Demonstration of the Multi-Server Queuing Model Using Big-M Method

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Abstract: ***This paper utilizes Linear Programming Problem (LPP) to determine optimal server count and expected number of customers in a multi-server queuing system, addressing uncertainties in arrival rate and service rate, server count. By employing the Big M method, the multi-server queuing model is formulated into LPP Model. Further, performance measures are also analysed.***

Keywords: Linear Programming Problem (LPP), Big M-method, Multi-Server queuing system, Performance measures

1. Introduction

Operations research (OR) is an analytical method of problem- solving and decision – making while linear programming is a subdivision of it. Queuing model is the concepts of queue lengths and waiting time. Some live examples are in waiting for line in ATM, Supermarkets etc., The utilization of the Big M method helps to solve the multi-server queuing model. Positioned within linear programming, this method tackles uncertainties linked with arrival rates, service rate, and server counts. By incorporating artificial variables, an initial variable solution is formulated. This process of foundation undergoes adjustments in the objective function, facilitated by the two of stages approach or by Big M method. Optimal selection of the value "M" is pivotal for the Big M method's success, though it is precise determination posses computational challenges .In contrast, the two state method bypasses this intricacy by maintaining discreteness between objective function across stages .This introductory insight paves, the way for a detailed exploration of how the Big M method enhances Performance measures and optimizes Cost in Multi- Server queuing scenarios.

2. Methodology

To study the servers involved in Multi-Server queuing model, Systematic approach is employed. which deals with the technique tools and procedures for analyzing arrival rates, service rates and server counts customer waiting times. In Linear Programming Problem, Big M Method or other optimization techniques might be used to determine optimal server numbers and also to determine waiting times. This section provides insights into the empirical and analytical basis of, revealing the challenges of the multi-server queuing model.

Linear Programming Problem:
Formulation of LPP is made by considering the required no of Server (s) and no of customers in the system (L_S) as variables. By letting the fixed service cost (C_S) for each server as 180, the average waiting cost of each customer(C_W) as 150,The Objective function of the linear Programming. Problem is mentioned as follows,

Min (z) = 180S +150L_S,

Subjects to the Constraints

S+ L_S ≥ 17, S≥2, L_S≥13, S, L_S > 0

Formulation of Linear programming
Min z = 180S + 150 L_S + $0x_1$+ $0x_2$+$0x_3$+0MA_1+0MA_2+0MA_3

Subjects to the Constraints
S+L_S-X_1+0X_2 + 0X_3 + A_1 + 0A_2+0A_3 = 17
0 S+L_S+0X_1-X_2 + 0X_3 + 0A_1 + A_2+0A_3 = 13
S+0L_S+0X_1+0X_2 - X_3 +0 A_1 + 0A_2+A_3 = 2
S, LS, X1, X2, X_3, A_1, A_2, A_3>0
All C_j- Z_j>0

Initial Basic solution is given by $AX = B$
$A = [11 - 100010 - 101000 - 1]; X = [x_1 x_2 x_3]; B = [17132]$

Adding the artificial variable,
$= [11 - 100100010 - 100101000 - 1001]$

The optimal solution of the LPP is as follow

Table 1: Iteration 1

C_j			180	150	0	0	0	M	M	M
B_V	CB_V	X_B	S	L_S	X_1	X_2	X_3	A_1	A_2	A_3
A_1	M	17	1	1	-1	0	0	1	0	0
A_2	M	13	0	(1)	0	-1	0	0	1	0
A_3	M	2	1	0	0	0	-1	0	0	1
Z_j			2M	2M	-M	-M	-M	M	M	M
C_j-Z_j			180-2M	150-2M	M	M	M	0	0	0

From iteration 1, we find that L_s enters the basis sand A_2 leaves the basis. It is observed that here 1 is a pivot element

Table 2: Iteration 2

C_j			180	150	0	0	0	M	M
B_V	CB_V	X_B	S	L_S	X_1	X_2	X_3	A_1	A_3
A_1	M	4	1	0	-1	1	0	1	0
L_S	150	13	0	1	0	-1	0	0	0
A_3	M	2	(1)	0	0	0	-1	0	1
Z_j			2M	150	-M	M-150	-M	M	M
C_j-Z_j			180-2M	0	M	-M+150	M	0	0

