/12
b) 3/12
c) 5/6 d.

Answer: (c)
When two dice are rolled: Possible outcomes = 6 x 6 = 36
Outcomes that lead to a sum equal to four are: (1, 3), (2, 2), (3, 1)
Outcomes that lead to a sum less than four are: (1, 1), (1, 2), (2, 1)
Thus 6 of the total 36 outcomes lead to a sum less than or equal to 4, which implies that the rest of the possible outcomes (30 outcomes) must lead to a sum greater than 4. Thus the desired probability would be 30/36 or ***5/6.***

2) **A fish tank has 5 golden fish and 8 red fish. A fish is taken out at random, checked and dropped back into the tank. Another fish is taken out similarly and dropped back. What is the probability that the first fish was red and the second one was golden?**
a) 64/169
b) 40/169
c) 25/169
d) 65/169

Answer: (b)
Probability of drawing a red fish, P(r) = 8/13 Probability of drawing a golden fish, P(g) = 5/13
Thus, Probability of getting a red fish first AND a golden fish second,
$P(r \cap g) = P(r) * P(g|r)$
Since the fish was replaced after the first draw both draws were independent events, and thus P(g|r) = P(r)

3) **The sum of the probabilities of all the elementary events of an experiment is:**
a) 0.5
b) 1
c) 2
d) 0

Answer: (b)
The sum of the probabilities of all the elementary events of an experiment is ***always 1****, since at least one of the events is bound to be the result of the experiment.*

4) ***In Delhi traffic, the probability that a vehicle is a bus is 40%, while the likelihood of spotting a white vehicle is 60%. If only 20% of all school buses are painted white and 50% of all buses are school buses. What is the probability that a white vehicle on the road is a school bus?***
a) 1/6
b) 1/5
c) 9/15
d) 1/15

Answer: (d)
Given, P(a vehicle is a bus) or, P(B) = 40% P(a vehicle is white) or, P(W) = 60% P(a school bus is white) or, P(W|S) = 20% P(a bus is a school bus) or, P(S|B) = 50%

Then, according to Bayes' Theorem,
*P(a white vehicle is a school bus) or, P(S|W) = (P(W|S) * P(S)) ÷ P(W)*
*= (0.2 * P(S)) ÷ 0.6*

Now, P(S) = P(a vehicle is a school bus)
*= P(a vehicle is a bus) * P(a bus is a school bus)*
*= P(B) * P(S|B)*
*= 0.4 * 0.5*
= 0.2

Therefore,
*P(S|W) = (0.2 * 0.2) ÷ 0.6*
= 0.04/0.6
P(S|W) = 1/15

5) **Rita is brilliant at Maths and also a skilled artist. Which of the following is more probable?**

A. **Rita is a Maths professor.**
B. **Rita is a Maths professor who also teaches painting to kids.**

a) A is more probable
b) B is more probable
c) Both are equally likely
d) Both are equally unlikely

Answer: (a)
While both scenarios are quite likely, statement ***A is more probable*** *because it is less specific than B.*
Mathematically speaking, if M: "Rita is a Maths professor", and P: "Rita teaches painting to kids" both have non-zero probabilities then
P (A) = P (M), but
*P (B) = P (M) * P (P)*
i.e. P (B) ÷ P (A) = P (P) Clearly, ***P (B) < P (A)****, since P (P) < 1*

Relations and Functions—

1) **The cardinality of the empty set ø is zero. What is the cardinality of the set {ø, ø}?**
a) 0
b) 1
c) 2
d) None of the above

Answer: (b)
Given, {ø, ø} = {ø}
Since {ø} contains a single element, ø, its ***cardinality is 1.***

2) **If f(x) = (3x + 2) ÷ (x - 3), what is (fof)(x)?**
a) x
b) -x
c) f(x)
d) -f(x)

Answer: (a)
Given, f(x) = (3x + 2) ÷ (x - 3) i.e. = 3 + 11/(x-3)

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Now, (fof)(x) = f{f(x)}
thus, = f{ 3 + 11/(x-3) }
= 3 + 11/{ 3 + 11/(x - 3) - 3 }
= 3 + 11/{ 11/(x - 3) }
= 3 + x - 3
Therefore ***(fof)(x) = x***