From iteration 2, we find that S enters the basis sand A_3 leaves the basis.it is observed that here 1 is a pivot element

Table 3: Iteration 3

C_j			180	150	0	0	0	M
B_V	CB_V	X_B	S	L_S	X_1	X_2	X_3	A_1
A_1	M	2	0	0	-1	(1)	1	1
L_S	150	13	0	1	0	-1	0	0
S	180	2	1	0	0	0	-1	1
Z_j			180	150	-M	M-150	M-180	M+180
C_j-Z_j			0	0	M	-M+150	-M+180	180

From iteration 3 we find that X_2 enters the basis sand A_1 leave the basis. it is observed that here 1 is a pivot element

Table 4: Iteration 4

C_j			180	150	0	0	0
B_V	CB_V	X_B	S	L_S	X_1	X_2	X_3
X_2	0	2	0	0	-1	1	1
L_S	150	15	0	1	-1	0	1
S	180	2	1	0	0	0	-1
Z_j			180	150	-150	0	-330
C_j-Z_j			0	0	150	0	330

$\therefore$ $all\ c_j - z_j \geq 0$ the optimum solution is obtained. Thus the solution,

L_S=15, S=2

Min (Z) =180S+150L_S=180(2) +150(15) =360+2250=2610

Min (Z) = 2610.

Sensitivity Analysis

The Multi-Server queuing model, is formulated into LPP model with the aid of Big M method. Here, it is assumed that 's' denoted the total no. of server required, and 'μ' denoted the service rate. Also the arrival rate is mentioned as 'λ'. Length of the system, Length of queue, waiting time of system, waiting time of queueare denoted as $L_s, L_q, W_s\ and\ W_q$ respectively. With the above considerations

(For the values of S =2 and μ =2>0 then expected arrival rate is 0<λ<Sμ (0<λ<4).

Now

By taking S=2 and μ=2 we calculate $L_s, L_q, W_s\ and\ W_q$ for the considering model

Table 5: Performance Measures with respect to $S = \mu = 2$

	S	λ	μ	L_S	L_q	W_S	W_q
1	2	1	2	0.5333	0.0333	0.5333	0.3333
2	2	2	2	1.3333	0.0333	0.6666	0.1666
3	2	2.5	2	2.0510	0.80104	0.8204	0.3204
4	2	3	2	3.4285	1.9285	1.1425	0.6428
5	2	3.1	2	3.8810	2.33101	1.2519	0.75196
6	2	3.2	2	4.4828	2.88288	1.40090	0.9009
7	2	3.3	2	5.2023	3.77055	1.6426	1.4258
8	2	3.4	2	6.012607	4.42607	1.80178	1.30178
9	2	3.5	2	7.4666	5.7166	2.21333	1.6333
10	2	3.6	2	9.4736	7.6736	2.6315	2.1315
11	2	3.7	2	12.8138	10.9638	3.4632	2.9632
12	2	(3.8)	2	19.4871	17.5871	5.1282	4.6282
13	2	3.9	2	39.4936	37.5436	10.1265	9.6265

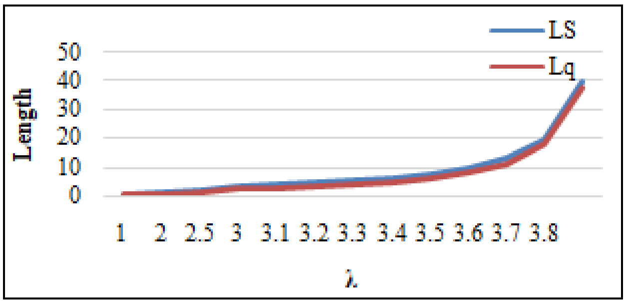

Figure 1: λ with respect to L_S,L_q

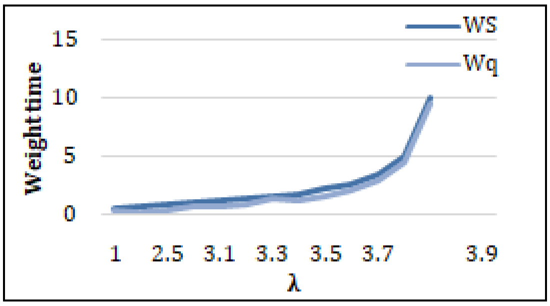

Figure 2: λ with respect to W_S, W_q

Form the above tabulation, we have assumed the total no .of server as S =2 and the arrival rate 'λ' varies from 1 to 3.9. (i. e) 0<λ<Sμ,whereas, the service rate 'μ' is assumed to be '2' throughout the model. The performance measures of the M/M/C queuing model are computed using formulae while as using Big M method. Form Iteration 4, it is obtained that L_S=15, S=2Thus from the above tabulation, for different 'λ' values, different L_s values are obtained but the nearest value for that L_S=15 in comparison with modi method is obtained when L_S=19.487for the corresponding λ =3.8. The above tabulations are represented graphically in Figure 1

From the above graph, the L_S values of the Table 5 are represented. It is observed that values of the Table 5 are represented. It is observed that L_S the value increase gradually from 0.5 to 39.49 as 'λ' value increases from 1 to 3.9. Similarly, 'L_q' values ranges from 0.033 to 37.5436 for λ varying from 1 to 3.9. On comparing L_S with L_q, it is observed that, Length of the system is more than that of the queue (L_q). From the above graph, the Ws values of the Table 5 are represented. It is observed that values of the Table 5 are represented. It is observed that W_S the value increase gradually from 0.5 to 10.12 as 'λ' value lies from 1 to 3.9. Similarly, 'W_q' values ranges from 0.033 to 9.62 for λ varying from 1 to 3.9. On comparing W_S with W_q, it is observed that, Length of the system is more than that of the queue (W_q).

Now by varying 'μ' value from '2' to 3, and 'λ' value from 1 to '5.9' i.e)0<λ<Sμ and by letting S=2, the performance measures are tabulated and obtained in Table 6

Table 6: Performance Measures with respect to $S = 2, \mu = 3$

	S	λ	μ	L_S	L_q	W_S	W_q
1	2	1	3	0.3466	0.01331	0.3466	0.0133
2	2	2	3	0.75	0.0833	0.375	0.0417
3	2	2.5	3	1.0169	0.1836	0.4068	0.0738
4	2	3	3	1.3333	0.3333	0.4444	0.1111
5	2	3.1	3	1.4183	0.38497	0.4575	0.1242
6	2	3.2	3	1.508	0.4413	0.47125	0.13795
7	2	3.3	3	1.577	0.477	0.47779	0.1446
8	2	3.4	3	1.6414	0.5080	0.4858	0.1495
9	2	3.5	3	1.7684	0.6018	0.5053	0.1719
10	2	3.6	3	1.875	0.675	0.5208	0.1875
11	2	3.7	3	1.9893	0.756	0.5376	0.2043
12	2	3.8	3	2.1150	0.84838	0.55659	0.2233
13	2	3.9	3	2.2510	0.95108	0.5772	0.2444
14	2	4	3	2.4000	1.0666	0.6	0.2666
15	2	4.5	3	3.2705	1.77047	0.7268	0.3934
16	2	5	3	5.4545	3.7878	1.0909	0.7575
18	2	5.1	3	6.1261	4.42612	1.2012	0.86787
19	2	5.2	3	6.96187	5.2286	1.3388	1.0055
20	2	5.3	3	9.0074	7.24076	1.6995	1.3662
21	2	5.4	3	9.4736	7.6736	1.7543	1.4210
22	2	5.5	3	11.482	9.6449	2.20869	1.7536
23	2	5.6	3	14.4827	12.6160	2.5862	2.2528
24	2	5.7	3	19.4827	17.5871	3.4188	3.0854
25	2	5.8	3	29.4915	27.5581	5.0847	4.7514