3) The number of binary operations that can be defined on a set of containing two elements is:

a) 4
b) 8
c) 16
d) 64

Answer: (c)
For sets A and B, with |A| = m and |B| = n, the number of maps from A to B = $|B|^{|A|} = n^m$.
Thus for a set S with p members the number of maps from S x S to S would be $p^{p \times p}$. *That is the number of binary operations on S would be* $p^{p \times p}$.
Therefore, for a set with two members, the number of binary operations that can be defined on it would be 2^{2x2} ***or 16.***

4) A binary operation * defined on R – {1} such that a * b = ab+1 is:

a) Associative but not Commutative
b) Commutative but not Associative
c) Both Associative and Commutative
d) Neither Associative nor Commutative

Answer: (b)
*a * b = ab + 1 = ba + 1 = b * a, thus* **** is commutative***
*a * (b * c) = a(b * c) + 1 = a(bc + 1) + 1 = abc + a + 1, while*
*(a * b) * c = (a * b)c + 1 = (ab + 1)c + 1 = abc + c + 1, therefore* **** is not associative***

5) For a function g, g(x) = g^{-1}**(x). What is cos((gog)(x)) at x = π/2 ?**

a. -1
b. 1
c. 1/2
d. 0

Answer: (0)
Since g(x) = $g^{-1}(x)$*, (gog)(x) = x*
Thus, cos((gog)(x)) = cos(x) At x = π/2, cos(x) = 0
Therefore, ***cos((gog)(x)) at x = π/2 is 0***

Continuity and Differentiability

1) f(x) = |x| and g(x) = 1/sin(|x - 1|) are both continuous at x = 0, but are they also differentiable?

a) f(x) is but g(x) is not
b) g(x) is but f(x) is not
c) Both f(x) and g(x) are
d) Neither of f(x) and g(x)

Answer: (b)
Given, f(x) = |x| = x when x > 0 and -x when x < 0 Thus f´(x) = 1 when x > 0 and -1 when x < 0 i.e. the left hand limit of f´(x) as x approaches 0 = -1 and, the right hand limit of f´(x) as x approaches 0 = 1 Clearly these limits do not match, implying that f´(x) is discontinuous at x = 0 Further implying that ***f(x) = |x| is not differentiable at x = 0***

As for, g(x)		*= 1/sin(|x - 1|)*
	g´(x)	*= -(x - 1)cos(|x - 1|)/(|x - 1| * sin²(|x - 1|))*
Thus,	*g´(0)*	*= cos(|-1|)/(|-1| * sin²(|-1|))*
		= -cos(1)/sin²(1)
		*= -cos(1) * cosec²(1)*
		= a finite number

Since 0 < 1 < π, in which range cosec(x) is finite and so is cos(x).
Thus, g´(0) is continuous at 0, implying that ***g(x) is differentiable at 0.***

2) On the real number line, tan(x) is discontinuous at

a) Exactly one point
b) Exactly two points
c) Infinitely many points
d) No points

Answer: (c)
*tan(x) is surely discontinuous at x = 0, and x = π/2, but since it is a periodic function, i.e. tan(x ± π) = tan(x), tan(x) is discontinuous at all x such that x = n * π or x = (2n + 1)* π/2 (where n is an integer). Thus, since there are infinite integers, n can take infinite values, making tan(x) discontinuous at* ***infinitely many points*** *on the real number line.*

3) If the derivative of a^x **= ln(a) *** a^x**, what would be ln(f´(x)) for f(x) =** e^x**?**

a) a^x
b) e^x
c) 1
d) 0

Answer: (c)
Given f(x) = e^x
f´(x) = e^x
Thus, ln(f´(x)) = ln(e^x*)*
i.e. ***ln(f´(x)) = 1***

4) If for the function ø(x) = µx^2 **+7x - 4, ø´(x) = 97, what is the value of µ?**

a) 9
b) 4
c) -1
d) 2 Answer: (a)

For	*ø(x)*	*= µ*x^2 *+ 7x - 4*
	ø´(x)	*= 2µx + 7*
Thus,	*ø´(5)*	*= 10µ + 7*
i.e.	*97*	*= 10µ + 7*

10µ = 90
Therefore ***µ = 9***

5) If f(x) and g(x) are both differentiable at x = 0, which of the following is not implied?

a) f(x) + g(x) is differentiable at x = 0

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b) f(x) * g(x) is differentiable at x = 0
c) f(x) ÷ g(x) is differentiable at x = 0
d) None of the above

Answer: (c)
Say f(x) = sin(x)
And, g(x) = x
Both f(x) and g(x) are differentiable at x = 0
But, f(x) ÷ g(x) = sin(x)/x is not differentiable at x = 0. Thus, (c) is not implied.

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