Again for the value S=2 and for μ=3>0 the expected arrival rate is consider to 0<λ<6The performance measures of the M/M/C queuing model are computed using formulated as .using Big M method. Form Iteration 4, it is obtained that L_S=15, S=2, μ=3

Thus from the above tabulation, for different 'λ' values, different values are obtained but the nearest value for that L_S=15 in comparison with modi method is obtained when L_S=19.487, λ=5.8 Since we know that from iteration 4, L_S=15, by using Big M method, on comparing with the table, the nearest value we obtain is L_S =19.48 which corresponds to λ=5.7. Thus from the above two tabulation, though there is a variation in the arrival and service rates, optimal L_S value obtained uniformly with that obtained from Big M method.

S=2, μ=3 0<λ<Sμ (0<λ<6)

In this table L_S lies in the range of 0.3 to 29.4

For λ=5.7 in L_S value is 19.4

The above tabulations are represented graphically in Figure 1&2

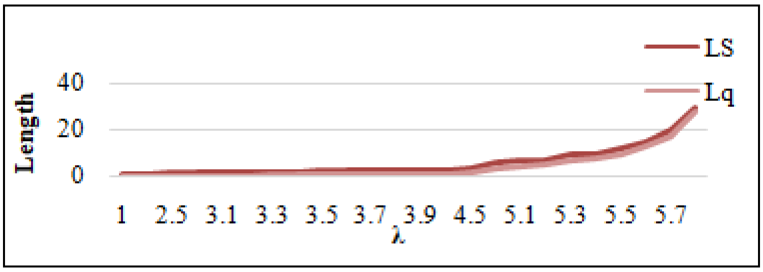

Figure 3: λ with respect to L_S, L_q

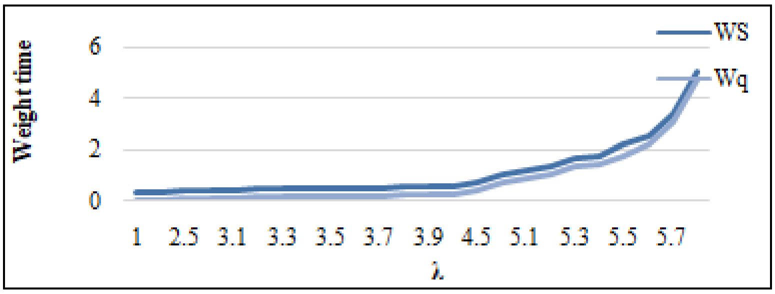

Figure 4: λ with respect to W_S, W_q

From the above graph, the L_S values of the Table 6 are represented. It is observed that values of the Table 6 are represented. It is observed that L_S the value differs from 0.3 to 29.49 as 'λ' value increases from 1 to 5.8. Similarly, 'L_q' values ranges from 0.013 to 27.55 for λ lies between1 to 5.8. On comparing L_S with L_q, it is observed that, Length of the system is more than that of the queue (L_q). From the above graph, the W_{S2} values of the Table 6 are represented. It is observed that values of the Table 6 are represented. It is observed that W_{S2} the value differs from 0.3 to 5.084 as 'λ' value increases from 1 to 5.8. Similarly, 'W_q' values ranges from 0.013 to 4.751 for λ lies between1 to 5.8. On comparing W_S with W_q, it is observed that, Length of the system is more than that of the queue (W_q)

3. Conclusion

This paper deals with the formulation of Multi-Server queuing model into LPP by using Big M method and also the various performance measures are evaluated. It is also observed that arrival rate (λ) and service rate (μ) can be calculated by letting either of them as independent. In Table 5&6, the arrival rates are found by taking a fixed independent service rate and compared with performance measures of the Multi-Server queuing model.

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Author Profile

Sharviya Sona S, graduated B.sc Mathematics from Nirmala College for women in 2022. Currently pursuing M.sc mathematics in Nirmala College for women, My area of research involves, formulation of multi-server queuing model as LPP by using Big M method